

Examining how children's use of math elicitation supports their own math learning

by

Kalina M. McNeil

Bachelor of Philosophy, University of Pittsburgh, 2023

Submitted to the Graduate Faculty of the
David C. Frederick Honors College in partial fulfillment
of the requirements for the degree of
Bachelor of Philosophy

University of Pittsburgh

2023

UNIVERSITY OF PITTSBURGH
DAVID C. FREDERICK HONORS COLLEGE

This thesis was presented

by

Kalina M. McNeil

It was defended on

April 13, 2023

and approved by

Darko Odic, University of British Columbia, Associate Professor, Department of Psychology

Diana Leyva, Associate Professor, Department of Psychology

James Morgante, Adjunct Professor, Department of Psychology

Thesis Advisor: Melissa Libertus, Professor, Department of Psychology

Copyright © by Kalina Maria McNeil

2023

Examining how children's use of math elicitations supports their own math learning

Kalina Maria McNeil, BPhil


University of Pittsburgh, 2023

Math abilities are related to outcomes including better health, greater chance of full-time employment, and higher income (Agarwal & Mazumder, 2013; Currie & Thomas, 2001; Reyna & Brainerd, 2007), with individual differences in math skills present as early as the beginning of kindergarten (Jordan et al., 2006). Previous work has found that parents' encouragement of math conversation supports children's math learning (Levine et al., 2010; Elliott et al., 2017). However, no work has looked at how children spontaneously discuss math, which may be an information-seeking technique used to shape their own learning. We examined children's math elicitations (questions or prompts used to encourage a response from the other person) during free play in both lab and home settings in parent-child dyads (RQ1: $n = 113$, 51% boys, M age = 3.9 years; RQ2: $n = 84$, 50% boys, M age = 3.9 years) in cases where parents were not previously discussing math, but children elicited math relevant to the conversation. RQ1: Children who used more of these spontaneous math elicitations had larger gains in math skills over 6 months, even controlling for a variety of covariates including children's overall elicitations and baseline math performance, $\beta=0.318$, $p=.004$, suggesting that children who take what their parents are discussing and make it math-related may promote their own math learning. Given the robust association between children's spontaneous math elicitations and their math performance, we were interested to explore predictors of children's use of spontaneous math elicitations. RQ2: We assessed children's spontaneous focusing on number (SFON) tendency (children's tendency to focus their attention on the number of objects in a set on their own, without any outside guidance or prompting) and

found that children's SFON did not significantly predict their later use of spontaneous math elicitations, $\beta=.088$, $p=.403$. This work stresses the importance of considering how children may directly shape the home environment and their opportunities to learn in addition to thinking about parents' influences on children's math learning. Future work should continue exploring factors that influence children's tendency to spontaneously seek out math information.

Table of Contents

Preface.....	x
1.0 Introduction:	1
1.1 The Current Study	4
2.0 Methods.....	6
2.1 Participants	6
2.2 Procedure	7
2.3 Measures.....	7
2.3.1 Spontaneous Math Elicitations	7
2.3.2 Children’s Math Performance	9
2.3.3 Children’s Spontaneous Focus on Number	9
2.3.4 Children’s Number Knowledge	10
2.3.5 Demographics	11
2.4 Data Analysis	11
3.0 Results	13
4.0 Discussion.....	15
4.1 SME as a Predictor of Children’s Math Performance.....	15
4.2 Associations between SFON and SME	17
4.3 The Role of the Parent and the Child	17
4.4 Limitations, Future Directions, and Conclusions	19
5.0 Figures.....	21
6.0 Tables	22

Appendix A SPIN Training Elicitation Coding Manual	25
Appendix A.1 Important Definitions	26
Appendix A.2 General Procedure	27
Appendix A.3 Precipitation Coding	30
Appendix A.4 Number and Spatial Identifiers 	41
Bibliography	51

List of Tables

Table 1 Descriptive Statistics for study variables for RQ1 (n=113).....	22
Table 2 Descriptive Statistics for study variables of RQ2 (n=84).....	23
Table 3 Regression analyses predicting gains in TEMA from lab visit 1 to lab visit 4	23
Table 4 Regression analyses predicting frequency of spontaneous math elicitations at lab visit 3	24
Table 5 SPIN Training Elicitation Important Definitions	26
Table 6 Number-Related Identifiers	41
Table 7 Spatial-Related Identifiers.....	42

List of Figures

Figure 1 Toys provided during the 10-minute free play at the lab visits.....	21
---	-----------

Preface

I would like to thank my thesis advisor, Dr. Melissa Libertus, as well as my graduate student mentor, Alex Silver, for all their help and guidance throughout my thesis process. I would also like to thank my committee members for their time and feedback, as well as everyone in the Kids' Thinking Lab who assisted with running participants, transcribing, and coding the SPIN Training Study.

1.0 Introduction:

Math is a skill used every day in both formal and informal settings: math is used while doing homework or math worksheets, as well as while cooking and telling time. Having stronger math skills is linked to better health outcomes, higher chances of employment, and greater income (Agarwal & Mazumder, 2013; Currie & Thomas, 2001; Reyna & Brainerd, 2007; Trusty et al., 2000). Children show variability in math skills as early as infancy as some infants are better at distinguishing between different quantities of objects than others, and these differences predict later math abilities when children are in preschool (Starr et al., 2013). Many explanations for these individual differences in young children's math skills have been explored, with some work focusing on the role of environmental influences, such as socioeconomic status (SES), engagement in math-related activities, and personal attitudes about math (Silver & Libertus, 2022).

Previous research focuses heavily on how parents can support their children's math learning. Parents' math engagement, such as talking about math and participating in activities that use math with their children, is typically found to be positively correlated with children's math performance on arithmetic, counting skills, comparing number magnitudes, comprehension of math language, and standardized math assessments (Benavides-Varela et al., 2016; Blevins-Knabe & Musun-Miller, 1996; Elliott & Bachman, 2018; King & Purpura, 2021; LeFevre et al., 2009; Silver & Libertus, 2022; Vasilyeva et al., 2018; Yildiz et al., 2018).

Math activities can be either formal or informal. Formal activities are done with the purpose to teach math concepts and can include math worksheets and flash cards, while informal activities are activities where math happens to appear but is not the direct purpose of the activity and can include playing card games and baking. The frequency of parents' use of both formal activities

and informal activities is related with young children's math skills (LeFevre et al., 2009; Ramani et al., 2015). Ramani et al. (2015) examined parents' use of math activities with 3- to 5-year-old children in low-income families and found that parents engaging in the direct teaching of math concepts as well as informal activities, such as board games, card games, and video games, were positively associated with children's performance on foundational number skills. Certain types of activities may be particularly beneficial for promoting children's math skills. Number games positively predict math knowledge of kindergarten children, while practicing number skills, playing number games, and engaging in indirect math activities positively predict fluency in math (LeFevre et al., 2009).

Another type of parental math engagement is discussing math concepts and numbers with their children regardless of the type of activity in which these conversations occur. Parents who use a higher frequency of number talk have preschoolers with greater math skills (Elliott et al., 2017; Levine et al., 2010). Additionally, Eason et al. (2021) observed 2- to 4-year-old children and their parents engaging in pretend play and found that parental number prompts (i.e., questions or statements inviting children to use number words) may be particularly beneficial in encouraging children's number talk. Increasing children's math talk may be advantageous in promoting their own math learning.

However, less research focuses on how children can support their own math learning in these interactions. We are interested in discovering why individual differences occur in children and what experiences lead to better math outcomes specifically by understanding how children might shape their own math learning. One study found that children's own skills are important to consider when predicting their math learning from parent-child interactions. Silver et al. (2021) found that children with better inhibitory control benefit more from instances of parent number

talk than children with lower inhibition. Specifically, in this study, parent-child dyads participated in a 10-minute free play with a set of toys in the lab. Following this play, children individually completed assessments and activities, including a standardized math test and a test measuring inhibitory control. They found that children's inhibitory control acts as a moderator between parents' frequency of number talk and children's math achievement such that only children with greater inhibitory control showed an association between parental number talk and children's math skills. This suggests that there are influences besides the parent that contribute to children's math skills and how much children benefit from learning opportunities. Thus, it is important to consider not just the math input children are receiving, but their own skills, when studying the benefits of parent-child math interactions.

