Tradeable Spectrum Interference Rights

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Abstract

Spectrum rights have gained increasing attention since Ronald Coase pointed out that the most efficient way to assign spectrum is to give it to those users who value it most through property-like rights and secondary markets. Defining spectrum rights turns out to be difficult due to the nature of electronic emissions[1]. As a result, it may be more practical to define interference rights (similar to pollution rights) rather than exclusive usage rights. Interference rights give a user the right to interfere with another user up to a specified level. In this paper, we develop the idea of a market in spectrum interference rights and, using some plausible use cases, illustrate its characteristics. The paper therefore includes a detailed description of interference rights along with some first order quantitative modelling of the use cases coupled with qualitative analysis.

1. Introduction

Spectrum, which supports the transmission of sound, data, and video, is critical to the implementation of a mobilly connected society. The original idea for spectrum assignment and allocation is exclusive usage. The dominant approach, command-and-control, grants fixed amounts of spectrum to wireless service providers for certain technology for 10 years. The command-and-control approach facilities spectrum management for the regulator and makes the spectrum usage fairly predictable. It largely prevents the harmful interference for licensees and provides for an exclusive usage environment. This static strategy served both the regulator and users well during the first 50 years of radio frequency regulation, when the spectrum demand could relatively easily be met through technological innovation that expanded the range of economically feasible frequencies. As the demand for access to spectrum grew dramatically, it became increasingly difficult to locate available spectrum for both new entrants and incumbents [2].The long licensing period, large geographic coverage, specific technology regulation, and exclusive usage offers predictable interference at the expense of spectrum utilization, a tradeoff that is worth revisiting in light of technological improvements.

Since radio spectrum has become scarcer, the FCC has been seeking policies that offer more spectrum access opportunities:

- They created the Spectrum Policy Task Force to study alternatives
- The secondary spectrum market opening in November 2000 is one method [3]. It partially changes the command-and-control strategy to a rights-based regulation, which permits organizations to transfer, purchase, and sell the rights to use spectrum in private market transactions.
- The FCC further liberalized the TV white space for unlicensed users in 2008.
- Interference Temperature (ITemp) is another proposed spectrum management strategy which is meant to allocate more spectrum access opportunities for secondary users and meanwhile maintain the existing Quality of Service (QoS) for primary users [4], especially for technologies such as Ultra-Wideband (UWB)

Although spectrum trading and other DSA approaches could increase spectrum usage efficiency, they have been adopted slowly. There are several reasons for this:

- The needed technology developments in radio systems,
- There is some evidence of a divergence of incentives among stakeholders [5], and
- License holders fiercely defend their rights against any kind of opportunistic use for fear of degrading the value of their investment (i.e., their license).

In simple terms, the value that a primary user obtains from their license depends on (1) their ability to earn revenues from that investment and (2) capital gains associated with its increasing value. The former depends on the ability of the license holder to deliver a suite of services to end users at a contracted quality level. The latter depends on the license holder's ability to defend the parameters of the license. Trading in interference rights allows primary users to explicitly manage both aspects of their license, since they can get revenue from bandwidth has not been fully utilized. Since they determine the parameters of the right, they can control the asset value of the license.

It is quite possible that mobile connectivity will follow the precident set by electric power in the early 20th century and the process that is arguably being played out in computing, moving from captive resource to shared resource to utility [6]. This notion was proposed recently in [7]. In this scenario, some infrastructure provider offers programmable spectrum infrastructure like the computational capability and storage in cloud computing, and users are charged by actual usage. Besides technology innovation and payment system, a tradable interference right may be a critical success factor for DSA since the spectrum trading needs to be as liquid as possible.

The rest of the paper is organized as follows. Section 2 illustrates the idea of define spectrum right as an exclusive usage right, includes the difficulties in defining the right and trading. Since define the spectrum right as exclusive right doesn't help in spectrum trading and sharing, we propose the spectrum right as interference right, which is introduced in section 3. Section 4 shows how to trade the spectrum under interference right and the case study. Section 5 states the open issues around trading in interference right and the conclusion.

2. Why Interference Rights?

Property rights associated with electromagnetic spectrum was first proposed by Coase [8], though it took many years for these notions to be incorporated, even partially, into government policy. In short, Coase pointed out that the most efficient way to assign spectrum is to give it to those users who value it most through property-like rights and secondary markets. As stated in [9] "[property rights] implies the ability to buy; hold; use; sell; dispose of, in whole or in part; or otherwise determine the status of an identifiable, separable and discrete object, right or privilege." Establishing a complete definition in this sense has occupied researchers, with mixed results, in the context of exclusive usage rights or exclusive right (see, for example [1] for a critical review). Even if spectrum exclusive usage right can be clearly defined, trading spectrum exclusive usage rights is quite challenging (see, for example [10]) . Lastly, the right transfer under exclusive right may require involvement of regulator which can decrease liquidity and increase transaction costs.

