ASYMMETRIC INFORMATION IN AN EVOLUTIONARY FRAMEWORK

by

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ABSTRACT

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This dissertation consists of three theoretical chapters. In the first chapter, I study an evolutionary model with a finite population of boundedly rational agents, who do not have access to the same amount of information. Time is discrete, and in each period two agents are paired to play a 2×2 symmetric coordination game. Each player can cross paths with two kinds of opponents: Neighbors or Strangers. If a player faces a Neighbor, she can access some information about her opponents past plays, and plays using a myopic best-response. But if she faces a Stranger, she does not have access to any information, and therefore plays according to a case-based decision rule. I show that in the short run, segregated localities emerge, to finally disappear in the long run, in favor of the Pareto Efficient convention. The main contribution of this chapter is that I show that agents coordinate in an evolutionary framework on an efficient outcome, even when information is asymmetric, without assuming any pre-play communication or mobility of the agents.

In the second chapter (with Alexander Matros) we consider K finite populations of boundedly rational agents whose preferences and information differ. Each period agents are randomly paired to play some coordination games. We show that several special (fixed) agents lead the coordination. In a mistake-free environment, all connected fixed agents have to coordinate on the same strategy. In the long run, as the probability of mistakes goes to zero, all agents coordinate on the same strategy. The long-run outcome is unique, if all fixed agents belong to the same population.

The last chapter (with Alexander Matros) extends the study of survival of Altruism in a

public good game similar to the one by Eshel, Samuelson and Shaked [14] to other circulant graphs. We describe short run outcomes and find sufficient conditions for the survival of Altruism in the long run.

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PREFACE

This dissertation is the result of a series of personal and professional events that lead me to enter the graduate school at the University of Pittsburgh. This work would have never been started without the support of my master's degree professors, E. Girardin, J.B. Lesourd, A.Peyguin-Feyssole and V. Marimoutou who were the firsts to encourage me to do a PhD. I have also greatly beneficiated from the support of Frank Giarratani and Jean-Francois Richard who encouraged me and supported me in joining the PhD program at the University of Pittsburgh.

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1.0 NEIGHBORS VERSUS STRANGERS IN A SPATIAL INTERACTION MODEL

1.1 INTRODUCTION

Case-based theory, as developed by Gilboa and Schmeidler [16; 17], suggests that in many situations of choice under uncertainty, the very language of expected utility models is inappropriate. It therefore assumes that agents make choices based on their own past performances in similar circumstances. In an analogous manner, evolutionary game theorists develop models with boundedly rational agents who use past information of other agents to either play best-response, as in Young [34] and, Kandori, Mailath and Rob [25], or to imitate, as in Robson and Vega-Redondo [31], and Josephson and Matros [24]. If case-based theory accommodates perfectly environments with asymmetric information, this is not the case for the above myopic best-response models of evolution. Evolutionary models using imitation can also accommodate environments with asymmetric information. But when agents imitate strategies that have been proven to be successful in the past, they do not necessarily take into account the similarity that exists between past and current situations, and do not desire to reach any specific utility level. In this paper, I use the attractive features of the evolutionary model developed by Young [34] in an environment where agents access different amounts of information, and propose an alternative to the imitation rule, inspired by case-based theory, in order to comprehend the asymmetry in the information structure. The main contribution of this paper is that I show that agents coordinate in an evolutionary framework on an efficient outcome, even when information is asymmetric, without assuming any pre-play communication or mobility of the agents.

Consider an arbitrary population, and notice that an agent does not only interact with agents she esteems. One can select her friends, but this selection is rather unlikely for colleagues, family, or next-door neighbors. Nevertheless, these connections carry some information, and the amount of information to which each agent has access depends on the number of connections this agent has with the rest of the population. This natural dichotomy between agents one can access information from, and those one does not know anything about, offers some interesting and very intuitive grounds for developing a foundational environment with asymmetric information.

In particular, this paper studies an evolutionary model where time is discrete and in each period agents from a finite population are paired to play a 2×2 symmetric coordination game. This game presents some tension between Risk Dominance, as defined by Harsanyi and Selten [19], and Pareto Efficiency. Each agent can cross paths with one of two kinds of opponents: Neighbors or Strangers. The definition of a neighbor used in this paper differs from the previous literature in Evolutionary Game Theory, for example by Ellison [12], Blume [9], or Ely [13]. However, it is similar to the definition adopted by Bala and Goyal [1]. I define an agent as a Neighbor if I can access this agents information about past plays. But I cannot get any information from a Stranger. In order to facilitate the visualization of such connections, I use Bala and Goyal [2] terminology and define a directed link from an agent i to an agent j as an information flow from agent i to agent j. This information flow contains past strategies and corresponding payoffs, and whether these strategies and payoffs were obtained against a Neighbor or a Stranger. Therefore, an agent considers another as a Neighbor if there exists a directed link from the latter to the former. Also, when two agents consider each other as Neighbors, the link is said to be mutual (double-sided), as in Jackson and Wolinsky [22].

Furthermore, I assume that agents are endowed with two decision rules. This allows me to capture the idea that playing against familiar versus unfamiliar opponents should be distinct. When an agent is matched with one of her Neighbors, she may want to adjust her behavior based on what she thinks her opponent will do. This is because she can try to deduce what her opponent will do from what her opponent has done in the past, as in Young [34]. Therefore, she draws a sample of strategies from her opponents past plays against Neighbors and plays a best reply against it. But when an agent is matched with a Stranger, she cannot anticipate her opponents behavior. This is due to the absence of information about her opponents past plays. In the last situation, one may simply suggest that the agent should sample some strategy-payoff pairs from her Neighbors past plays, and imitate the strategy that gave the highest average payoff, as in Josephson and Matros [24]. Instead, I assume that she follows a case-based decision rule, where she uses her aspiration level as a benchmark to determine which strategy to play. More precisely, she first computes her aspiration level by averaging her past payoffs obtained against Strangers. Then, she samples from her Neighbors some strategy-payoff pairs obtained against Strangers and computes the average payoff of the sample. If it turns out that the sample average payoff is beyond her aspiration level, she plays the strategy that gave the highest average payoff in the sample she has drawn from her Neighbors. But if the sample average payoff is below her aspiration level, she refers to her own past plays and uses the strategy that gave her the highest average payoff against Strangers.

Given this environment, I show that in the short run, segregated localities similar to the segregated neighborhoods of Schelling [32] emerge. However, the segregation disappears in the long run in favor of a situation where all agents play the Pareto Efficient strategy. This outcome contrasts with the results obtained by Ellison [12], Young [34], and Kandori, Mailath and Rob [25], but is consistent with Matros [28], and Robson and Vega-Redondo [31]. It also shows that Pareto Efficiency can be reached without assuming any pre-play communication, as in Kim and Sobel [26], and Matsui [29], and without any mobility of the agents, as in Bhaskar and Vega-Redondo [7], Ely [13], Oechssler [30], and Blume and Temzelides [8] among others.

The formal description of the model and an example are presented in Section 1.2. Section 1.3 is devoted to the study of the asymptotic properties of the model. Finally, Section 1.4 concludes and discusses extensions of the model.