In addition to inhibitory control, children's tendency to spontaneously focus on number (SFON) may also play a role in how much they benefit from math-related input. SFON captures children's tendency to focus their attention on the number of objects in a set on their own, without any outside guidance or prompting. Children's SFON in preschool predicts their counting skills and arithmetic skills two years later, as well as their standardized test scores seven years later (Edens & Potter, 2013; Hannula & Lehtinen, 2005; Hannula et al., 2007; Hannula et al., 2010; Hannula-Sormunen et al., 2015). Importantly, Braham et al. (2018) found that parents' increased use of number talk improved children's SFON. In their study, parents were randomly provided with one of two booklets that contained prompts to guide their pretend grocery shopping play with their children. In the Budget condition, parents were prompted to shop for a meal on a \$20-budget. In the Healthy Eating condition, parents were prompted to shop for a meal that included all food groups. As expected, parents in the Budget condition used significantly more number talk than parents in the Healthy Eating condition. Critically, children's SFON was assessed before and after

the guided play, and children in the Budget condition had a greater degree of SFON than children in the Healthy Eating condition at post-test. However, in this study they did not examine whether children's own baseline skills might play a role in the benefit they received from the increased number talk in the Budget condition. More generally, it remains unknown how children's SFON might relate to their ability to benefit from parental math input.

1.1 The Current Study

In the current study, we are interested in how children seek out math information while playing with their parents and whether they learn from it. Math information includes both numerical (i.e., functions such as “add”, “subtract”, etc.; numbers such as “one”, “two”, etc.; ordinals such as “first”, “second”, etc.) and spatial concepts (quantifiers such as “some”, “any”, “lots”, etc.; orientation such as “on top of”, “turn”, “below”, etc.; shapes such as “circle”, “oval”, etc.; sequences such as “next”, “after”, etc.). Seeking out math information may be one way in which children show that they spontaneously focus on math during play. We propose to study this topic by looking at times when children prompt their parent for mathematical information during play, specifically in instances when the parent was not previously talking about math. These instances may be especially beneficial as they show children are scaffolding their learning experiences by pulling math concepts into the conversation on their own, which may show an increased focus and interest in math learning. Furthermore, we will test whether children's inclination to seek out mathematical information during play with a parent is related to children's tendency to spontaneously focus on number in other contexts.

Specifically, we will first ask how children's information-seeking through math elicitations (i.e., questions or prompts used to encourage a response from the other person) relates to their own math learning (RQ1). Specifically, we will examine how the frequency of spontaneous math elicitations (SME) posed by children to their parents during a free play interaction is related to children's gains in math skills over the course of 6 months. Second, we will examine what factors predict children's use of SME, specifically by exploring whether children's SFON is related to their frequency of SME (RQ2). For RQ1, we hypothesize that children who use a greater frequency of SME will have greater gains in math skills over the 6 months. For RQ2, we hypothesize that children with higher SFON tendency will use a higher frequency of SME.

2.0 Methods

2.1 Participants

113 parent-child dyads (50% boys) participated in this longitudinal study. Children ranged in age from 3 years 9 months to 4 years 0 months at the first lab visit (M age = 3.9 year). Parents were 95% mothers and were highly educated (89% had at least a bachelor's degree). Families were recruited from a mid-sized city in the United States through a combination of flyers, online postings, and mailings. Parents signed an informed consent form to participate in the study and were compensated \$8 an hour for their time. Participants were told the study was designed to examine how parents support their children's early learning but were not told that the study was specifically focused on math to observe their natural behavior and interactions. Due to missing data, only a subset of the sample was used to answer RQ2 (84 parent-child dyads; 51% boys; M age = 3.9).

We ran a series of chi-squared tests comparing the full sample to those included in the subsample for RQ2, and found that those dyads who were missing data and excluded from the RQ2 subset did not significantly differ from the full sample in children's gender ($p = .79$), children's age at lab visit 1 ($p = .79$), parent education ($p = .07$), children's gains in math over the 6-month period ($p = .29$), children's baseline math score ($p = .25$), training condition ($p = .73$), children's SME at lab visit 1 and home visit 1 combined ($p = .84$), children's overall elicitations at lab visit 1 and home visit 1 combined ($p = .67$), children's SFON at lab visit 2 ($p = .25$), children's overall talk during the SFON task at lab visit 2 ($p = .83$), children's SME at lab visit 3 ($p = .41$), children's overall elicitations at lab visit 3 ($p = .69$), or children's number knowledge at lab visit 2 ($p = .42$).

2.2 Procedure

The SPIN Training Study was a longitudinal study that observed children and their parents over six months beginning right before the child's fourth birthday. Parents and children participated in four lab visits (every two months), with virtual home visits held every two weeks between the first and second lab visit, and between the second and third lab visit. In total, dyads participated in four lab visits and six virtual home visits. During all lab visits and virtual home visits, parents and children were observed in 10-minute naturalistic free play sessions. Additionally, during each lab visit children completed measures of their math performance including a standardized math assessment and a measure of SFON. Parents also completed surveys and math assessments during each lab visit that are irrelevant for the purposes of the present study. The current study uses data from the free play from the first lab and first home visit as well as the third lab visit, SFON from the second lab visit, and children's math performance at the first and fourth lab visits.

2.3 Measures

2.3.1 Spontaneous Math Elicitations

During the lab visits, dyads were provided with a standardized set of toys which could possibly elicit math talk. These toys included a cash register, coloring supplies, puppets, a balancing game, pretend food and dishes, two books, and some wooden vehicles (Figure 1). During the virtual home visits, dyads were able to play with any toys they wanted, and in both settings,

they were told to play how they normally would. All these interactions were videotaped, transcribed, and then tallied for instances of children's math elicitations (including both number and spatial elicitations) and non-math elicitations. Examples of math elicitations that either the parent or child could ask are "How many cars are there?" (number) or "Where does this go?" (spatial), while an example of a non-math elicitation is "What color is that car?". We double coded 32% of the interactions at lab visit one ($r = .96, p < .001$), 28% of the interactions at home visit one ($r = .88, p < .001$), and 24% of the interactions at lab visit 3 ($r = .73, p < .001$) to ensure reliability.

Beyond the overall frequency of math and non-math elicitations, we coded the frequency of SME that children used, as well as their relevance to the ongoing conversation. For example, if the parent said, "I am going to buy a cheeseburger and broccoli", a relevant spontaneous math elicitation the child could use would be "How many dollars do you have?" As shown in this example, the parent did not use math talk in their prior statement, but the child elicited a conversation about math in a relevant manner. In other words, the child continued the ongoing conversation by discussing the same topic and sought out math information. However, if the parent said, "I am going to buy a cheeseburger and broccoli", and the child responded with "Do you want fries with that?", the child is still eliciting in a relevant manner, but not including any math content. Additionally, if the parent said, "I am going to buy two cheeseburgers" and the child responded with "How many dollars do you have?", the child is eliciting in relevant manner about math information, but the elicitation is not spontaneous as the parent mentioned math in their previous statement. It is important to note that only the last utterance prior to the elicitation is considered to establish relevancy and spontaneity. Therefore, it is possible that the parent discussed math a few

utterances before the elicitation, but the elicitation was still counted as being spontaneously-math focused if the utterance immediately preceding it is not related to math.

2.3.2 Children's Math Performance

Children's math ability was tested using the third edition of the Test of Early Mathematics Ability (TEMA-3) (Ginsburg & Baroody, 2003) at all lab visits, allowing us to track children's gains in math skills throughout the study. The TEMA-3 tests children's knowledge on numbering skills, number comparison, numeral literacy, mastery of number facts, calculation skills, and understanding of concepts. It is normed for children between 3 and 8 years of age.

2.3.3 Children's Spontaneous Focus on Number

Children's tendency to spontaneously focus on number (SFON) was measured using two imitation tasks, the Dinosaur Task and the Puppet Task. Each SFON task was video-recorded and coded, and children's total points for all tasks were summed to create a composite SFON score. SFON was not tested at all lab visits; children completed the Dinosaur Task and the Puppet Task at the second lab visit.