2.1 Property rights in spectrum cannot be defined clearly

As summarized in [11], DeVany *et.al.*[12] proposed a multidimensional set of rights called TAS, which means "the owner of the TAS-based rights would have the exclusive right to produce electromagnetic waves for a specified period of time (T), over a specified geographic area (A), and in a specified range of frequencies (S)."

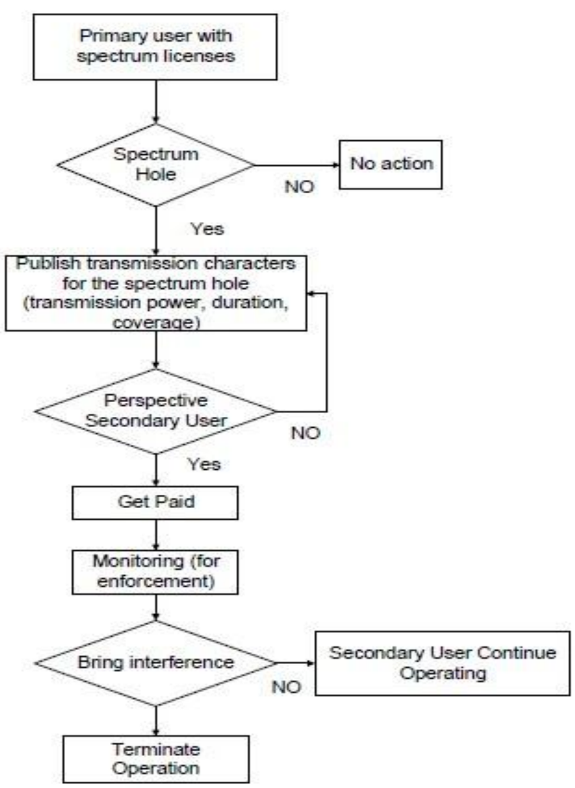
This definition brought technical difficulties, since electronic waves are not restricted by area and bands. In particular, it is hard to prevent radio signal from trespassing to other area. Although electronic waves degrade with distance, it is very hard or impossible to prevent them at boundaries. Therefore, the exclusive usage right is hard to set geographic boundaries for spectrum exclusive right holders. Besides the geographic trespassing, radio signal also spill over to adjacent channel. It acts as interference to other frequencies, and the degree of harmful depends on the desired signal strength and receiver's performance.

Matheson [13] developed "TAB" as a seven dimensional model to surpass the difficulties in geographic and adjacent channel spillover. It includes three dimensions of location (latitude, longitude, and elevation), time, and two possible directions of arrival (azimuth and elevation angles).Together with the zoning method proposed by Hatfield and Weiser [1], which "recognize that certain bands are more conductive to some services and other bands are more conducive to other ones", these are all approaches designed to define a right that excludes others. This kind of right, sometimes called an exclusive usage right, can be achieved by using guard bands, transmission power caps, and other methods which prevent others to access the available spectrum. However, Interference and noise are stochastic processes and vary with time and location. Even if spectrum right are defined as specifically as Matheson's seven dimensional TAB, unexpected interference and noise cannot be prevented nor guarantee can the quality of the resource be guaranteed.

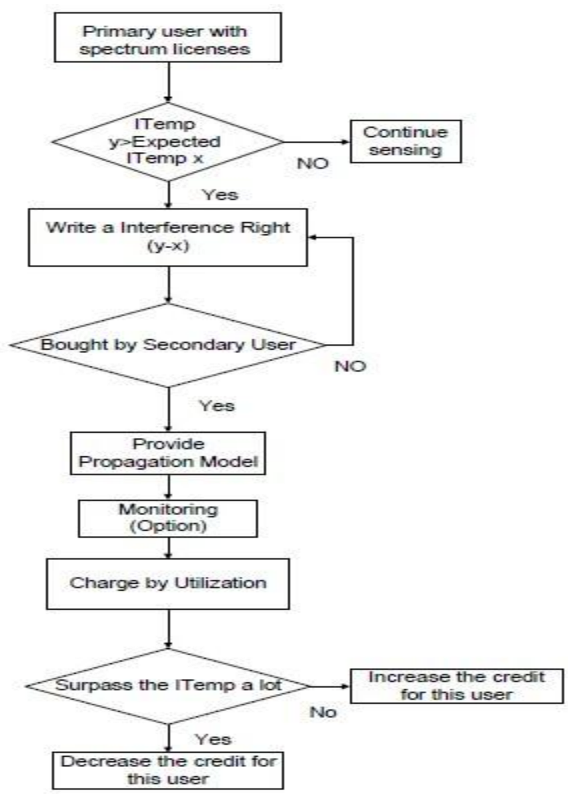
2.2 Property rights as an "all or nothing" kind of sharing