1.2 THE MODEL

1.2.1 Information Structure.

Consider a finite population of $2n$ agents. It is possible to represent the information structure within this population by a directed graph G, where the vertices represent the agents and the edges correspond to the information flows between agents. A directed graph is a pair of disjoint sets of vertices $V = (1, 2..., i, ..., 2n)$ and edges E, together with two maps $init : E \to F$ and $ter : E \to F$ assigning to every edge e an initial vertex $init(e)$ and a terminal vertex $ter(e)$. For each agent i, all vertices with an adjacent edge e to i such that $ter(e) = i$ are called i's Neighbors. Denote by N_i the set of is Neighbors. If agent $j \notin N_i$, agent j is considered a Stranger by agent i. More precisely, if agent i considers agent j as a Neighbor, agent i has access to all past plays of agent j, for some finite number of periods. But if agent j is a Stranger to agent i, agent i does not know anything about agent j. Note that it is possible that agent i considers agent j as a Neighbor, but agent j considers agent i a Stranger. An example for this kind of situation can be given by considering public figures whose information is widely spread.

For each agent, I adopt the following notation:

(i) $N_i = \{j \ s.t \ j \to i\}$ is the set of agent i's Neighbors

(ii) $R_i = \{j \ s.t \ i \to j\}$ is the set of agents who consider i as their Neighbor

(iii) $I_i = N_i \bigcap R_i = \{j \ s.t \ i \to j\}$

(iv) $S_i = \overline{N_i}$ is the set of Strangers for agent i

A strongly connected component of a directed graph is a maximal set of vertices such that for any two vertices, i and j, in the set, there is a directed path (a sequence of directed edges from i to j and from j to i.[1] Let L^x be a strongly connected component of G and refer to it as a locality (the subscript x is used to differentiate localities - L^x and L^y therefore refer to two distinct localities). Since every directed graph is a dag (a directed acyclic graph) of its strongly connected components, it is possible to decompose any directed graph into its strongly connected components. This decomposition will help describe the short run and

[1] A connected component is a maximal set of vertices such that for any two vertices, i and j, in the set, there is a path from i to j or from j to i.

long run outcomes of the model.

A strongly connected component can be source (no incoming edge), sink (no outgoing edge) or both. More formally:

Definition 1 *A strongly connected component* $V' \subseteq V$ *of a graph* $G = (V, E)$ *is said to be source if there is no* $e \in E$ *such that* $init(e) = i \in V \setminus\ V'$ *and* $ter(e) = j \in V \setminus\ V'$.

Therefore, if there exists a directed path from an agent $i \in L^x$ to an agent $j \notin L^x$, but no directed path from any agent $j \notin L^x$ to an agent $i \in L^x$, L^x is a source strongly connected component of G. In a similar way, a sink locality can be defined as follows:

Definition 2 *A strongly connected component* $V' \subseteq V$ *of a graph* $G = (V, E)$ *is said to be sink if there is no* $e \in E$ *such that* $init(e) = i \in V'$ *and* $ter(e) = j \in V \setminus\ V'$.

In this latter case, if there exists a directed path from an agent $j \notin L^x$ to an agent $i \in L^x$ but there is no path from any agent $i \in L^x$ to an agent $j \notin L^x$, L^x is a sink strongly connected component of G. It is of importance to note that these two types of strongly connected components or localities are not exclusive, and that a locality may well present both characteristics. If a locality is neither source nor sink, it is said to be disconnected. Finally, I define two connected localities as follows:

Definition 3 *Two localities* L^x *and* L^y *are said to be connected if for some* $i \in L^x$ *and* $j \in L^y$ *there exists an edge* $e \in E$ *such that either* $init\,(e) = i$ *and* $ter\,(e) = j$ *or* $init\,(e) = j$ *and* $ter\,(e) = i$.

Example. 1 *The diagram in Figure 1 illustrates the notions of sink and source localities.* $L^1 = 1$ *and* $L^3 = 8, 9, 12$ *are sink localities, whereas* $L^4 = 6, 7, 10, 11$ *is a source locality and* $L^2 = 2, 3, 4, 5$ *is both, sink and source. All these localities are connected to one another. Note that in order to simplify the picture, edges from an agent to herself have not been drawn, even though an agent is assumed to consider herself as a Neighbor.*

Finally, I impose two assumptions on the population structure:

Assumption. 1 $Card(N_i) > 1, \forall i$

Assumption. 2 *If there is more than one locality,* $\exists i \in L^x$ *and* $j \in L^y \neq L^x$ *such that* $j \in S(i)$*, but if there is only one locality, the condition becomes:* $\exists i \in E$ *such that* $S(i) \geq 1$.

The first condition stipulates that for each agent there exists a Neighbor other than the agent herself. The second condition insures that in each locality L^x there is an agent who is considered a Stranger by an agent in another locality. For the case where the information structure is connected (i.e. there is only one locality), the second assumption insures that there exists at least one agent who is considered a Stranger by another agent.

1.2.2 Dynamics of the Game.

Consider Γ the 2×2 symmetric coordination game in strategic form in Table 2. The set $X = \{x_1, x_2\}$ is the set of pure strategies available to all agents and $a, b, c, d \in \Re$, with $a > c, d > b$ and $a - c < d - b$. This implies that Γ contains a Pareto Efficient equilibrium (x_1, x_1) and a Risk Dominant equilibrium (x_2, x_2), as defined by Harsanyi and Selten [13]. Next, let $t = 1, 2...$ be successive time periods and denote by m the memory size of an agent. I assume that m is finite and identical across agents. A triple $\{Strategy, Payoff, Type\}_i^t$ refers to the play of agent i at time t. By play of agent i at time t, I mean the strategy and corresponding payoff obtained by agent i while playing in period t, and whether her opponent was a Neighbor or a Stranger. The variable $Type$ takes the value Ng if the opponent was a Neighbor, and St if the opponent was a Stranger. Therefore, each agent i is characterized at time t by her memory Q_i^t, which consists of her m most recent plays $Q_i^t = \left\{\{Strategy, Payoff, Type\}_i^{t-m+1}, ..., \{Strategy, Payoff, Type\}_i^t\right\}$. Finally, the finite history of length m at time t is denoted by the vector $H_t = (Q_1^t, ..., Q_m^t)$.

In each period agents are randomly paired to play a 2×2 symmetric coordination game. Given her opponent, each agent selects which decision rule to use as follows. If agent i is paired with one of her Neighbors, she draws a sample of size s of strategies previously played by her opponent against Neighbors and plays using the myopic best-response rule. If agent i is paired with a Stranger, she plays according to the following case-based decision rule. First, she evaluates her aspiration level by averaging her most recent payoffs against Strangers. Then, she samples from some of her Neighbors s plays which have the value St

in their third component. This means that the agent samples from her Neighbors some of their past experiences against Strangers. She then computes the sample average payoff and compares it to her aspiration level. If the sample average payoff is below her aspiration level, she prefers to trust her own experience, and plays the strategy that gave her the highest average payoff against Strangers. On the other hand, if the sample average payoff is beyond her aspiration level, she trusts her Neighbors experience more than her own, and plays the strategy that was the most successful on average in the sample she drew. If aspiration level and sample average payoff coincide, I assume that the agent can choose either mechanism described above with the same probability.