In the Dinosaur Task (Hannula & Lehtinen, 2005), researchers stamped a specific number of spikes onto a printed picture of a dinosaur and children were instructed to make their dinosaur "look like" the researchers without specifying what "looking alike" meant. Researchers stamped three dinosaurs with different numbers of stamps (trial 1: two blue triangles, trial 2: four red diamonds, trial 3: two green squares and three green rhombi). Children received a score of 1 if they stamped the correct quantity of spikes in each trial. For trial 3, children were counted as

spontaneously focusing on number if they correctly stamped one of the subsets (i.e., two green squares and two green rhombi) or the total number in the set (i.e., three green squares and two green rhombi), allowing for a total score up to 3 for the three trials.

For the Puppet Task (Hannula & Lehtinen, 2005), children observed the researcher feeding marbles to a puppet and were instructed to “do as [the researcher] did”. The researcher fed the puppet three times with a different number of marbles each time (trial 1: two marbles, trial 2: three marbles, trial 3: one marble). Children received a score of 1 if they fed the same quantity of marbles per trial, resulting for a total score of up to 3.

Furthermore, for all tasks children were also coded as focusing on number if they verbally mentioned number or quantity during a trial, even if they did not reproduce the exact number of actions. SFON talk was defined as any talk about numerical information that the child used before, during, or after the trial that was related to the trial (i.e., “I put three spikes on my dinosaur” even if the researcher only put two spikes.). To account for differences in children’s overall frequency of talking, children’s general talk, coded as their word count during each trial, was also coded and will be controlled for in the analyses.

2.3.4 Children’s Number Knowledge

Children’s variability in SFON tendency may be influenced by how much they know about numbers. To account for children having variability in their number knowledge, we controlled for it in RQ2. Children’s number knowledge was tested using the Give-a-Number (GAN) task (Wynn, 1992; Ansari, Donlan, Thomas, Ewing, Peen, and Karmiloff-Smith, 2003; Sarnecka and Carey, 2008) at lab visit 2. Researchers placed 14 toy fish on a table and showed the child an animal puppet. The researcher told the child the animal is hungry and asked the child to help “feed” the

puppet. Children were instructed to create sets of fish for the animal to “eat”: “Can you give [animal] [number] fish?” Across twelve trials, children were asked to create sets of one through six fish, where they were asked for each number twice in a pseudo-randomized order. After creating a set, the experimenter asked, “Is that [number]?” If the child said yes, the puppet “ate” the fish. If the child indicated this was not the number, they were told: “Well, [animal] wants [number]. Can you give [number]?” and were allowed to update their response.

2.3.5 Demographics

Parents reported the age and gender (0 = female, 1 = male) of their children at the first lab visit, which will be included as controls in our analyses.

2.4 Data Analysis

As part of the training study, families were randomly assigned to one of five training conditions after the second lab visit (see Ribner et al., 2022, for more information). The training conditions were composed of four training games (a parent-child number board game, a parent-child shape board game, a parent-only non-symbolic number training game, and a parent-only trivia game) and a business-as-usual control group. Ribner et al. (2022) found that children in the parent-only non-symbolic number training group had significantly lower math scores than children in the other training conditions at lab visit 4. As a result, we ran all analyses to address our research questions twice: first we ran the analyses with all the participants and then again while excluding the children in the parent-only non-symbolic number training group (RQ1: $n = 90$; RQ2: $n = 68$).

To test whether children who use more SME had larger gains in their TEMA scores (RQ1), we ran a regression analysis to predict children's gains in TEMA scores over the six months of the study (lab visit 1 to lab visit 4) from children's use of SME following parent non-math talk while controlling for children's use of overall elicitations (combined from lab visit 1 and home visit 1), baseline math performance at the first lab visit, training condition, age at lab visit 1, and gender. Controlling for these variables allowed us to rule out the possibility that children are using more SME because they elicit more in general, or that children are not showing greater gains because they are performing better in math already. We first examined whether children's frequency of SME was correlated at the first lab and home visit. We found that children's use of SME at lab visit 1 was not significantly correlated with their use of SME at home visit 1 ($r = .03$, $p = .69$); however, we combined the variables from the sessions for our analyses to give us more variability.

To test if there is an association between children's SFON and children's SME (RQ2), we ran a regression analysis predicting SME from SFON when controlling for overall talk during the SFON tasks, overall number of elicitations, training condition, number knowledge, and gender. As we did not collect SFON at lab visit one, we used SFON scores from lab visit two. Additionally, as we are predicting SME from SFON, we used SME from a later timepoint (lab visit three) to answer the second research question.

Another possible concern is that children are engaging in more SME if their parents engage in greater SME, possibly modeling this behavior. To account for this, we ran a correlation between children's frequency of SME and parents' frequency of SME. These variables were not significantly correlated at any time point (Lab Visit 1: $r = .15$, $p = .109$; Home Visit 1: $r = .18$, $p = .057$; Lab Visit 3: $r = .03$, $p = .770$), so we did not control for them in our analyses for RQ1 or RQ2.

3.0 Results

Descriptive statistics for RQ1 are shown in Table 1. Children showed some variability in their usage of overall elicitations, as well as SME at home visit 1 and lab visit 1. Descriptive statistics for RQ2 are shown in Table 2. Children showed variability in their SFON tendency and their use of SME at lab visit 3.

To address RQ1, we tested whether children who use more SME had larger gains in their TEMA scores across the longitudinal study. The regression predicting children's gains in TEMA scores over the six months of the study from children's use of SME following parent non-math talk while controlling for children's use of overall elicitations (combined from lab visit 1 and home visit 1), baseline math performance at the first lab visit, training condition, age, and gender revealed a significant positive association between children's SME and gains in math skills ($\beta = .318, p = .004$; Table 3). Children who seek out more math information from their parents showed greater improvement in their math skills over time than children who use fewer SME.

To address RQ2, we tested if there is an association between children's SFON and children's SME. The regression analysis predicting SME from SFON controlling for overall talk during the SFON tasks, overall number of elicitations, training condition, number knowledge, and gender did not yield a significant association between children's SFON at lab visit two and SME at lab visit 3 ($\beta = .088, p = .403$; Table 4).

Since Ribner et al. (2022) found that children in the parent-only non-symbolic number training group had significantly lower TEMA scores than children in the other training conditions, we first ran the analyses described above with all the participants and then ran them again while excluding the children in the parent-only non-symbolic number training group, controlling for

training conditions both times. Overall, the pattern of findings remained the same. We still found a significant positive association between children's SME and gains in math skills ($\beta = .340, p = .006$). In addition, there was again no significant association between children's SFON at lab visit two and SME at lab visit 3 when controlling for overall SFON talk, overall number of utterances, number knowledge, training condition, and gender ($\beta = .054, p = .655$).

4.0 Discussion

Math is an important skill that children and adults use every day, and early math abilities are predictive of long-term outcomes (Duncan et al., 2007). Previous work emphasizes how parental math talk and engagement in math activities benefits children's math learning, as well as the role of environmental factors such as SES and math attitudes (Silver & Libertus, 2022). More recent work has begun looking at the role the child plays in promoting their own math learning and found that children's own characteristics do play a role in their math learning (Silver et al., 2021). Here, we found that children's gains in math are predicted by their tendency to engage in spontaneous math elicitations, i.e., how frequently children use math questions and prompts while playing with their parent when the parent was not previously talking about math concepts. In this way, children who seem to seek out opportunities to discuss math-related topics may be promoting their own math learning. While spontaneous math elicitations seem to be a critical way for engaging in math learning, it remains unclear what factors predict children's likelihood of using SME, as we did not find SFON to be a predictor of the variability seen in children's usage of SME.