As mentioned by Coase, defining a spectrum right is the first step in improving spectrum utilization efficiency. The second step is to trade this right. Following [14], flow chart 1 describes the spectrum trading process under a exclusive rights regime. Under exclusive usage right, primary users only sell spectrum that they don't operate on. Since they determine the transmission power threshold for specific area, we assume that the primary user has awareness about the spectrum hole in its license region. Secondary spectrum users get the information about the transmission power cap and the operating duration from the primary users and make the decision on whether to purchase this spectrum hole or not. If they decide to operate on this spectrum hole, they need to pay the fee to primary user and obey primary users' requirements. For the sake of enforcement, primary users could continue sensing the spectrum utilization and determine whether the secondary users bring harmful interference or not during secondary user' transmission period. Secondary users must terminate their transmission if primary users detect harmful interference and ask them to do so. This process is not necessary, though it gives the primary user with more certainty about their QoS, it also brings the cost of sensing [5, 15].

As stated above, there are two major problems associated with trading exclusive rights. First and foremost, it is "all or nothing" trading for both primary and secondary users. Primary users only sell frequencies on which they do not operate, which means there is little or no underlay with primary users. In addition to the frequency bands that have no primary user transmissions, there are also some bands only have little primary usage which can underlay with secondary users to improve the spectrum usage efficiency. Second, there is a fixed transmission power cap for spectrum usage, which gives secondary users less flexibility and may lead to unnecessary waste of spectrum.



Flow Chart 1. Trading under Esclusive Rights



Flow Chart 2. Trading under Interference Right

3. What are Interference Rights?

Because of these challenges, we propose that it may be more practical to define interference rights (similar to pollution rights) rather than ownership rights. An interference right gives a user the right to interfere with another user up to a specified level. Under an interference rights regime, existing license holders can write interference rights on their licenses, which can be traded, combined, or exchanged with other users. These users don't need to get permission from FCC and existing licensees control their interference rights freely.

3.1 Explicit permission to degrade a resource

In contrast to exclusive usage rights, an interference right explicit allows other spectrum users to interfere existing services to a certain level. A primary user would accept interference from secondary users to their wireless systems in terms of time, geographic area, and frequency. This interference degrades the resource (spectrum) but does not necessarily degrade the QoS since it is possible, within limits, to design receivers (at higher cost) that are less sensitive to prevailing interference and noise.

The interference temperature (which proposed by Spectrum Policy Task Force [16]) and spectrum usage rights (which were described by UK Office of Communications [17]) expressed similar ideas. Instead of rigid guard bands and geographic separation they increase the spectrum utilization in different ways. The interference temperature metric allows more spectrum access and the spectrum usage rights give licensees' the authority to change their technology and deployment, as long as the total amount of interference they bring to others are less than some threshold. Both of them explicitly permit users other than license holders to degrade a resource.

3.2 Pollution Rights Analogy

The Interference Right is analogous to a pollution right. For receivers, Radio Frequency Interference (RFI) is a kind of pollution which cannot be restricted by geographic area and other arbitrary boundaries. Its sources are natural and man-made, just like air pollution. There are two types of measures that have been used to regulate air pollution for the past several years, price-based measures and rights-based measures. A price-based measure persuades polluters to reduce their emissions by charges, taxes and subsides. A rights-based measure sets a predetermined "pollution" limit, then assigns the pollution right to qualified organizations, and finally allows them to trade these rights. It has been shown that the rights-based measure, which is a market-based policy, controls the consumption of non-excludable goods more efficiently than the price-based measure [18]. The best part of this method is that, although the purpose of the pollution rights trading is to minimize costs for firms rather than maximize environmental gains, past experience has showed that the pollution level is much less than what was guaranteed [19] .

Likewise, interference is an outcome when we generate electrometric waves, either purposefully or incidentally. It cannot be exterminated as long as we use the wireless communication. While spectrum pollution is a problem that ultimately limits capacity, it is also not our goal to keep the spectrum sufficiently quiet.

3.3 Properties

The interference right we proposed here combines the interference temperature, spectrum usage rights, and Hazlett's liberalized spectrum right [20]. Under the interference right, current license holders have full control over their spectrum, which means primary users can partially share their spectrum with others without regulators' involvement and secondary users can negotiate the desired amount of EM energy with primary users. Secondary users can combine an interference right with another one to create a service area. The tradeoff for primary users is between their own coverage and the gain from sharing spectrum with others, while the tradeoff for secondary users is between the spectrum utilization and the spectrum sharing fee.