Some comments need to be made here regarding the best reply rule. The best reply rule described above takes into account the similarity of the situations: since the agent is playing against a Neighbor, she only samples past plays against Neighbors from her opponents memory. Nonetheless, it is also acceptable that a player chooses to play a best reply to a sample that has not been drawn conditional upon Type $= Ng$. This can happen if the opponent does not have any play against a Neighbor in her memory, or if the agent believes that her opponents frequency of strategies does not vary from Neighbor to Stranger. An example with the use of the first best reply rule will be given, but the use of either of these best reply rules is authorized, and does not impact the models predictions.

Further comments, this time considering the use of the case-based decision rule also need to be made. First, regarding the aspiration level. I assume that the level of utility one may expect against a Stranger can be calculated, by averaging only the payoffs obtained against Strangers. But it is perfectly possible that an agent never played against a Stranger, or that she simply wants to reach a utility level higher than her overall average payoff. In these two cases, the agent will therefore compute her aspiration level as the average of all past payoffs in her memory, unconditional on the value of the variable Type. Also, instead of choosing the strategy which gave the highest average payoff, it is perfectly possible that an agent chooses the strategy that gave the highest payoff. And here also, if an agent does not believe that only play against Strangers are relevant for choosing her strategy, she can draw her sample unconditional on the value of the variable Type. Finally, if an agent does not want to compute any aspiration level, and simply wants to imitate the strategy that gave the highest (average) payoff, it is perfectly acceptable and does not influence the results.

These comments show that the decision rules used by the agents can adapt their preferences regarding the importance they give to the similarity of the situation or the aspiration level presented in case-based theory. This flexibility underlines the robustness of the results, and allows the model to encompass a wider range of behaviors as long as the use of the decision rules involves the following: when paired with a Neighbor (respectively a Stranger), the agent uses one or a mixture of the best reply rules (respectively one or a mixture of the case-based decision rules or imitation rules)described above.

Since the main focus of this study is to find which strategy is chosen and by whom, I introduce the two following definitions in order to facilitate the description of the outcomes by focusing only on the strategies.

Definition 4 *A state at time t is defined by a vector $\Theta^t = \{A_1^t, A_2^t, ..., A_{2n}^t\}$, where A_i^t represents the vector of the last m strategies played by agent $i \in V$.*

Without loss of generality, let agents from 1 to $k \leq 2n$ belong to the locality L^x.

Definition 5 *A local state at time t within the locality L^x is defined by a vector $\theta^t = \{A_1^t, A_2^t, ..., A_k^t\}$, where A_i^t represents the vector of the last m strategies played by agent $i \in L_x$.*

Given a state Θ^t at time t, the process will move to a state Θ^{t+1} in the next period. For each $x_j \in X$, let $p_i(x_j|\Theta)$ be the probability that agent i chooses x_j in state Θ, for $j = 1, 2$ and some $i = 1, ...2n$. The assumptions imply that $p_i(x_j|\Theta)$ is independent of time and that $p_i(x_j|\Theta) > 0$ if and only if x_j is the strategy played by agent i, which is selected by a myopic best-response, and agent i faces a Neighbor, or x_j is the strategy inferred by agent i using a case-based decision rule described above, and agent i faces a Stranger.

If Θ' is a successor of Θ the transition probability becomes:

$$P^{m,s}(\Theta'|\Theta) = \Pi_i p_i(x_j|\Theta).$$

If Θ' is not a successor of Θ, then $P^{m,s}(\Theta'|\Theta) = 0$. This defines a Markov process on the finite space of states, which I denote by $P^{m,s}$, and refer to as selection play with memory m and sample size s.

Example. 2 *Consider the* 2×2 *symmetric coordination game in Table 3, and assume that the structure of the population is described by Figure 2.*

Now, suppose that agents are in period $t+3$*. They are assumed to have a finite memory of length 4 and be able to sample two plays from the history. Assume that the history* H^{t+3} *is given by Table 4.* [2]

Agents are now in period $t+4$ *and the matching is such that agents 1 and 2 are paired, and so are agents 3 and 4. This means that agents 1 and 2 will use a myopic best-response rule, whereas agents 3 and 4 will use a case-based decision rule.*[3] *If agent 1 samples what agent 2 did in periods* $t+2$ *and* $t+3$*, she plays strategy* x_2 *next period. Now if agent 2 samples agent 1s strategies in period* $t+1$ *and* $t+3$*, her best response will be to play* x_2 *next period.*[4] *Consider now agents 3 and 4. They consider each other as Strangers. Therefore, they both compute their aspiration level, i.e. their average payoff level from their past experience against Strangers. This gives* $\frac{20}{3}$ *for agent 3 and* $\frac{19}{3}$ *for agent 4. Agent 4 and agent 3 only Neighbor (other than themselves) is agent 1, who presents an average payoff of 10 while playing against Strangers. Since 10 is above the aspiration level of agents 3 and 4, they both imitate agent 1 and play strategy* x_1 *next period. Therefore, the finite history* H^{t+4} *is given by Table 5.*

This example illustrates the model mechanisms under the assumption that agents do not make mistakes. Nonetheless, it is more than possible that an agent makes the mistake of playing another strategy than the one assumed by the theory. It is also possible that one agent decides to experiment, and instead of playing the strategy that she computed, decides to play another strategy. Therefore, I define a perturbed version of the process as follows. In each period, there is some probability $\epsilon > 0$ that an agent uses a strategy at random. I assume that agents experiments are independent from one another. I then denote the perturbed process by $P^{m,s,\epsilon}$ and refer to it as selection play with memory m, sample size s, and experimentation probability ϵ.

[2] The indexes i and t of the triple $\{Strategy, Payoff, Type\}_i^t$ have been omitted in the table for clarity purposes.

[3] The myopic best-response and case-based decision rules illustrated here, are the ones presented first in the model.

[4] Agent 1 mixes with probability $\frac{1}{2} < \frac{7}{8}$, which makes strategy x_2 a best response for agent 2.

1.3 ASYMPTOTIC BEHAVIOR

1.3.1 Properties of the unperturbed process.

A recurrent class of the unperturbed process $P^{m,s}$ is defined as a set of states such that there is zero probability of moving from any state in the class to any state outside, and there is a positive probability of moving from any state in the class to any other state in the class. A singleton recurrent class is referred to as an absorbing state. Here are the definitions of some particular states:

Definition 6 *A convention Θ_A is a state of the form $\Theta = (A, ..., A)$ where A is a strategy vector of length m played by all agents $i \in V$ and $A = (x_1, ..., x_1)$ or $A = (x_2, ..., x_2)$.*

In a similar manner, I define a local convention as follows:

Definition 7 *A local convention θ_A within a locality L^x is a local state of the form $\theta = (A, ..., A)$ where A is a strategy vector of length m played by all agents $i \in L^x$, and $A = (x_1, ..., x_1)$ or $A = (x_2, ..., x_2)$.*