4.1 SME as a Predictor of Children's Math Performance

As hypothesized, we found that children's tendency to spontaneously elicit math content when their parents were not previously discussing math significantly predicted their gains in math ability six months later, even when controlling for other predictors. Children who used a greater frequency of SME had larger gains in math skills over six months. Past studies have shown that

parents' use of math talk and their questions about math support their children's learning (Levine et al., 2010; Elliott et al., 2017), and that children's own personal characteristics play a role in the development of their math learning (Silver et al., 2021). However, no work has looked at children's use of math elicitation in general and the relation to their math performance. Importantly, here we focused on children's spontaneous use of math elicitation that were relevant to the on-going conversation as it reflects children's ability to spontaneously seek out math-related information in the context of various play activities.

To ensure the robustness of these results, we included a number of important covariates and follow-up analyses. First, as this study is part of a larger, longitudinal intervention study, we ran the same analyses while excluding children whose parents completed a non-symbolic number training because these children had significantly lower math scores at lab visit 4 compared to children in the business-as-usual control condition (Ribner et al., 2022). However, even with the exclusion of these dyads, we found that children's SME frequency continued to predict their gains in math skills six months later.

In addition, as past research has shown that parents' own use of math talk and questions influences their children's math abilities, we wanted to ensure that children were not just copying their parents' use of spontaneous math elicitation. Therefore, we ran a correlation analysis to evaluate if children's SME and parents' SME were correlated. We found that children's SME was not significantly correlated with parents' SME at any time point. This suggests that children are not just imitating their parents' use of spontaneous math elicitation but are independently seeking out math information in their environment and bringing math into the conversation in a relevant manner. What remains to be seen is what accounts for the variability in children's use of SME.

4.2 Associations between SFON and SME

We had originally hypothesized that children with stronger spontaneous focusing on number would be more likely to use SME during play. In contrast to our hypotheses, we did not find any significant relations between children's SFON and their SME in any of our analyses. As we found SME is predictive of children's math skills, and previous work has found that SFON is related to future math abilities (Edens & Potter, 2013; Hannula & Lehtinen, 2005; Hannula et al., 2007; Hannula et al., 2010; Hannula-Sormunen et al., 2015), this suggests that usage of SME may not explain why children with greater SFON tendency tend to perform better in math. One possible explanation could be that children with greater SFON tendencies do not necessarily express their SFON tendencies verbally or may not do so by eliciting math-related information from others. Another possibility may be a lack of power in our study, since only a subset of dyads have been coded for SME. Finally, SFON was only measured via two imitation tasks which may have resulted in limited variability in children's SFON scores.

4.3 The Role of the Parent and the Child

While specific factors leading to children's SME are still unknown, parents and children can continue to shape their environment to assist math learning and potentially lead children to engage in more SME. The materials that parents have in their homes for children to play with may be influential. Some toys, such as a cash register, may elicit more conversation about math, while other toys, such as dolls may elicit other types of learning (e.g., socio-emotional skills). In our study, we included a variety of toys, some of which could elicit math talk, but did not have to. For

example, we had a cash register where children and parents could pretend to play shopping at a grocery store. Additionally, we had several sets of colorful items, such as silverware, cars, and eggs. Dyads could either count the sets or focus more on the different colors and uses of the objects. It may be beneficial for parents to have similar toys in their homes to allow for children to elicit math content if they desire, but without forcing the conversation in that direction.

Parents engaging in guided play may also be beneficial to the child. A study by Braham et al. (2018) found that differences in parents' guided play prompts influenced their children's SFON tendency. While we did not find that SFON was predictive of children's SME, specific guided play prompts may be beneficial to promoting children's SME. An example of this could be providing children with a set of blocks with a variety of shapes and sizes. Parents should ask questions that elicit the child to use math elicitation and math talk without using math themselves. For example, the parent could ask "What do you think we should do?", "What do you want to build?", "Which blocks should we use?", "What can I do to help?". Guided questions like these allow for the child to choose how they would like to play, and which blocks they would like to use, while also eliciting how the child wants to play with the blocks.

Additionally, parents can allow for children to shape their own environment. One way this can occur is giving children the ability to lead the conversation. If a child is interested in a topic, even one seemingly unrelated to math, parents should allow their children to discuss the topic and engage with their children. While this may not always lead to SME, it is possible that there is math hidden in the topic which children may end up focusing on. When this occurs, parents should be mindful and respond in a relevant manner.

4.4 Limitations, Future Directions, and Conclusions

Although these results are exciting, a few limitations are worth discussing. Our sample comprised primarily White, educated mothers and their children living in the Pittsburgh area. As such, our findings may not be generalizable to broader populations. Additionally, past work has found significant associations between socioeconomic status and children's math performance (Jordan & Levine, 2009), and therefore it is imperative to replicate our findings with more diverse samples.

Furthermore, although data for this study were derived from a larger longitudinal intervention study, we cannot draw causal conclusions because of the correlational nature of our results. Moreover, we only considered the last utterance said before the child's elicitation to determine if the child's elicitation spontaneously related to math or not. However, it is possible that we missed larger contexts of math conversations where parents may have been discussing math in the conversation, just not in the most recent utterance. Additionally, certain play activities may have been more routinized and even though we may not have observed parents' math conversations, children may remember those from past play times. Future studies may want to consider coding more than the single utterance before the elicitation to ensure the elicitation is truly spontaneous or only consider play contexts that are truly novel to parents and children.

It is important to note that we did not find a correlation between children's SME at lab and home visit 1 suggesting that children seek mathematical information at different rates when they are playing with their parents in the lab and home settings. A possibility for this may be due to the toys provided in the lab setting, as they may not be generalizable to what children are playing with in the home setting. Future work should systematically explore how materials and environmental context relate to children's SME.

Another limitation to keep in mind is the difference between warm up versus actual play. Previous research shows that the first three minutes of play can be considered as warm up for the parent and child, while the rest of the interaction is actual play (Shanley & Niec, 2011). Future studies should add an additional three minutes to the beginning of the free-play interaction that is not coded or counted towards the analyses to account for this. It would be interesting to explore in future work how SME may differ in the warmup versus actual play periods of an interaction.

Excitingly, our results open a myriad of possibilities for future research directions. First, future work should continue to look for factors that predict children's SME. Since we found that children's SME is associated with their math learning, we should try to find ways to increase their SME. However, this is difficult on its own as prompting children to ask about math removes the spontaneity of their elicitations. Therefore, it is vital to discover what factors increase children's use of SME, and ways we may be able to tailor the environment to promote SME.

Another future direction includes further examining the conversation in which the child engages in SME. Is it possible that certain types of conversations invite children to use more SME? Additionally, the quality of the parents' responses may be another important aspect to consider, as perhaps children seek more information when parents respond enthusiastically and appropriately to their children's SME.

Overall, our findings suggest that it is important to take children's own characteristics into account when examining how they learn math from conversations with their parents. Our results demonstrate that children's own spontaneous elicitations of math-related information while playing with their parents predicts gains in their math abilities. Understanding the role of the parent, child, and the way they interact with each other is critical in supporting children's early math skills.

5.0 Figures



Figure 1 Toys provided during the 10-minute free play at the lab visits

6.0 Tables

Table 1 Descriptive Statistics for study variables for RQ1 (n=113)

Variable	<i>M</i>	<i>SD</i>	Min	Max
Children's SME at lab visit 1	1.4	1.5	0	5
Children's SME at home visit 1	1.5	1.7	0	7
Children's SME at lab visit 1 and home visit 1 combined	2.8	2.3	0	10
Children's Overall Elicitations	28.4	13.4	5	71
TEMA at lab visit 1	11.0	5.5	0	27
TEMA gains from lab visit 1 to lab visit 4	5.8	4.4	-2	17

Table 2 Descriptive Statistics for study variables of RQ2 (n=84)

Variable	<i>M</i>	<i>SD</i>	Min	Max
Children's SME at lab visit 3	2.6	2.1	0	10
Children's Elicitations at Lab Visit 3	16.3	9.3	2	56
Children's SFON at Lab Visit 2	4.5	1.6	0	6

Table 3 Regression analyses predicting gains in TEMA from lab visit 1 to lab visit 4

Variable	B (S.E.)
SME Lab and Home Visit 1	0.61** (0.21)
Overall Elicitations Lab and Home Visit 1	-0.07 (0.04)
TEMA Lab Visit 1	0.01 (0.07)
Training Condition	-0.56 (0.29)
Child Age Lab Visit 1	4.25 (6.60)
Child is Male	1.04 (0.81)
Constant	-9.43 (25.91)