3.3.1 License holder control

The major drawback of Interference temperature and spectrum usage rights is that the threshold is determined by a regulator. This leads to three problems. First, although models are easy to construct compared to conducting measurements, models are less accurate. Second, primary users have less incentive to trade with either secondary users or their counterparts because of the elevated transaction cost associated with following necessary regulatory procedures and the time it takes to receive approvals. Third, license holders have less control over their sub-licensees under this approach and more interference doesn't bring any economic gain for them. Thus, they are less likely to be willing participants, since *any* interference degrades the value of their license

In order to address these concerns, license holders have the full authority to control the spectrum that is listed in their licenses under the proposed interference right. While the value of the license is degraded when a right is issued, the license holder receives a monetary payment that is (presumably) compensatory. They have the flexibility to define the spectrum right by time, frequency, coverage and price. Thus, they can make tradeoffs between their system operation and income from the sale of interference rights. The license holder themselves determine the operating time, bandwidth, center frequency, transmission power, and geographic coverage with secondary users (limited, of course, by the original license terms). Following the outcomes of [14], they can manage transaction costs by being able to determine with whom bargain. They trade, exchange, or share the spectrum with other users according to their willingness, not because of a requirement from regulator. In other words, they will trade the spectrum if the gain is higher than not trading.

3.3.2 Partial sharing of spectrum

From the statement above, primary users can partially trade their spectrum to other users as an interference right. That means that primary users not only sell bands that they don't use but also underlay with secondary users. They balance the gain from trading the spectrum with service loss and then determine how to share the spectrum. This trading may not guarantee the existing QoS. It also doesn't limit to any technology deployment. It is possible that license holders sacrifice certain geographic coverage to the economic gain. It is also possible that licensees improve their receivers' performance for more spectrum sharing. This behavior in essence reflects the concept of assign the spectrum to customers who value it the most.

3.3.3 Rights of buyers of interference rights

A buyer of an interference right may transmit a maximum amount of EM energy into the frequency/time/space defined by the right according to their service requirement. Fig.1-Fig.3 illustrate the idea of the benefit and flexibility of interference rights buyers. The first operational situation for secondary user is to operate a single transmitter with relatively high transmission power as shown in Fig.1. This case is preferred when secondary user has limited transmitter and coverage requirements. Receivers for its system may be less sensitive and need large SINR to differentiate desired signal from interference and noise. In the second case, the secondary user deploys several transmitters as shown in Fig.2. In this case, each one has less transmission power but can have a relatively large coverage. This might be appropriate for secondary users who target large system availability and have high performance receivers. The third case illustrates a secondary user who aggregates interference rights from various license holders as shown in Fig.3. Secondary users can build their own service territory.

At the same time, primary user also needs to state its own transmission power in the contract with secondary users. This gives secondary users a clear understanding about the interference level that they may encounter. Without that, secondary users would not have a clear definition of what they are buying, since in the wireless communication transmission power by itself does not define QoS. The SINR, which combines transmission power and interference level, affects the service level agreement (SLA) that secondary users can make with their subscribers.