In the situation where a locality is disconnected, a local convention is the only possible absorbing state. Nonetheless, when localities are connected recurrent classes that are not singleton exist. In particular, source localities need to follow a local convention, whereas localities which are only sink can oscillate between states that are part of a recurrent class. And if a locality is both sink and source, it has to be in a convention to be part of an absorbing set. The intuition is as follows: if a locality L^x is source, it means that agents from other localities can directly sample from the memory of some agents who belong to L^x. Therefore, if the state within L^x is not a local convention, the state of the sink localities connected to L^x can be anything, contradicting the fact that the system is in an absorbing set. On the other hand, sink localities do not influence agents other than their own. Agents in an sink locality can access information from agents in other localities, but do not provide them with any information. Therefore, localities that are only sink do not play the regulation role that source localities need to play. This means that a locality which is only sink adopts a local convention only if all the source localities to which it is connected follow the same local convention. Otherwise, if a sink locality is influenced by many source localities that do not follow the same local convention, the sink locality's state evolves over time among states that are part of a recurrent class. The following theorem describes the set of absorbing states of our process:

Theorem 1 *For $s \leq \frac{m}{2}$, a set of states is a recurrent class if and only if the local state within all source and disconnected localities is a local convention.*

Proof. It is obvious that, if the local state within each source or disconnected locality is a local convention, the state of the process is a recurrent class. Therefore, I need to show that these recurrent classes are the only absorbing sets.

Let $\Theta = (A_1^t, A_2^t ..., A_{2n}^t)$ be an arbitrary state, and consider L^x, one of the source or disconnected localities. There is a positive probability that an agent within L^x, denoted agent i, is matched with the same Neighbor (given assumption 1 in the population) and samples the same string of strategies $(x^{t-s+1}, ..., x^t)$ in every period from $t+1$ to $t+s$ inclusive. Without loss of generality, assume that agent i plays a unique best response x_i^* , where $x_i^* = x_1$ or x_2. With positive probability, agent i can be matched with an agent who belongs to R_i, denoted agent j, and agent j samples the same string of strategies $(x_i^*, ...x_i^*)$ from period $t+s+1$ to $t+2s$ inclusive. Since agent j samples only x_i^*, she plays the best response to x_i^* which is x_i^* in every period from $t+s+1$ to $t+2s$ inclusive (this is possible since $s \leq \frac{m}{2}$. Therefore, agent i and agent j have a positive probability to have a memory that contains only the strategy x_i^*. Considering now an agent from R_j and repeating the same reasoning as the one described above, one can see that L^x has reached a local convention. This can be proven for any source or disconnected locality. **End of Proof.**

Example. 3 *Short run outcomes when the information structure considered is the one presented in figure 1.*

The only source localities are $L^2 = (2, 3, 4, 5)$ (which is also sink) and $L^4 = (6, 7, 10, 11)$. Therefore, there exist four possible short run outcomes: two absorbing states, and two absorbing sets. The first absorbing state is when all agents from L^2 and L^4 play strategy x_1, thus leading the whole population to follow a convention Θ_A where $A = (x_1, ..., x_1)$. Similarly, the second absorbing state is the convention Θ_A with $A = (x_2, ..., x_2)$. The third possible short *run outcome arises when all agents from L^2 play strategy x_1 and all agents from L^4 play play strategy x_2. In this case, agents from L^1 and L^3 can play either x_1 or x_2 depending on the samples they draw. Finally, the last possible short run outcome arises when all agents from L^2 play strategy x_2 and all agents from L^4 play play strategy x_1.*

Note that if all localities are disconnected, some information about successful or unsuccessful plays are never seen by their agents forbidding them to learn something new. Therefore, if none of them makes a mistake, they will always play the same strategy. Furthermore, if the local convention adopted in one disconnected locality differs from the one adopted in another, the population is partitioned into segregated localities similar to Schellings [32] segregated neighborhoods.

The learning mechanisms that arise from the unperturbed process, lead agents to conform to a certain uniformity in the way they play the game. But this uniformity is acquired more easily when localities are connected in both direction. Therefore, for arbitrary information structures which do not insure connectedness, the unperturbed process recognizes many possible states as possible outcomes, without being able to choose among them. In order to select the most probable outcome among this multiplicity of states, I introduce the possibility that agents make mistakes and study the behavior of the perturbed process as $\epsilon \to 0$.

1.3.2 Limiting distribution of the perturbed process.

By an argument similar to Young [34], the perturbed process $P^{m,s,\epsilon}$ is a regular perturbation of $P^{m,s}$ and hence it has a unique stationary distribution μ^ϵ satisfying the equation $\mu^\epsilon P^{m,s,\epsilon} = \mu^\epsilon$. Moreover, by theorem 4 in Young [34], $lim_{\epsilon\to 0}\ \mu^\epsilon = \mu^0$ exists and μ^0 is a stationary distribution of $P^{m,s}$. Similarly to Foster and Young [15], a state Θ is said to be stochastically stable relative to the process $P^{m,s,\epsilon}$ if $lim_{\epsilon\to 0}\ \mu^\epsilon = \mu^\epsilon(\Theta) > 0$.

Now, let Θ' be a successor of Θ, and consider a directed graph with one vertex for each absorbing state. Then, the resistance for any two states Θ and Θ' is the total number of mistakes involved in the transition $\Theta \to \Theta'$, if Θ' is a successor of Θ. Note that if Θ' is not a successor of Θ, $r(\Theta, \Theta') = \infty$. Furthermore, the stochastic potential ρ of an absorbing state Θ is defined as the minimum resistance of all trees rooted at Θ. The following lemma states that an absorbing state is stochastically stable if it has the least resistance.

Lemma 1 *The only stochastically stable states of $P^{m,s,\epsilon}$ are the absorbing states with minimum stochastic potential.*

Proof. This follows from theorem 1 and theorem 4 in Young [34]. **End of proof.**

Suppose first that the Pareto Efficient and the Risk Dominant equilibria are distinct. Suppose also that the number of localities is Z. One needs to consider rooted trees, where each vertex represents a state where $\lambda + Z$ of the Z localities respect the Pareto Efficient local convention, and $(1-\lambda)Z$ of them follow the Risk Dominant local convention, with $\lambda \in [0,1]$.

The introduction of the possibility for agents to make mistakes allows agents in sink or disconnected localities to access some information that differs from what they can sample within their locality. In particular, if a source or disconnected locality follows a local convention there is no way out unless agents make mistakes. The following theorem states the main result of this chapter: the Pareto Efficient convention is stochastically stable.

Theorem 2 *For $\frac{2(a-b-c+d)}{a-c} < s \leq \frac{m}{2}$, the unique stochastically stable state is the Pareto Efficient convention.*

Proof. See Appendix. **End of proof.**

This non-standard result in evolutionary game theory necessitates two major comments: one regarding the reason why Pareto Efficiency is reached and the second one on the disappearance of the segregated localities.