$F(6,106) = 2.56, p = .023; R^2 = 0.13$

** $p < .001$

Table 4 Regression analyses predicting frequency of spontaneous math elicitations at lab visit 3

Variable	B (S.E.)
Overall Elicitations Lab Visit 3	0.09 (0.03)
SFON Lab Visit 2	0.12 (0.14)
Word Count During SFON Task Lab Visit 2	0.002 (0.003)
Training Condition	0.18 (0.16)
Child is Male	.47 (0.44)
Number Knowledge Lab Visit 2	-0.63 (1.26)
Constant	.23 (1.40)

$F(6,77) = 3.21, p = .007; R^2 = 0.20$

Appendix A SPIN Training Elicitation Coding Manual

Updated 05/14/2021 by AMS

Overview: parents and children engaged in a 10-minute free play interaction in the lab with a standardized set of toys or a 10-minute free play interaction at home with their own toys. All interactions have been transcribed, and highlighted for instances of number and spatial talk, as well as number and spatial function words. This coding pass will investigate what was happening in the conversations directly prior to and directly after the elicitation.

Aim: to characterize and code the precipitations and responses to all elicitations between parent-child dyads during the free play interaction.

Appendix A.1 Important Definitions

Table 5 SPIN Training Elicitation Important Definitions

Term	Definition
Elicitation	<p>Any question or statement that is intended to evoke a response from the other person. Most questions are elicitations. (<i>Exception:</i> When reading a passive question from a storybook and the other person does not actively respond, that question is not an elicitation. Questions that are not intended to evoke a response are not elicitations).</p> <p>Additionally, commands, prompts or suggestions may also be elicitations if they are used in a way that is intended to evoke a response/action from the other person.</p> <p>Number/Spatial elicitations specifically aim to evoke a response that requires using math concepts (i.e., number/spatial words or gestures).</p>
Gesture	<p>Here, we'll define a gesture as any non-verbal form of communication that is used to either draw attention to or express an idea or concept.</p> <p>Examples include pointing, nudging, nodding, shrugging, shaking head, rolling eyes, holding up fingers, handing an object, etc.</p>
Precipitation	<p>The utterance or action occurring directly before the elicitation.</p> <p>Precipitations can be verbal or non-verbal.</p>

	Note: We will be coding the precipitation for both the parent and the child (i.e., the last utterance and/or action each of them produced before the elicitation).
Relevance	<p>Whether the utterance continues in the same context as whatever preceded it. Relevance can be either to the activity (continuing to play with the same toy or talk about the same toy/activity) or the concept (continuing to discuss the same topic or use the same skill).</p> <p>Note: We will be coding for the relevance of the elicitation to the precipitation, and for the relevance of the response to the elicitation.</p>

Appendix A.2 General Procedure

1. Open the copy of the coding spreadsheet that is saved to your computer. You will do all of your coding on your computer and save it locally. Only once you have finished coding will you copy and paste your coding into the master file on the server (see step 10).
2. Identify the subject and visit number that you are assigned to code on the google doc SPIN Training Coding Spreadsheet (Sheet name “New Elicitations Coding”)
 - a. Create a new sheet in your excel file and rename it with “S” followed by the subject number (ex: subject 1’s sheet should be labeled “S1”)
 - b. Open the “Template” sheet and copy at least the first and second row
 - c. Paste these rows into the first and second rows of your new sheet
3. Open the compiled transcript for that free play interaction

- a. For lab visits: (Libertus > Studies > SPIN Training Study > Free Play Coding > Lab Free Play > Word Counts and Coding > Compiled Transcripts)
 - b. For home visits: (Libertus > Studies > SPIN Training Study > Free Play Coding > Home Free Play > Word Counts and Coding > Compiled Transcripts)
4. Open the video file for that free play interaction
 - a. For lab visits: (Libertus > Studies > SPIN Training Study > Free Play Coding > Lab Free Play > Videos > Free Play Videos)
 - b. For home visits: (Libertus > Studies > SPIN Training Study > Free Play Coding > Home Free Play > Videos)
 5. Begin by watching the video and following along in the compiled transcript
 6. Identify the first elicitation and copy the transcription of it into the “Elicitation Transcribed” column
 - a. Enter the speaker of the elicitation in the “Speaker of Elicitation” column
 - i. If the speaker was the parent, enter “P”
 - ii. If the speaker was the child, enter “C”
 - b. Identify whether the elicitation was a question or not in the “Elicitation Was Question” column
 - i. If it was a question, enter “1” here
 - ii. If it was not a question, enter “0” here
 - c. Identify whether the elicitation was math-related in the “Elicitation Math” column

- i. If it was math-related, enter “1” here
 - ii. If it was not math-related, enter “0” here
 - d. Identify whether the elicitation was number-related in the “Elicitation Number” column
 - i. If it was a number-related elicitation, enter “1” here
 - ii. If it was not number-related, enter “0” here
 - e. Identify whether the elicitation was spatial-related in the “Elicitation Spatial” column
 - i. If it was a spatial-related elicitation, enter “1” here
 - ii. If it was not spatial-related, enter “0” here
 - iii. Note, if it was already coded as number-related in the step above, it cannot be coded as spatial-related here, so enter “0”
 - f. Identify whether the speaker gestured with/as they were speaking the elicitation in the “Gesture with Elicitation” column
 - i. If the speaker gestured, enter “1” here
 - ii. If they did not gesture, enter “0” here
7. Using the coding scheme detailed below, code the precipitation and response to this elicitation.
8. Continue to the next elicitation in the interaction, and repeat steps 6-8.
9. When you have coded all elicitations in the entire interaction, save the file locally.
10. When you have finished coding a subject, connect to the server, open the master coding spreadsheet, make a new tab for the subject you have coded, and copy and paste your coding into the master file on the server (then save!).

- a. Find the master coding spreadsheet for Lab V1 on the lab server here
(Libertus > Studies > SPIN Training Study > Data > Excel Files > Elicitations > Lab V1 > LabElicitationConversation_V1_Updated.xlsx).
 - b. Find the master coding spreadsheet for Home V1 on the lab server here
(Libertus > Studies > SPIN Training Study > Data > Excel Files > Elicitations > Home V1 > HomeElicitationConversation_V1_Updated.xlsx).
11. **Reliability procedure:** 20% of the interactions will be double coded. If you are the second person coding the interaction follow all of the same steps, EXCEPT name the new sheet in Step 10 with “S” followed by the subject number followed by “_R” (ex: subject 1’s reliability sheet should be labeled “S1_R”)
12. Continue to the next subject and repeat this process for the next interaction.

Appendix A.3 Precipitation Coding

For all elicitations, we will be coding the precipitation and response from both the parent and the child.

Begin by coding the Precipitation, walking through each column. You’ll start by coding the Parent’s precipitation. Then, you’ll code the child’s precipitation.

Column H

Precipitation Column Name: Parent Last Speech Transcribed

Description: The last utterance that the parent spoke prior to the elicitation.

If the parent gestured instead of verbally speaking, describe the gesture in brackets.

Examples of gestures:

P: [pointed at car]

P: [handed child crayons]

How to Code:

- Copy the transcribed speech from the compiled transcription and paste the utterance
- If the parent gestured instead of verbally speaking, describe the gesture in brackets
- If the elicitation was the very first thing said in the interaction, enter NA

Column I

Precipitation Column Name: Parent Last to Speak

Description: Identify whether the parent was the most recent person to speak immediately prior to the elicitation.

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction, enter NA

Column J

Precipitation Column Name: Parent Gesturing Prior

Description: Identify whether the parent was gesturing immediately prior to the elicitation or if they gestured during their last verbal utterance (transcribed in Column H).