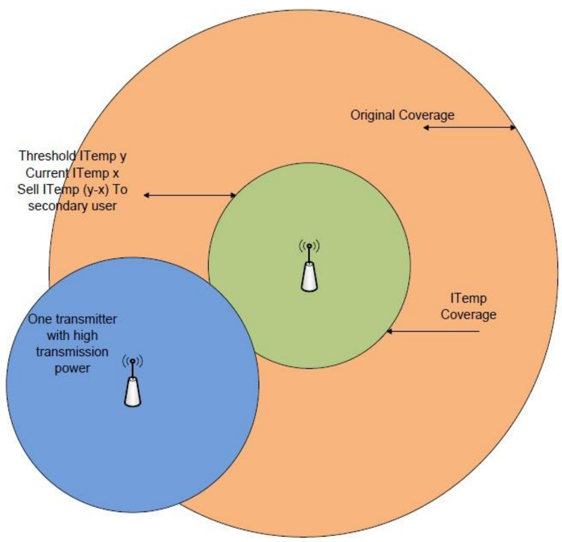


Fig.1 One transmitter

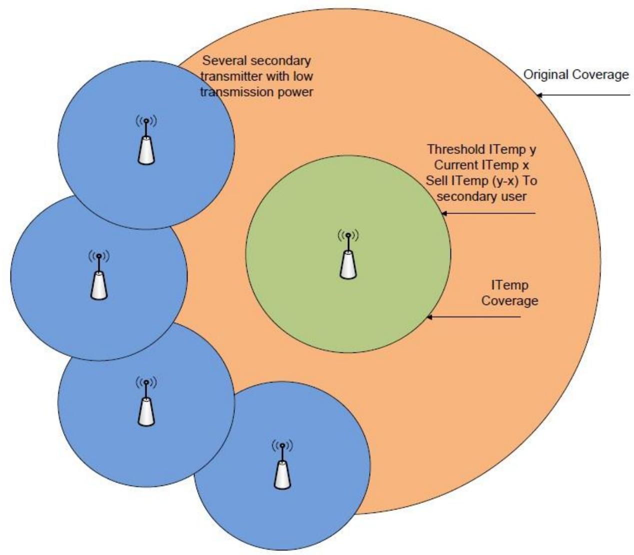


Fig.2 Several Transmitter

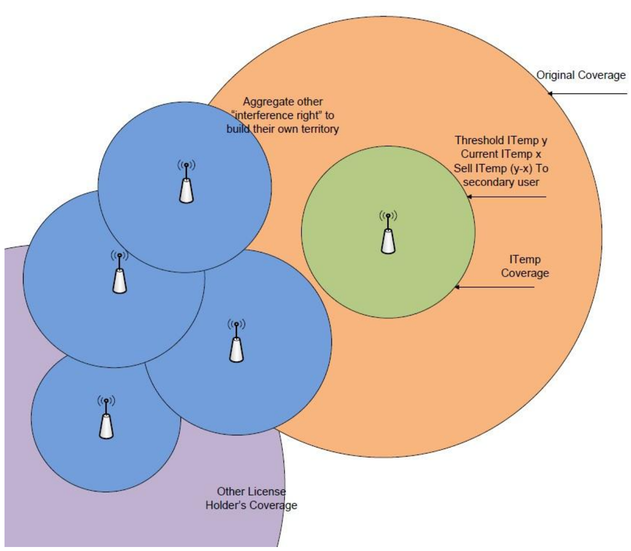


Fig. 3 Combine several Interference Right

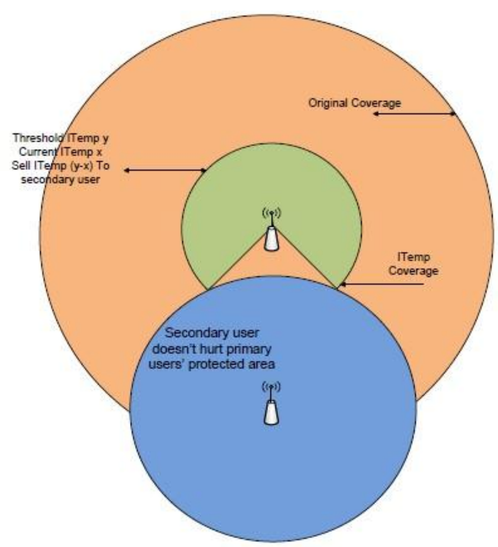


Fig. 4 Directional antenna w/o harmful interference

3.4 Definition of an interference right

As with financial options, license holders determine the definition of the right. In financial investments (e.g., stocks), an option may be written to increase the return on that investment. For example, an investor may write a call option on stock if she expects its value to be stable for a time period, collecting the premium for the option and keeping the underlying investment and earning any dividends that accrue to it (if the ex ante expectation turns out to be reflected in reality). In the same way, suppose that a license holder determines that a contractual QoS can be achieved with ITemp x and the expected ITemp is y (where x>y) in a particular 3-tuple of geographic region, time period and frequency band. The license holder could then write an interference right of (x-y) on the 3-tuple without affecting the contractual QoS and thereby increasing the return on their spectrum license. This right could be bought by a potential secondary user who could then cause interference to the license holder's receivers up to that level. Interference rights can be aggregated by secondary users (or brokers) to build a service area over one or more dimensions of the spectrum 3-tuple

3.4.1 Interference rights and "covered" financial options

Interference rights are created by license holders in a manner very similar to that used by owners of stocks to write "puts" and "calls". This differentiates this approach from other spectrum trading methods, most which propose different auction systems. In our trading methods, primary user writes an interference right based on their user and technological deployment and it is ready to trade. This method decreases the transaction cost and should make for a more liquid market due to timely trading minimal procedures.

Financial Option	**Trading right**
Strike Price	EM energy permitted in time/space/frequency
Expiration	Expiration
Premium	Price

3.4.2 Regulatory engagement

Unlike the exclusive right, interference has less regulatory involvement since licenses are not exchanged. From the point of view of trading, regulatory participation in spectrum trading should be minimal under interference rights. In reality, users may need to register with regulator and then take part in spectrum trading. As stated above, the definition and trading terms of interference rights are completely determined by license holders. The negotiation, implementation and enforcement of the contract will be reflected in future trading preferences. The punishment for violations of the interference threshold may be determined in the negotiated contract and may result in higher prices in future to compensate for the higher transaction costs. As in financial markets, trustworthy secondary users should benefit from better terms and lower prices in future trading.

4. How Would Interference Rights Work?

In this section, we will discuss how to trading in interference rights might work. We will also qualitatively compare the trading under exclusive usage right with trading under interference right. Then two scenarios for trading under interference right are given. The first case deploys the isotropic antenna and the second one use directional antenna.

4.1 Trading Procedures

There are several ways to implement the interference right, which include the interference temperature. The interference temperature metric, which aims at allow unlicensed users to operate in licensed bands, failed after 5 years from it was initially proposed by SPTF. We will use ITemp to show how to trade the interference right in this paper. ITemp is definitely not the only way to implement the spectrum trading through interference rights. Similar methods includes interference threshold, SINR, and so on. We choose ITemp to illustrate our idea here since it is a relatively familiar concept to most people.