First, in evolutionary model using imitation, it is shown that imitation not only leads the agents to coordinate on the most efficient outcome, but imitation does it in a faster way than best reply does. Assuming that the population follows the Risk Dominant convention, one can see that 2 mistakes only are needed with imitation to go from one local convention to another within a locality. This is also the same number of mistakes that one needs to make with the use of the case-based decision rule. The reason why these numbers of mistakes coincide is as follows. When looking for the minimum number of mistakes agents should make to go from an absorbing state to another, it is sufficient to consider events which happen with positive probability, no matter how small this probability is. Therefore, consider a path from a Risk Dominant local convention to a Pareto Efficient one within a disconnected locality. Once one agent experiences a higher payoff than her Neighbors, her aspiration level necessarily increases, leading her to consider only her own experience when paired with a Stranger next period. Also she can be matched with a Stranger for many periods to come (there exists a positive probability for this event to happen). Consequently, the way she plays the game is similar to the fact that she imitates her most successful strategy on average. The aspiration level threshold in this case actually encourages her to do so, creating a strong similarity with the imitation rule.

The second aspect of the results that requires some comments is about the uniformity of the convergence and the disappearance of local conventions. With the use of the case-based decision rule, it is easier to convert a locality that respects the Risk Dominant local convention to Pareto efficiency than the opposite. Therefore, since stochastic stability requires the sum over all rooted trees of the number of mistakes needed to go from one state to the other, it is clear that converting every locality that is either disconnected or source from Risk Dominance to Pareto Efficiency, is easier than converting them from Pareto Efficiency to Risk Dominance. This explains why, in the long run, the stochastically stable outcome is the Pareto Efficient convention.

Finally, one word should be said about the assumptions on the population structure. If the graph $G = (V, E)$ is complete, i.e. assumption 2 is violated, the stochastically stable state is the Risk Dominant convention.

1.4 CONCLUSION AND EXTENSIONS

In this chapter I have shown that in the short run segregated localities can coexist, but that in the long run, they tend to disappear in favor of the Pareto Efficient convention. The efficiency of the outcome can be seen as a virtue of the asymmetric information structure. In particular, the existence of Strangers motivates the use of a case-based decision rule, which facilitates the propagation of the efficient outcome.

Given the analysis presented in this chapter, several possible extensions can be considered. First, what would be the consequence of having an endogenous network? Since the presence of Strangers in the population leads to the selection of the most favorable outcome, one should expect that agents will maintain the incompleteness of the network structure (they will not create links with every agent), but this is not obvious. The second question deals with the main assumptions and the results of this chapter. Would an experiment corroborate the assumptions regarding the selection of the decision rules and the theoretical predictions of the chapter? Finally, which type of absorbing sets and stochastically stable outcomes one can expect in a similar environment when a population is heterogeneous?

2.0 LOCATION, INFORMATION AND COORDINATION

2.1 INTRODUCTION

Numerous studies have shown the importance of social interactions and neighborhood effects in explaining phenomenon such as levels of education, income, production, crime, arising of gangs, ghettos and so on.[1] Therefore, the incorporation of social interactions on behavior is of primary interest. In particular, the influence that some agents exert on others can have a profound impact on the selection of economic outcomes.

In this chapter, we consider a coordination problem among heterogeneous agents. Our objective is to analyze the importance of information flows among agents and see whether or not heterogeneous agents are able to find a way to coordinate and on what outcomes. In every day life, social norms, national traditions, and focal points ease agents coordination problems. For example, when facing the choice of driving on the right or the left side of the road, an agent follows the social norm adopted in her country. Therefore, it is natural to assume that agents can use their neighbors and own past experience for guidance in future coordination.

We consider K finite populations of agents, each of which represents a group of agents who share common preferences. Each period, all agents are randomly paired to play some $K \times K$ coordination games. Each population prefers to coordinate on its preferred strategy. By preferred strategy we mean that when two agents from the same population are matched, a unique Pareto efficient outcome can be obtained if both agents coordinate on the so-called preferred strategy. But when the matching involves two agents from different populations and the two of them coordinate, several Pareto efficient outcomes can be reached. The

[1]See e.g Becker [3], Benabou [4], Glaeser and al. [18], Jankowski [23], Venkatesh [33] among many others.

information available to each agent differs from one agent to the other. In order to capture this asymmetry, we define a neighbor as an agent from whom one can sample information about past plays, whereas a stranger is an agent one does not have any information about. A convenient way to represent these relationships is as follows. A directed link from agent i to agent j is an information flow from agent i to agent j, as in Bala and Goyal [1; 2] and Masson [27]. It means that agent j considers agent i as a neighbor. If the link between agent i and agent j is mutual, as in Jackson and Wolinsky [22], it means that both agents consider each other as neighbors. Note that contrarily to what is commonly assumed in the literature on social interactions, we do not impose that neighbors share common preferences (are from the same population).

When an agent faces a neighbor, she can access her opponents past plays and payoffs. Therefore, we assume that she samples some of her opponents past plays and uses a myopic best response to this sample that she considers as her opponents strategy distribution for the current period. This approach is common in the literature and can be found in Kandori, Mailath, and Rob [25] and Young [34; 35] among many others. However, the situation differs when an agent is matched with a stranger. In that case, the agent does not have any information about her opponent. The only information at her disposal comes from a sample of her neighbors past plays. In this case, we assume that the decision rule she uses satisfies the following requirements: the decision rule can either select a strategy present in the sample or a best response against some subsample.[2] Our motivation for this approach is intuitive: if an agent does not know anything about her current opponent, she may believe that one of her neighbors could have played against her current opponent in the past. Therefore, any past experience from her neighbors could indeed carry some useful information. Notice that any imitation rule and any belief-based rule satisfy our requirement.

The main question we answer in this chapter is how coordination arises among agents with different preferences when the game they play is what Schelling calls "a mixture of mutual dependence and conflict of partnership and competition." [3] Our model encompasses many situations. For example, consider a Battle of the Sexes game where men and women

[2]Note that the or is not exclusive. The strategy present in the sample can also be a best response against some subsample

[3]See Schelling [32]

from different finite populations are paired to play each period. Note that any two agents can be matched each period. A question iswhether men and women coordinate on the same strategy? Another example where agents might find it difficult to coordinate is when they are faced with the choice of a product. In the academic world, for example, where coauthoring makes it critical to coordinate, which of SWP[4] or LaTex should be used for the first draft of a paper. One might prefer LaTex but accommodate SWPs coauthors and the question is: why? What is more important for coordination: the proportion of agents sharing similar preferences, or the information agents can access?

Our main result shows that there exist some special agents leading the coordination. Similar agents and their impact on the social behavior of others have been observed by Glaeser and al. [18] in a study on crime. They called them Fixed agents. Bala and Goyal [1] also studied these peculiar agents, and called them the members of the royal family.

Our first result shows that connected fixed agents have to coordinate on the same strategy choice in the short run. This prediction is independent from the decision rule agents use when they are matched with a stranger and might lead to the existence of segregated neighborhoods where agents from the same neighborhood play the same strategy.[5] This result implies that for complete, directed star, wheel or double sided links information structures, all agents play the same strategy in the short run. If we assume that there is a small probability that agents can make their choices at random, we obtain a sharp prediction as this probability goes to zero. In particular, if all agents use the following imitation rule: imitate the strategy which gives the highest payoff in the sample, all agents coordinate on the same strategy in this noisy environment.[6] Moreover, if all fixed agents are from the same population (members of the same royal family), the long-run (noisy) outcome is unique: all agents coordinate on the preferred strategy of the royal family. Our assumption that imitation is a sensible rule to use when access to information is limited is supported by experimental evidence (see Huck and al., [21]) and Hayek's theory of cultural evolution.[7]. Theoretical analysis and impact of

[4]Scientific WorkPlace.