How to Code:

- If yes, enter 1

- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the parent, enter NA

Column K

Precipitation Column Name: Parent Prior Speech Number

Description: Identify whether the parent’s last utterance (transcribed in Column H) used an instance of number talk. (See last page of manual for list of number words)

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the parent, enter NA

Column L

Precipitation Column Name: Parent Prior Speech Spatial

Description: Identify whether the parent’s last utterance (transcribed in Column H) used an instance of spatial talk. (See last page of manual for list of spatial words)

How to Code:

- If yes, enter 1
- If no, enter 0
- Note, if it was already coded as number-related in the step above (aka if Column K has a “1”), it cannot be coded as spatial-related here, so enter “0”
- If the elicitation was the very first thing said in the interaction or by the parent, enter NA

Column M

Precipitation Column Name: Elicitation Relevant Parent

Description: Identify whether the elicitation (transcribed in Column A) was overall relevant to the last utterance/action from the parent (transcribed in Column H). Ask yourself, did the elicitation continue in the same context as the parent's precipitation that preceded it?

Relevance can be either to the activity (continuing to play with the same toy or talk about the same toy/activity) or the concept (continuing to discuss the same topic or teach the same skill).

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the parent, enter NA

****If the "Elicitation Relevant Parent" is a 0, the next three columns will automatically be coded as NA****

Column N

Precipitation Column Name: Elicitation Relevant Parent Activity

Description: IF the elicitation is relevant to the parent (coded as a 1 in Column M), identify whether the elicitation was relevant because it refers to the same activity.

Examples:

P: Hm, we have crayons and colored pencils.

P: What should we draw?

P: I want to read this book.

C: What book is that?

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the parent, enter NA
- If the elicitation was not overall relevant to the parent's last utterance (aka there is a 0 in Column M), enter NA

Column O

Precipitation Column Name: Elicitation Relevant Parent Number

Concept

Description: IF the most recent utterance/action used an instance of number talk (coded as 1 in Column K), identify whether the elicitation was relevant to the last utterance from the parent because it is using or describing the same number concept discussed in the most recent utterance (Look back to "Parent Prior Speech Number" in Column K).

Examples:

P: There's seven cars.

P: Can you count them?

P: I see four eggs.

C: How many are there all together?

How to Code:

- If yes, enter 1
- If no, enter 0

- If the elicitation was the very first thing said in the interaction or by the parent, enter NA
- If the parent’s most recent utterance was not coded as number-related (aka there was a 0 in Column K), enter NA

Column P

Precipitation Column Name: Elicitation Relevant Parent Spatial

Concept

Description: IF the most recent utterance/action used an instance of spatial talk (coded as 1 in Column L), identify whether the elicitation was relevant to the last utterance from the parent because it is using or describing the same spatial concept discussed in the most recent utterance (Look back to “Parent Prior Speech Spatial” in Column L).

Examples:

P: That’s a lot of shapes.

P: Where is the triangle?

P: I have more blueberries than you.

C: Can you give me more?

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the parent, enter NA
- If the parent’s most recent utterance was not coded as spatial-related (aka there is a 0 in Column L), enter NA

Column Q

Precipitation Column Name: Child Last Speech Transcribed

Description: The last utterance that the child spoke prior to the elicitation.

If the child gestured instead of verbally speaking, describe the gesture in brackets.

Examples of gestures:

C: [pointed at car]

C: [handed parent crayons]

C: [nodded head yes]

C: [held up three fingers]

How to Code:

- Copy the transcribed speech from the compiled transcription and paste the utterance
- If the child gestured instead of verbally speaking, describe the gesture in brackets
- If the elicitation was the very first thing said in the interaction or by the child, enter NA

Column R

Precipitation Column Name: Child Last to Speak

Description: Identify whether the child was the most recent person to speak immediately prior to the elicitation.

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction, enter NA

Column S

Precipitation Column Name: Child Gesturing Prior

Description: Identify whether the child was gesturing immediately prior to the elicitation or if they gestured during their last verbal utterance (transcribed in Column Q).

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the child, enter NA

Column T

Precipitation Column Name: Child Prior Speech Number

Description: Identify whether the child's last utterance (transcribed in Column Q) used an instance of number talk. (See last page of manual for list of number words)

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the child, enter NA

Column U

Precipitation Column Name: Child Prior Speech Spatial

Description: Identify whether the child's last utterance (transcribed in Column Q) used an instance of spatial talk. (See last page of manual for list of spatial words)

How to Code:

- If yes, enter 1
- If no, enter 0
- Note, if it was already coded as number-related in the step above (aka there is a 1 in Column T), it cannot be coded as spatial-related here, so enter “0”
- If the elicitation was the very first thing said in the interaction or by the child, enter NA

Column V

Precipitation Column Name: Elicitation Relevant Child

Description: Identify whether the elicitation (transcribed in Column A) was overall relevant to the last utterance/action from the child (transcribed in Column Q). Ask yourself, did the elicitation continue in the same context as the child’s precipitation that preceded it?

Relevance can be either to the activity (continuing to play with the same toy or talk about the same toy/activity) or the concept (continuing to discuss the same topic or teach the same skill).

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the child, enter NA

****If the “Elicitation Relevant Child” is a 0, the next three columns will automatically be coded as NA****

Column W

Precipitation Column Name: Elicitation Relevant Child Activity

Description: IF the elicitation is relevant to the child’s precipitation (coded as 1 in Column V), identify whether the elicitation was relevant because it refers to the same activity.

Examples:

C: Hm, we have crayons and colored pencils.

P: What should we draw?

C: I want to read this book.

C: What is the title?

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the child, enter NA
- If the elicitation was not overall relevant to the child's last utterance (aka there was a 0 in Column V), enter NA

Column X

Precipitation Column Name: Elicitation Relevant Child Number

Concept

Description: IF the most recent utterance/action used an instance of number talk (coded as 1 in Column T), identify whether the elicitation (transcribed in Column A) was relevant to the last utterance from the child because it is using or describing the same number concept discussed in the most recent utterance (Look back to "Child Prior Speech Number" in Column T).

Examples:

C: Look it's a seven.

P: So what number comes next?

C: I have two crayons.

C: [picks up another crayon] Now I have three.

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the child, enter NA
- If the child's most recent utterance was not coded as number-related (aka if there is a 0 in Column T), enter NA

Column Y

Precipitation Column Name: Elicitation Relevant Child Spatial

Concept

Description: IF the most recent utterance/action used an instance of spatial talk (coded as 1 in Column U), identify whether the elicitation (transcribed in Column A) was relevant to the last utterance from the child because it is using or describing the same spatial concept discussed in the most recent utterance (Look back to "Child Prior Speech Spatial" in Column U).

Examples:

C: That's a lot of shapes.

C: Where is the triangle?

C: I have more blueberries than you.

P: Can you give me more?

How to Code:

- If yes, enter 1
- If no, enter 0
- If the elicitation was the very first thing said in the interaction or by the child, enter NA
- If the child’s most recent utterance was not coded as spatial-related (aka if there is a 0 in Column U), enter NA

Appendix A.4 Number and Spatial Identifiers

We will be coding if an utterance is either number- or spatial-related. These identifiers should already be highlighted in the compiled transcripts.

Table 6 Number-Related Identifiers

Number-Related			
Functions	Numbers		Ordinal
Number	One	Forty	First
Count	Two	Fifty	Second
Add	Three	Sixty	Third
Subtract	Four	Seventy	Fourth
Multiply	Five	Eighty	Fifth
Divide	Six	Ninety	Sixth
Sum	Seven	Hundred	Seventh
Take away	Eight	Thousand	Eighth
Plus	Nine	Million	Ninth
Minus	Ten	Billion	Tenth
Times	Eleven		Eleventh
How much	Twelve		Etc..
How many	Zero		
	Teen		
	Twenty		
	Thirty		

Table 7 Spatial-Related Identifiers

Spatial-Related				
Quantifiers		Orientation and Transformation	Shapes	Sequence
Much	Amount	Upside down	Circle	Order
Many	Quantity	Right side up	Oval	Next
Little	None	Upright	Ellipse	Last
All	Any	Orientation	Semicircle	Before
Enough	Lot(s)	Turn	Triangle	After
Some	Plenty	Flip	Square	Then
Few	Total	Rotate	Rectangle	
More	Couple	Behind	Diamond	
Less	Fraction	Between	Pentagon	
Most	Whole	On top of, on	Hexagon	
Least	Half	Underneath, under	Octagon	
Several	Quarter	Below	Parallelogram	
Majority	Third, etc.	Over	Quadrilateral	
Minority		Inside, in	Rhombus	
		Outside, out of	Polygon	
		Next to	Shape	

Number Words

Number words are just what they sound like: words that describe a quantity of objects. The most important number words to think about are “one”, which can be used in a non-numeric sense, and “five”, which is not counted when “high five” is said.