ITemp was defined in the NOI as “a measure of the RF power generated by undesired emitters plus noise sources that are present in a receiver system (I+N) per unit of bandwidth. More specifically, it is the temperature equivalent of this power measured in units of Kelvin (K).” [4]The formula proposed by the FCC for calculating ITemp is $IT = \frac{P}{B \times k}$. Where, IT is the Interference Temperature in Kelvin (K), P is the total amount of power received by an antenna in Watts, B is the RF bandwidth in Hertz, and k is the “Boltzman’s constant” (1.38×10^{-23}watt-sec/K). Primary user determines an Interference Temperature threshold for a geographic boundary on a frequency band, and then secondary users operate on that band. Secondary users may continue sensing the spectrum to obtain a more accurate understanding of spectrum usage. They can increase their interference to primary users when the noise and other interference is low; while they need to constraint their transmission when the noise and other interference is relatively high.

Trading under interference rights can be initiated by license holder or secondary user. Flow Chart 2 describes the trading procedures under interference right when initiated by the license holder. The primary user determines whether the current interference temperature is less than expected according to past statistics. If it is less than the expected interference temperature, the license holder may write an "interference right" which describes the spectrum hole according to geographic area, time period, and interference level that can be tolerated by primary receivers. This is then advertised in a market or through a broker. After this interference right is bought by a secondary user, primary users may or may not provide propagation model for their area to the secondary user. As an option, primary user may continue sensing the spectrum and charge the secondary user by utilization. There is a tradeoff for this, since primary users may charge based on secondary spectrum usage, but they also need to pay for sensing.

It is also reasonable to assume that the primary user could collect spectrum usage information from their subscribers, such as the interference level, SINR, Interference Temperature, and so on. Technically, there is a control channel between primary transmitters and receivers which allows them to communication in both directions. Economically, primary users can get more accurate spectrum utilization without the sensing cost. What's more, primary users cares more about how many subscribers are affected by secondary users instead of how large the geographic area that has been interfered by secondary users since they gain profits from subscribers not from coverage. Therefore, collecting spectrum usage characters from subscribers provides more useful information for primary users.

Secondary users can also initiate spectrum trading by requesting an "interference right" for certain price. This interference right is the same as the previous one. After the license holder receives the proposed interference right, they compare the gain with coverage lost and decide whether to write the option and sell it to the secondary user or not.

4.2 Qualitative comparison with exclusive usage right

We have stated the trading procedures under exclusive usage right and interference right. Table 1 briefly compares the difference between these two approaches. Compare to the trading under exclusive usage right, the benefit for primary user under interference right is that they don't need to determine the transmission power threshold for secondary user, and they may earn more revenue since the guard band is less in interference right than in the exclusive usage right. The additional cost for primary user under interference rights is that the QoS may degrade and the protected region where they have service level agreement may be violated. Benefits for secondary user mainly depend on the free choice of transmission parameters, and cost may also come from dynamic spectrum access and monitoring.

	Trading under Exclusive Usage right	Trading under interference right
definition	Exclusive usage of spectrum	Allow transmitter to interfere others transmission to a level
Mechanisms	Set transmission Power Cap for users	SIR regulation at receiver side, learning algorithm
Benefit for Primary users	Income from releasing spectrum	Income from releasing spectrum
Cost for Primary users	(1) Determine transmission power threshold; (2) Monitoring (option); (3) potential spectrum waste;	(1) Monitoring (option); (2) Potential interference;
Benefit for Secondary users	Operate	(1) operate; (2) build service area; (3) choose proper power and center frequency by their own;
Cost for Secondary users	Leasing charge	(1) leasing charge; (2) dynamic spectrum access; (3) monitoring (option);

Table 1. Comparison of trading under interference right and trading under exclusive usage right