[5]See Schelling [32] for the first discussions about segregated neighborhoods.

[6]We modify the imitation rule to take into account heterogeneity among agents. We assume that an agent imitates a strategy which gives the highest payoff to an agent from the same population. This is always possible since an agent can imitate herself.

[7]See Hayek [20]

imitation rules are presented by Robson and Vega-Redondo [31] and Josephson and Matros [24].

The chapter is organized as follows. A detailed description of the model is given in Section 2.2. Short run predictions are presented in Section 2.3. Section 2.4 describes the long-run outcomes and Section 2.5 concludes.

2.2 THE MODEL

2.2.1 Heterogeneous Populations and Payoffs Matrices.

Suppose that there exist K finite populations with $n_k \geq 1$ agents in population k, such that $n_1 + ... + n_K = 2n$, for $k = 1, ..., K$. Time is discrete, and in each period, all agents are randomly paired to play some $K \times K$ coordination games. Each agent from population k faces the payoff matrix in table 6.

Payoffs $a^k_{hh} > a^k_{lh}$ for all $h, k, l = 1, ..., K, h \neq l$; and $a^k_{kk} > a^k_{ll}$ for any $k \neq l$. The first condition $a^k_{hh} > a^k_{lh}$ insures that agents favor coordination. The second condition $a^k_{kk} > a^k_{ll}$ stipulates that an agent from population k prefers to coordinate on strategy k. Therefore, an agent plays different coordination games with agents from different populations. In particular, each population k has a preferred strategy k that leads to a unique Pareto Efficient outcome when two agents from this population are matched. This is the case when two men are matched in the Battle of the Sexes game. In this situation, both men have a preferred strategy which is to watch a football match. On the other hand, when two agents from different populations are matched, there are at least two Pareto Efficient outcomes. This corresponds to the case where a woman and a man are matched in the Battle of the Sexes Game.

Denote a pure strategy $x \in \{1, ..., K\}$ of an agent from population k by a vector $x = (0, ..., 0, 1, 0, ..., 0)$. Suppose that an agent from population k is matched with an agent from population l. If the agent from population k plays strategy $x \in \{1, ..., K\}$ and the agent from population l plays strategy $y \in \{1, ..., K\}$, then the two agents obtain the following payoffs

$$\pi_k(x,y) = \pi_k(x,y) = xA^k y^T$$

and

$$\pi_l(y,x) = \pi_l(y,x) = yA^l x^T$$

The following example illustrates this point.

Example. 4 ***Battle of the Sexes*** *Suppose that there are two populations: men and women. In each period, all agents are randomly paired to play some* 2×2 *coordination games. Each agent from the mens population has the payoff matrix in Table 7, and each agent from the womens population has the payoff matrix in Table 8. Let us call the first strategy, (1, 0), watch a Football match, and the second strategy, (0, 1), watch a Movie. Agents can be matched in three ways: man and man, woman and woman, and man and woman. If two men are matched, they play the coordination game in Table 9. If two women are matched, they play the coordination game in Table 10, and if a man is matched with a women, they play the coordination game in Table 11.*

This example illustrates how the populations heterogeneity is introduced into the payoffs matrices. We now focus on the information structure and adopt some necessary terminology for the description of its asymmetry.

2.2.2 Information Structure.

At time t, each agent from population i chooses a strategy x_i^t from the set $X = \{1, ..., K\}$ according to some behavioral rules (described below) based on past plays available information. Therefore, the play at time t can be defined as $x^t = (x_1^t, x_2^t, ..., x_{2n}^t)$, and the history of plays up to time t can be represented by the sequence $h^t = (x^{t-m+1}, ..., x^t)$ of the last m plays.

Define the information structure $\langle \nu, \tau \langle$, where ν is the set of agents and τ is the set of information links. Our information structure is similar to the information structure in Bala and Goyal [1] and Masson [27]. We call agents from whom agent q can access past plays neighbors of agent q, and any other agent a stranger to agent q. Denote by $Nb(q)$ the set of neighbors of agent q; $St(q)$ the set of strangers of agent q; and $A(q)$ the set of agents who can access agent qs past plays. Note that agents in $Nb(q)$ do not need to share common preferences.

The dichotomy neighbor/stranger can be represented by a directed graph (the information structure $\langle \nu, \tau \langle$ where a directed link (flow of information) from agent q to agent g, $\{q \rightarrow g\} \in \tau$, means that agent g can access information about past plays of agent q, or $q \in Nb(g)$ and $g \in A(q)$. We assume that $q \in Nb(q)$ for any q. The following example illustrates the above definitions.

Example. 5 *Suppose that the information structure of the Battle of the Sexes game is as in Figure 3, where agents 1 and 2 are men, and agents 3 and 4 are women. For each agent* $i = 1, 2, 3, 4$ *we can define a set of neighbors,* $Nb(i)$*; a set of agents who can access agent is information about past plays,* $A(i)$*; and a set of strangers,* $St(i)$*:*

$$Nb(1) = \{1\}, \ (1) A = \{2\}, \ St(1) = \{2, 3, 4\};$$

$$Nb(2) = \{1, 2, 3\}, \ A(2) = \{3\}, \ St(2) = \{4\};$$

$$Nb(3) = \{2, 3, 4\}, \ A(3) = \{2\}, \ St(3) = \{1\};$$

$$Nb(4) = \{4\}, \ A(4) = \{3\}, \ St(4) = \{1, 2, 3\}.$$

In the next subsection, we describe the decision rules agents are assumed to follow.

2.2.3 Decision Rules.

It is important to note that each agent has her own preferences (belongs to a particular population) as well as her own amount of information (from a set of neighbors). Any two agents can be paired in each round and each agent chooses her strategy as follows. Fix integers s and m, where $1 \leq s \leq m$. At time $t+1$, each agent q inspects a sample $(x^{t_1}, ..., x^{t_s})$ of size s taken without replacement from her neighbors history of size m of plays up to time t, where $t_1, ..., t_s \in \{t - m + 1, t - m + 2, ..., t\}$. We assume that samples are drawn independently across agents and time.

If an agents is matched with one of her neighbors, she has information about her opponent past plays and plays the best reply against the opponents strategy distribution in the sample.

This approach is intuitive: agents are boundedly rational and expect the play of the game to be almost stationary. See Kandori, Mailath, and Rob [25] and Young [34; 35] for discussions.

However, the situation is different if an agent is matched with a stranger. Since no information is available about the opponent, the agent has to select a strategy based on the available information about her neighbors. We assume that the decision rule the agent uses in order to select a strategy satisfies the following requirement: it can either select a strategy present in the sample or a best response against some subsample. Our motivation for this approach is intuitive: if an agent does not know anything about her current opponent, she believes that one of her neighbors could have played her current opponent in the past. Therefore, it is plausible to believe that she considers her neighbors information as valuable for her current play. Note that any imitation rule and any belief-based rule satisfy our requirement.