- “One” Rules
 - The number “one” is counted when it is part of a count, refers to one object, and could be followed by “of” or “more”
 - Ex: “I want one”
 - Ex. “I want one of those”
 - Ex. “There isn’t one”
 - Ex. “There is one we are missing”

- Ex: “Which one of these eggs is sad?”
- “One” is counted when referring directly to money, so pay attention to context, especially in grocery.
 - Ex. “I need ten ones back”
 - Ex: “Give me one [dollar]”
- If “one” is said in isolation without context, always count it, we assume it is used in a numeric sense
- DO NOT count one when it can be replaced with the word “thing” or “person” – this can get a bit tricky when “no one” is said.
 - Ex. “Can you hand me that one” – can be replaced with “egg”
 - Ex. “I need a green one”
 - Ex. “No one is here” (This is effectively “Nobody is here”)
- Any occurrence of “high five” should NOT be counted as a number word.

Ordinal Terms

Ordinal terms describe the position or rank of something. When used in an elicitation they will be considered NUMBER rather than spatial.

- DO NOT count the ordinal terms “third, fourth” etc. when referring to fractions. When these words refer to a fraction, they are counted in quantifiers.
 - Ex. “The third thing on my grocery list is corn” – this is counted as ordinal
 - Ex. “It’s his fourth birthday” – this is counted as ordinal
 - Ex. “Can I buy a third of the pizza?” – this is NOT counted as ordinal, but it should be counted as a quantifier in spatial coding.

- Ordinal terms are counted when order or sequence is referred to in some way, and this can be determined by replacing the term with another ordinal term and seeing if the sentence makes sense
 - Ex. “What should we play with first?” can be replaced with “what should we play with second?” and therefore “first” is counted as an ordinal term.

Function Terms

Functions are words or phrases that elicit number talk or conversations about math. This includes asking or alluding to doing addition, subtraction, or other math problems, or asking to count or state the amount of something.

- “Number” should be coded in all instances.
- DO NOT count the word “count” when used in a way that is synonymous with “matter”
 - Ex. “I dropped the die so that one didn’t count” – this is not counted
 - Ex. “Can you count this for me?” is counted
- DO NOT count “plus” when adding ideas to a phrase. “Plus” should only refer to objects or quantities.
 - Ex. “I think we played with the blocks enough, plus I wanted to check out this book” – this should not be coded
 - Ex. “I would like to buy this pizza, oh and plus this corn” is counted
 - Ex. “These animals plus two more” is counted
- Likewise, DO NOT count “add” when referring to ideas. “Add” should only refer to the addition of objects or quantities.
 - Ex. “I would like to add, that we are still missing some pieces”. – this is not counted.

- Ex. “Which pieces need to be added yet?” is counted.
- Ex: “Add more animals to the picture.” Is counted
- “Take away” should be counted when referring to subtraction. It should not be counted when referring to taking something away something as a punishment, or taking away something that does not elicit a numeric discussion.
 - Ex. “Now take away two pieces” is counted
 - Ex. “I am going to take away your toy” should NOT be counted
 - Ex. “Let’s take away this game since you’re done playing with it” should NOT be counted
- DO NOT count “times” when referring to a non-mathematical expression.
 - Ex. “How many times did that happen? Two times?” Both of these are NOT counted

Spatial Coding Rules

Function term

- “Where” should be coded in all instances (though it is not highlighted in the compiled transcripts)

Quantifiers

Quantifiers are words that estimate or describe a quantity of objects.

- “Much” should be coded when it could be interchanged with “many”.
 - Ex. “so much” and “too much” are counted
 - Ex. “thank you very much” and “much better” are NOT counted
- DO NOT count “whole” when you cannot replace it with another fraction like “half” and have the sentence makes sense. “Whole Foods” is not counted.

- Ex. “whole lot”, “whole bunch of” are NOT counted, as “half” is not a sufficient replacement
- DO NOT count “little” when referring to the size of individual objects. “Little” should be counted when referring to the size of a set. In general, count “little” when you can replace it with “a little bit of” and the sentence still makes sense (this rule does not always work). “Little” should be counted when referring to time or distance, even if it’s implicit.
 - Ex. “a little while” – this is counted because it refers to time
 - Ex. “Could I have a little milk?” – this is counted due to “little bit of” replacement
 - Ex. “the little car” is NOT counted because it’s the size of a single object
- DO NOT count “more” when is used to compare qualities. “More” should only be counted when quantifying a set of objects.
 - Ex. “I have more money than you” is counted
 - Ex. “Is there another piece that looks more like that?” is NOT counted because it is a quality
- Like “more”, DO NOT count “a lot” or “lot” when referring to a quality. “Lot” should be counted when referring to a quantity of objects
 - Ex. “that piece looks a lot like that piece” is not counted
 - Ex: “I have a lot of blocks” -is counted
- DO NOT count “quarter” when referring to the coin or a period in a sports game. “Quarter” should only be counted when used as a fraction.
- “Most” and “Least” should be counted when referring to a quantity of things. “Most” should be counted when it can be replaced with “the majority of”. Do not count “most” or “least” when referring to qualities of things.

- Ex. “Do you know anything that is blue and white that you like the most?” is not counted (quality)
- Ex. “most importantly” is not counted
- Ex. “Except most banks now, what do we do? We insert the chip huh?” is counted because “the majority of banks”
- “At least” as an expression should NOT be counted
 - Ex. “At least we have other toys to play with”.
- Only count “enough” when referring to a quantity of objects, not when used as a reprimand or expression, such as “fair enough”. “Close enough” should be counted when referring to distance or quantity of items.
- “Couple” should not be counted when used as a noun to describe a group of people.
 - Ex. “what a cute couple” is not counted
- Rules for “all”
 - Count “all” when referring to members of a set. Also, count “all” when referring to the entirety of a whole object (this **excludes actions**).
 - Ex. “But he did all the decorating. Do you do all the decorating on your birthday? – for both of these instances, “all” should not be counted, action
 - Ex. “I need two fives, that’s all I need” – this is counted, refers to fives
 - Ex. “They will eat all the cake” – counted, referring to entirety of whole cake.
 - In general, “all” should NOT be counted when used as part of an expression: “all done,” “all right,” “all the time,” “all over,” “at all,” “all around,” “after all,” or “all out”, “all by himself”, “all ready”, “all along”. DO NOT count “that’s all”

when part of an expression, but something like “that’s all I have” or “that’s all you need” should be counted, as it is referring to a set quantity of objects.

Sequence

Sequence words contextualize and describe the order of objects or events

- Only count “then” when referring to the order of events and patterns. “Then” should be counted when it can be replaced with “next.” “Then” should not be counted when it can be replaced with “in that case” or “in addition”. With the “in addition” rule, if you can switch the order of the things described and the sentence makes sense, “then” should not count because it is not being used in a sequential way.
 - Ex. “well then, I will hold onto the money since I have to pay you”. NOT counted
 - Ex. “Put the treasure chest on first, and then put the cannonballs on.” this is counted
- Only count “next” when referring to the order of events. “Next” should be counted when referring to items in a sequence/pattern.
 - Ex: “1,2,3 what number comes next?”
 - Ex: “Which egg comes next?”
- Only count “order” when used in a sequential sense. DO NOT count “can I take your order” or “order” when referring to tidiness “let’s get this room back in order”
- DO NOT count “last” to refer to something that happened previously. If “last” refers to a final action in a sequence, then it should be counted. Think hard about which of these is the case when “last time” is said. Last should not be counted when referring to the duration of something. Count “lastly” as a sequence word when referring to the final action in a sequence.