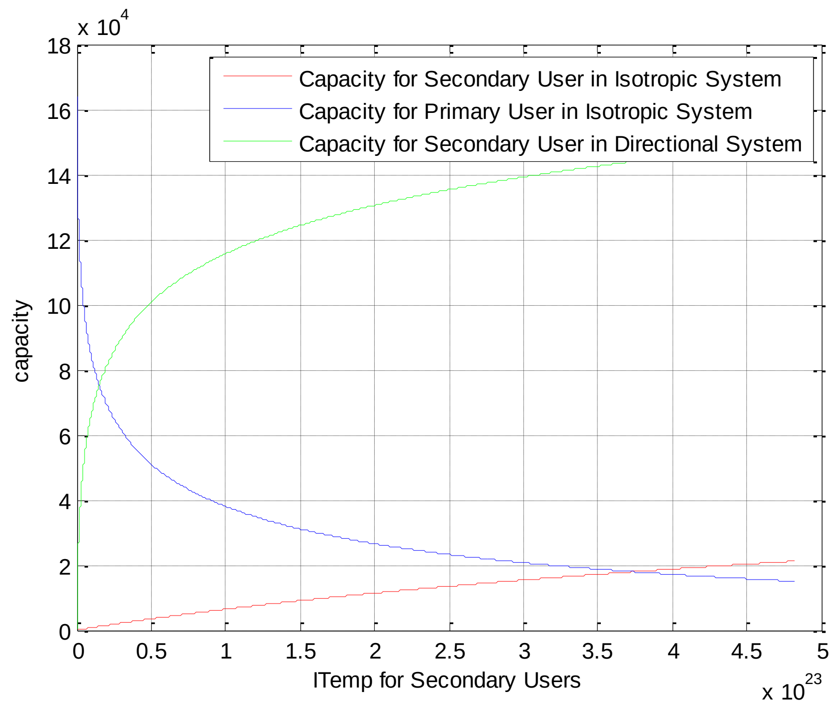
4.3 Specification and analysis

According to the trading procedures stated above, primary user will calculate the ITemp that is potentially free for a trade in a region **R** of his or her coverage area. The primary user might write an interference right if s/he anticipates "free capacity" for a time period in all or part of his or her licensed frequency band. For example, if the maximum ITemp for primary user is y and s/he expects an ITemp of no more than *x* from their own operations in region **R** for the time period *T*, then the primary user can write an interference right of level (*y*-*x*) for this region and time period, with a guarantee that the primary user's ITemp will be no larger than *x*. This means that a secondary user could generate as much as (*y*-*x*) interference to the primary user. The secondary users can choose their own transmission parameters that correspond to the right and, under truthful operations, could expect to be suffer interference from the primary user of no more than *x*.

In selling the interference right, the primary user's capacity is diminished. We can estimate this at a high level using the Shannon-Hartley theorem, which states that the channel capacity C is a function of bandwidth B and signal-to-noise ratio S/N, specifically, $C=B \log_2 (1+S/N)$. When a carrier concludes that they can sell an interference right at the level (y-x), they are basically allowing an increase in N of the amount $(y-x)Bk$. The primary user basically trades capacity in the amount of $C=B \log_2 (1+S/(y-x)Bk)$. If the primary user's average signal power is expressed in terms of temperature (i.e., $s = S/Bk$), then the capacity traded is $C=B \log_2 (1+s/(y-x+n))$. For the secondary user, the acquired capacity becomes $C=B \log_2 (1+(y-x)/(s+n))$, because the primary user's signal acts as interference to the secondary user.

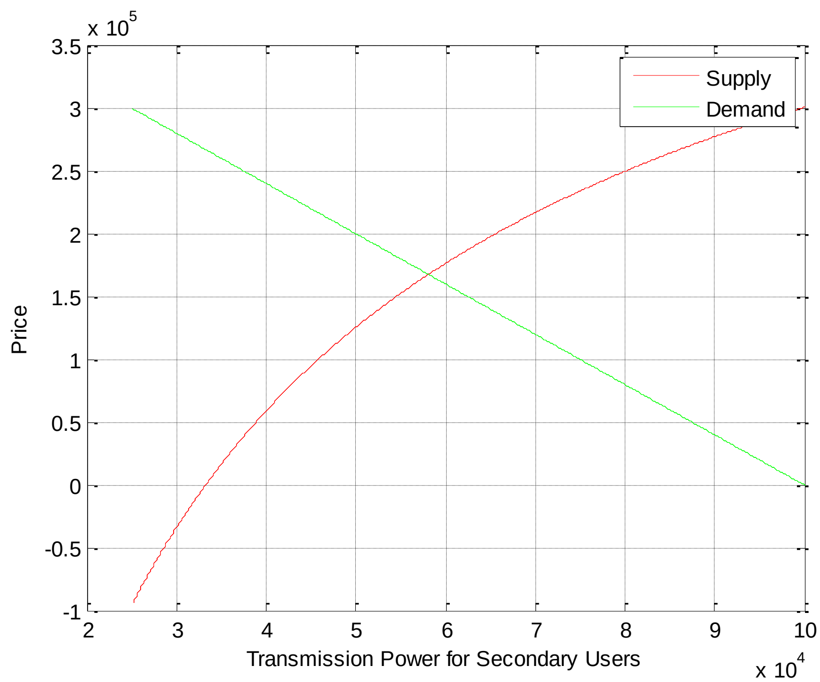
We consider two cases in this paper. In the first, the primary user deploys a directional antenna as shown in Fig.4. Among others, this situation models a scenario in the broadcast industry where the broadcaster (primary user) knows for certain that there are no over-the-air receivers in a certain sector of their service area. In this case, secondary users and primary users are not collocated and the primary user is willing to sell the entire spectrum in that sector. In the second case, we consider the primary user with an isotropic antenna as shown in Fig.1. The green circle is the protected region where primary user has service level agreement with subscribers. The second case is more complicated but more realistic, and shows the benefits of the interference right over an exclusive usage right. In the isotropic infrastructure, secondary subscribers could collocate with primary subscribers even inside primary users' protected region.