2.2.4 Markov Process.

Let the sampling process begin in period $t = m + 1$ from some arbitrary initial sequence of m plays, h^m. We define a finite Markov chain (call it $B^0 \equiv B^{\nu,\tau,m,s,0}$) on the state space $[(X)^{2n}]^m = H$ (of sequences of length m drawn from the strategy space X), with the information structure $\langle \nu, \tau \langle$ and an arbitrary initial state h^m. The process B^0 moves from the current state h to a successor state h' in each period, according to the following transition rule. For each $x_i \in X$, let $p_i\,(x_i|h)$ be the conditional probability that agent i chooses x_i, given that the current state is h. We assume that $p_i(x_i|h)$ is independent from t.

The perturbed version of the above process can be described as follows. In each period, there is a small probability $\epsilon > 0$ that any agent experiments by choosing a random strategy from X instead of applying the described rule. The event that one agent experiments is assumed to be independent from the event that another agent experiments. The resulting perturbed process is denoted by $B^\epsilon \equiv B^{\nu,\tau,m,s,\epsilon}$. As we will see below, the resulting process B^ϵ is ergodic, making the initial state irrelevant in the long run.

2.3 SHORT RUN

In order to characterize the short-run outcomes of the model, we first need to adopt some terminology.

Definition 8 *The information structure* $\langle \nu, \tau \langle$ *is connected if for any two agents* $q, g \in \nu$ *there exists a sequence of agents* $f_1, ..., f_k \in \nu$ *such that* $\{f_m - f_{m+1}\} \in \tau$, $m = 1, ..., k-1$, *where link* $-$ *is either link* $\rightarrow$, *or link* $\leftarrow$, *or both; and* $q = f_1$ *and* $g = f_k$.

We assume that the information structure is connected for the remainder of the chapter. Next, we define some special agents, called fixed agents, who have the ability to influence other agents. Formally,

Definition 9 $q*$ *is a fixed agent, if*
(i) $\{q*\} = Nb\,(q*)$, *or*
(ii) for any $g \in Nb\,(q*)$ *, there exists a sequence of agents* $g_1, ..., g_k$
such that $\{g_m \rightarrow g_{m+1}\} \in \tau,\ m = 1, ..., k-1,$ *and* $g_1 = q*$ *and* $g_k = g$.

Denote by $F \subseteq \nu$ the set of fixed agents. Fixed agents propagate information within the information structure. For example, agents 1 and 4 are fixed agents and agents 2 and 3 are not in Figure 1. The following Lemma establishes the existence of fixed agents.

Lemma 2 *Each information structure has a fixed agent.*

Proof. Since each information structure is finite, the claim follows immediately. **End of proof.**

Some information structures can have several fixed agents who might share information. Therefore,

Definition 10 *Two fixed agents* $q* \in F$ *and* $g* \in F$ *are connected, if there exists a sequence of agents* $q_1, ..., q_k$ *such that* $\{q_l \rightarrow q_{l+1}\} \in \tau,\ l = 1, ..., k-1,$ *and* $q_1 = q*$ *and* $q_k = g*$.

It is obvious that each agent in the sequence of agents $q_1, ..., q_k$ is a fixed agent too. The following example illustrates the definition.

Example. 6 *Suppose that the information structure in the Battle of the Sexes game is as in Figure 4. Agents 2 and 3 are two connected fixed agents in Figure 2.*

It is important to note that there may exist many distinct groups of connected fixed agents within the information structure. There are finitely many, $1 \leq L < \infty$, disjoint groups of connected fixed agents. We denote such groups of connected fixed agents by $F_1, ..., F_L$, where $F_i \cap F_j = \emptyset$, for any $i \neq j$, and $F_1 \cup ... \cup F_L = F$.

2.3.1 General Information Structures.

Whereas the previous definitions were focused on the information structure, the following definition describes the important states of the Markov process B^0.

Definition 11 *A partial convention, $h_{y_1,...,y_L}$, is a set of states where all connected fixed agents in F_j played strategy y_j for the last m periods in each state of the set $h_{y_1,...,y_L}$.*

Note that agents from different groups of connected fixed agents could play different strategies in a partial convention, thus leading the non-fixed agents to play different strategies. The following example illustrates how the strategies of non-fixed agents can vary in partial conventions.

Example. 7 *Consider the Battle of the Sexes game with the information structure from Figure 1. We assume that $m = 2,\ s = 1$. Suppose that agents 1 and 2 are from the mens population, and agents 3 and 4 are from the womens population. Agents are matched at random to play the 2×2 coordination games described in section 2.1. Note that agents 1 and 4 are fixed agents, but they are not connected. In a partial convention, each fixed agent played the same strategy both times in the past. If both fixed agents coordinated on strategy A, then the following set of states is a partial convention:*

$$h_{A,A} = \{((A, A), (w, x), (y, z), (A, A))\},$$

where $w, x, y, z \in \{A, B\}$ and the first bracket represents the strategy choices of agent 1 in the last two periods, the second bracket shows the strategy choices of agent 2 in the last two periods, and so on. If both fixed agents coordinate on strategy B, then the following set of states is a partial convention:

$$h_{B,B} = \{((B,B), (w,x), (y,z), (B,B))\},$$

where $w, x, y, z \in \{A, B\}$.

However, fixed agents who are not connected do not need to coordinate in a partial convention. For example, agent 1 could play strategy A in the last two periods, and agent 4 could play strategy B in the last two periods. The following set of states is therefore also a partial convention:

$$h_{A,B} = \{((A,A), (w,x), (y,z), (B,B))\},$$

where $w, x, y, z \in \{A, B\}$. *Analogously, if agent 1 played strategy B in the last two periods, and agent 4 played strategy A in the last two periods, the following set of states is also a partial convention:*

$$h_{B,A} = \{((B,B), (w,x), (y,z), (A,A))\},$$

where $w, x, y, z \in \{A, B\}$.

Note that non-fixed agents 2 and 3 can switch from strategy A to strategy B (and vice versa) in any partial convention. Note also that partial conventions contain either exactly one absorbing state or exactly one absorbing set of the unperturbed process B^0. *In our example, the partial convention* $h_{A,A}$ *(resp.*$h_{B,B}$*) contains one absorbing state where all agents play strategy A (resp. B). And the partial convention* $h_{A,B}$ *(resp.* $h_{B,A}$*) contains one absorbing set where non-fixed agents 2 and 3 can play either A or B. In this latter case, the partial convention and the absorbing set do coincide. But this does not have to be the case. In a similar example, where agents could be of three types A, B and C the partial convention* $h_{A,B}$ *(resp.* $h_{B,A}$*) allows non-fixed agents 2 and 3 to play anything from A, B or C, whereas the absorbing set contained in* $h_{A,B}$ *(resp.* $h_{B,A}$*) is identical to the one described above.*

We are now able to characterize all absorbing sets and states - short-run outcomes. An absorbing set of the unperturbed process B^0 is a minimal set of states such that there is zero probability for the process B^0 of moving from any state in the set to any state outside, and there is a positive probability for the process B^0 of moving from any state in the set to any other state in the set. A singleton absorbing set is called an absorbing state.

Definition 12 *A uniform convention,* $h_x = h_{x,\dots,x}$, *is a state where all agents play the same* *strategy x.*

Note that there exist K uniform conventions.