- Ex. “last year I learned this in school” – not counted
- Ex. “this is the last block” – this is counted
- Ex. “It didn’t really last too long.” – not counted (duration/time)
- DO NOT count “after” when used to mean “chase”
 - Ex. “Do you think he will go after the bird?”
- DO NOT count “after” when used as an expression, such as “happily ever after” and “after all”.
- DO NOT use any of the sequence words when they are only in reference to TIME
 - Ex: Put this block on before that one” -counted
 - Ex: How long are we staying here after we play? -not counted

Orientation and Transformation

Orientation and transformation words describe the relative orientation or transformation of objects and people in space. They include words that encode direction or location of movement, and transformation around an axis.

- “Turn” and “flip” should not be coded when saying “turn/flip the page” or “turning four” referring to birthdays. “It’s your turn” should not be counted
- “Between” should only be used in a spatial context, not to express choosing something
 - Ex: “I’m stuck between having pizza or a cheeseburger” – NOT counted
 - Ex: “Can you grab the egg between the sad and angry?” – is counted
- Only use “on top of” “underneath/under” “below” “over” “inside” “outside” and “next to” when referring to a POSITION IN SPACE
 - Ex: “I’m going to look under the table” -counted
 - Ex: “I want this to be over” -NOT counted

- Ex: “Draw a circle below the square” -Counted

Ex: “Stay on top of things!” and ”I’m on top of it” -are NOT counted (used as ‘in control of’)

Bibliography

- Agarwal, Sumit, and Bhashkar Mazumder. (2013). "Cognitive abilities and household financial decision making." *American Economic Journal: Applied Economics*, 5 (1): 193-207.
- Ansari D., Donlan, C., Thomas, M.S., Ewing, S.A., Peen, T., Karmiloff-Smith, A. (2003). What makes counting count? Verbal and visuo-spatial contributions to typical and atypical number development. *J Exp Child Psychol.* 85(1):50-62. doi: 10.1016/s0022-0965(03)00026-2
- Benavides-Varela, S. et al. (2016). Numerical activities and information learned at home link to the exact numeracy skills in 5–6 years-old children. *Front. Psychol.* 7, 94.
- Blevins-Knabe, B. & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Dev. Parent.* 5, 35–45.
- Braham, E. J., Libertus, M. E., & McCrink, K. (2018). Children's spontaneous focus on number before and after guided parent–child interactions in a children's museum. *Developmental Psychology*, 54(8), 1492–1498. <https://doi.org/10.1037/dev0000534>
- Currie, Janet and Duncan Thomas. (2001). "Early test scores, school quality and SES: Long run effects on wage and employment outcomes." *Research in Labor Economics: Worker Wellbeing in a Changing Labor Market*, 20: 103-132.
- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428–1446. <https://doi.org/10.1037/0012-1649.43.6.1428>
- Eason, S. H., Nelson, A. E., Dearing, E., & Levine, S. C. (2021). Facilitating young children's numeracy talk in play: The role of parent prompts. *Journal of Experimental Child Psychology*, 207, 105124. <https://doi.org/10.1016/j.jecp.2021.105124>
- Elliott, L. & Bachman, H. J. (2018). How do parents foster young children's math skills? *Child. Dev. Perspect.* 12, 16–21.
- Elliott, L., Braham, E. J., & Libertus, M. E. (2017). Understanding sources of individual variability in parents' number talk with young children. *Journal of Experimental Child Psychology*, 159, 1–15. <https://doi.org/10.1016/j.jecp.2017.01.011>
- Edens, K. M., & Potter, E. F. (2013). An exploratory look at the relationships among math skills, motivational factors and activity choice. *Early Childhood Education Journal*, 41(3), 235–243. 10.1007/s10643-012-0540-y

- Ginsburg, H. P., & Baroody, A. J. (2003). TEMA-3: Test of early mathematics ability-Third Edition. Austin, TX: Pro-Ed.
- Hannula-Sormunen M. M, Lehtinen E, & Räsänen P (2015). Preschool children's spontaneous focusing on numerosity, subitizing, and counting skills as predictors of their mathematical performance seven years later at school. *Mathematical Thinking and Learning*, 17(2–3), 155177. 10.1080/10986065.2015.1016814
- Hannula M. M, & Lehtinen E (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction*, 15(3), 237–256. 10.1016/j.learninstruc.2005.04.005
- Hannula M. M, Lepola J, & Lehtinen E (2010). Spontaneous focusing on numerosity as a domain specific predictor of arithmetical skills. *Journal of Experimental Child Psychology*, 107(4), 394406. 10.1016/j.jecp.2010.06.004 [PubMed: 20643417]
- Hannula M. M, Rasanen P, & Lehtinen E (2007). Development of counting skills: Role of spontaneous focusing on numerosity and subitizing-based enumeration. *Mathematical Thinking and Learning*, 9(1), 51–57. 10.1207/s15327833mtl0901_4
- Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews*, 15(1), 60–68. <https://doi.org/10.1002/ddrr.46>
- King, Y. A. & Purpura, D. J. Direct numeracy activities and early math skills: Math language as a mediator. *Early Child. Res. Q.* 54, 252–259 (2021).
- LeFevre, J.-A. et al. Home numeracy experiences and children's math performance in the early school years. *Can. J. Behav. Sci.* 41, 55–66 (2009).
- Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2011). "What counts in the development of young children's number knowledge?": Correction to Levine et al. (2010). *Developmental Psychology*, 47(1), 302.
- Mutaf Yildiz, B., Sasanguie, D., De Smedt, B. & Reynvoet, B. (2018). Frequency of home numeracy activities is differentially related to basic number processing and calculation skills in kindergartners. *Front. Psychol.* 9, 340.
- Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development*, 35, 15–33. <https://doi.org/10.1016/j.cogdev.2014.11.002>
- Reyna, V. F., & Brainerd, C. J. (2007). The importance of mathematics in health and human judgment: Numeracy, risk communication, and medical decision making. *Learning and Individual Differences*, 17(2), 147–159. <https://doi.org/10.1016/j.lindif.2007.03.010>
- Ribner, A., Silver, A., Elliott, L., Libertus, M. (2022). Exploring effects of an early math intervention: The importance of parent–child interaction. *Child Development*

- Sarnecka BW, Carey S. (2008). How counting represents number: what children must learn and when they learn it. *Cognition*. 108(3):662-74. doi: 10.1016/j.cognition.2008.05.007.
- Shanley, J. & Niec, N. L. (2011) The contribution of the dyadic parent-child interaction coding system (DPICS) warm-up segments in assessing parent-child interactions, *Child & Family Behavior Therapy*, 33:3, 248-263, DOI: 10.1080/07317107.2011.596009
- Silver, A. M., Elliott, L., & Libertus, M. E. (2021). Parental math input is not uniformly beneficial for young children: The moderating role of inhibitory control. *Journal of Educational Psychology*. Advance online publication. <http://dx.doi.org/10.1037/edu0000679>
- Silver, A.M., Libertus, M.E. (2022). Environmental influences on mathematics performance in early childhood. *Nat Rev Psychol*. 1, 407–418. <https://doi.org/10.1038/s44159-022-00061-z>
- Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *PNAS Proceedings of the National Academy of Sciences of the United States of America*, 110(45), 18116–18120. <https://doi.org/10.1073/pnas.1302751110>
- Trusty, J., Robinson, C. R., Plata, M., & Ng, K.-M. (2000). Effects of gender, socioeconomic status, and early academic performance on postsecondary educational choice. *Journal of Counseling & Development*, 78(4), 463–472. <https://doi.org/10.1002/j.1556-6676.2000.tb01930.x>
- Vasilyeva, M., Laski, E., Veraksa, A., Weber, L. & Bukhalenkova, D. (2018). Distinct pathways from parental beliefs and practices to children’s numeric skills. *J. Cogn. Dev.* 19, 345–366.
- Wynn, K. (1992). Children’s acquisition of the number words and the counting system. *Cognitive Psychology*, 24(2), 220–251. [https://doi.org/10.1016/0010-0285\(92\)90008-P](https://doi.org/10.1016/0010-0285(92)90008-P)