To perform a first-order analysis, we computed the capacity according to the Shannon-Hartley theorem for both primary and secondary users for each scenario. Figure 5 illustrates the results, with the y-axis representing the capacity in terms of number of users, and the x-axis is the Interference temperature that can be used by secondary users (in degrees Kelvin), which is (y-x). To simplify, we assume that both the primary and secondary users have only one transmitter each and that the noise floor for receivers is 2.4155×10^{20}K, which is equivalent to 50W. For the directional antenna system, we only analyze the capacity for secondary users shown in green line, since primary users' capacity does not change due to secondary users' operation. It is shown that with 0.5×10^{23}K (which equivalent to 10350W transmission power), secondary users can achieve 10^5 customers. For isotropic antenna system, we assume the worst case where the primary subscriber is collocated with secondary transmitter and secondary subscriber is inside the protected zone (since the secondary user know where is the primary transmitter during the negotiation with primary user, there is no need for secondary subscribers to be collocated with the primary transmitter). For simplicity, we use 0.6 as the path loss for the primary transmitter to secondary subscriber signals; as before, a more second order model could use a more sophisticated path loss model. The red line is the capacity for secondary users in the isotropic system and has a nearly linear increase with the Interference Temperature. The blue line is the capacity for the primary user in isotropic system. It decreases with the increase of the secondary's operation. The rate of decrease in capacity is largely due to the interference from secondary users, therefore if primary subscribers are not collocated with secondary users it decreases more slowly than the blue line shows.



We will elaborate on the isotropic case since it highlights the benefits of tradeable interference rights more clearly. Suppose the primary user has 3×10^4 subscribers in their service area, as shown in Fig.5 and the maximum ITemp desired by the secondary user is 4.83×10^{23}K, equivalent to 10^5Watt (100 KW). We further assume that the cost for primary user losing one subscriber is equivalent to the cost for secondary user to provide service to one subscriber, which is \$20. Using a linear demand function, we can write that the demand $D = -4(ITV)+4\times10^5$, and that the supply function is $S = 20\times(3\times10^4-15000\times\log2(1+10^5/(ITV+50)))$, where ITV is the ITemp value for secondary users to operate. For the demand function, we assume that secondary user's maximum expected capacity is 2×10^4, and the cost to provide service to one subscriber is \$20. The supply function reflects the cost of losing subscribers for the primary user. We plot the demand and supply according to secondary users' transmission power for the simplicity using Matlab, and the value from ITemp to Power is computed using IT =P/(Bk).

As indicated by the figure below, the primary user can sell 1.6×10^{23}K, equivalent to approximately 33 KW, to secondary users without diminishing her/his own capacity. In other words, the best strategy for primary user is to sell as much as they can up to 1.6×10^{23}K, and the revenue for spectrum selling will be a net economic gain (shown in the figure as negative price). It is also possible that a secondary user wishes to purchase additional interference rights (i.e., they would like to purchase a higher Interference Temperature for their QoS). In this circumstance, the primary user faces a tradeoff between economic gain from selling interference rights and the cost of losing subscribers (or paying penalties for violating SLAs). In order to cover this cost, the primary user needs to charge at least as much as the cost of degrading their subscribers' QoS. Primary and secondary users have the same expected value for spectrum, which is 1.7×10^5, at the transmission power of 6.7×10^4 Watt for secondary users where two lines meet. Thus, voluntary spectrum transactions *could* take place up to this point. After this point, secondary users would have to pay too much to make the service worthwhile, so, absent strategic necessities, no transactions would take place.



From this figure we can see that a primary user's expected price increases with secondary users' transmission power, since the more secondary user transmits, the more that will hurt primary users' capacity. However, the marginal cost for the primary user decreases with the increase of secondary users' transmission power, since the capacity doesn't decrease linearly with the power increase.

5. Conclusion

In this paper, we have presented the concept of interference rights as instruments analogous to covered financial options that may (but need not) be written by license holders. We illustrate this idea using Interference Temperature and perform a first order evaluation using a case study. As seen from our first-order analysis, the interference rights offer some advantages to exclusive usage rights from three perspectives. First, primary users can partially sell their spectrum depending on the tradeoff between losing customers and earning leasing fees. Second, regulators have less in the trading process, which reduces the transaction cost. Third, it improves the spectrum utilization since primary and secondary subscribers can collocate and diminished guard bands. This proof-of-concept evaluation indicates that additional research is warranted to further develop the concept and analyze its implications. More more sophisticated modelling, such as that found in [14], should be performed to understand this kind of bargaining more fully.

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