Proposition. 1 *Each uniform convention is an absorbing state.*

Proof. It is evident. **End of proof.**

Proposition. 2 *Each partial convention contains a unique absorbing state or set.*

Proof. Note that each agent in a group of connected fixed agents has to coordinate on the same strategy for the last m periods in any absorbing set. Any partial convention has this property. **End of proof.**

Proposition. 3 *If all fixed agents are coordinated on the same strategy x in a partial convention, then this partial convention contains the absorbing state, uniform convention, h_x.*

Proof. It is evident. **End of proof.**

We now take a closer look at several information structures that have been of particular interest in the network literature.

2.3.2 Common Information Structures.

If the information structure is such that all agents are fixed, or there is just one fixed agent, then the short-run prediction is a state. There are several common information structures where all agents are fixed and connected. For example, (1) if all directed edges are double-sided (the information goes in both directions), (2) if the information structure is a directed wheel, or (3) if the information structure is complete.

Definition 13 *The connected information structure $\langle \nu, \tau \rangle$ i is a directed wheel, if any agent $q \in \nu$ has exactly one agent who can access agent q's past plays, $\|A(q)k\| = 1$.*

Definition 14 *The information structure $\langle \nu, \tau \rangle$ is complete, if for any two agents $q, g \in \nu$*

$$\{q \to g\}\,,\ \{g \to q\} \in \tau$$

Similarly, there are several information structures with just one fixed agent. For example, (4) a directed star, or (5) a directed chain.

Definition 15 *The connected information structure* $\langle \nu, \tau \rangle$ *is a directed star, if there exists an agent* $q \in \nu$ *such that every other agent* $g \neq q$ *has exactly two neighbors,* $Nb(g) = \{g, q\}$, *and* $Nb(q) = \{q\}$.

Definition 16 *The connected information structure* $\langle \nu, \tau \rangle$ *is a directed chain, if there exists an agent* $q \in \nu$ *such that every other agent* $g \neq q$ *has exactly two neighbors,* $\|Nb(g)\| = 2$, *and* $Nb(q) = \{q\}$.

The following example illustrates the last definition.

Example. 8 *Suppose that the information structure is as in Figure 5. Agent 2 is the fixed agent.*

The following proposition gives the short-run outcomes for some common information structures.

Proposition. 4 *Suppose that one of the following conditions is true*
(i) *all edges are double-sided*
(ii) *the information structure is a directed wheel*
(iii) *the information structure is a complete*
(iv) *the information structure is a directed star*
(v) *the information structure is a directed chain*
Then the Markov process B^0 *converges with probability one to a uniform convention.*

Proof. Note that when the first, second, or third condition holds, all agents are connected fixed agents. If the forth, or fifth condition holds, there is a unique fixed agent. The statement follows from Proposition 3. **End of proof.**

2.4 LONG RUN

Many short-run outcomes are candidates for the long-run outcomes when agents can make mistakes. As usual in the literature, we will describe properties of the unique stationary distribution of the perturbed Markov process $B^{epsilon}$. The main result of this section is that only K states can be the long-run outcomes. These are the uniform conventions.

We use the following technical definitions.

Definition 17 ***Stochastic Stability:*** *A state $h \in H$ is stochastically stable relative to the process B^{ϵ} if $lim_{\epsilon \to 0}\ \mu_h^{\epsilon} > 0$ where μ_h^{ϵ} is the unique stationary distribution of the process B^{ϵ}.*

Definition 18 ***Resistance:*** *For any two states h, h' the resistance $r(h, h')$ is the total number of mistakes involved in the transition from h to h', if $h'$is a successor of h; otherwise $r(h, h') = \infty$.*

A directed tree is a directed graph (V, E). The vertices, V , represent all possible absorbing sets and the edges, E, represent the transition from one absorbing set to the other. Each edge is assigned a weight which is equal to the corresponding resistance. The resistance of such a directed tree is equal to the sum of the resistances of its edges.

The stochastic potential ρ of an absorbing set is the minimum resistance of the tree rooted at this absorbing set. We will use the following well-known result.

Theorem 3 *The only stochastically stable sets of the perturbed Markov process B^{ϵ} are the absorbing sets with minimum stochastic potential.*

We can now describe the long-run outcomes.

Theorem 4 *For any absorbing set $h \in H$ different from a uniform convention, there exists a sample size $s*$ and a uniform convention $h_x \in H$, such that for any $s \geq s* > m/2 \rho(h) > \rho(h_x)$.*

Proof. First, note that all connected fixed agents have to coordinate on the same strategy in any absorbing set. Second, if all fixed agents coordinate on the same strategy, the absorbing set is a uniform convention from Proposition 3. Third, it takes at least two mistakes to leave any uniform convention. It is enough to show now that it takes just one mistake to leave an absorbing set different from a uniform convention. It becomes obvious once we see how to leave such a set where all but one (connected) fixed agent(s) coordinate on one strategy.

Consider an absorbing set $h \in H$ different from a uniform convention. It must be the case that at least two strategies are played by the fixed agents. Find a group of connected agents, F_i, who are playing the most popular (among fixed agent) strategy, $\{x\}$. Consider another set of fixed agents, F_j, who are playing another strategy, y. There is a positive probability that all connected fixed agents in this set, F_j , are matched with the fixed agents who play strategy x. Suppose that this matching takes place for $s-1$ periods. As a result, all connected fixed agents in F_j have $s-1$ miscoordinated plays. Suppose that one of the fixed agents in F_j, agent q, makes a mistake and coordinates on the strategy x. There is a positive probability that she and every member from the set $A(q)$ sample her last s plays for the next s periods. Suppose that all connected fixed agents from $A(q)$ and agent q are matched with fixed agents they consider strangers. This will lead to another absorbing set where more fixed agents play strategy x. Continuing in this way, we can see that it takes one mistake to move from one absorbing set to the next until we reach the uniform convention h_x. Hence, $\rho(h) > \rho(h_x)$. **End of proof.**

In a partial convention, connected fixed agents from the same group coordinate on the same strategy. So, if not all fixed agents follow the same strategy and two fixed agents from different groups who play different strategies are matched, only one mistake is needed to have them both coordinating on the same strategy. Therefore, a partial convention is never as stable as a uniform convention, since in order to leave a uniform convention, two mistakes are needed.

From Theorem 4 it follows that a uniform coordination must be reached in the long-run. But a sharper prediction can be obtained, if we assume that agents imitate the most successful play in their samples when they are matched with strangers. Call this Markov process BI^{ϵ}.

Theorem 5 *Suppose that all fixed agents belong to the same population i and there exist at least two distinct groups of fixed agents. Then there exists a sample size $s*$, such that for any $s > s*$, the perturbed process $BI^{epsilon}$ converges with probability one to a uniform convention $h_{i...i}$.*

Proof. We have to select the long-run outcome among different uniform conventions from Theorem 4. Since all fixed agents are from the same population i, they obtain the highest payoff if they coordinate on the strategy i. The imitation rule drives the selection in favor of the uniform convention $h_{i...i}$. Similar results were obtained in Robson and Vega-Redondo [31] and Josephson and Matros [24].

Consider a uniform convention $h_{j...j}$, $jneqi$. It takes just two mistakes in order to leave a uniform convention $h_{j...j}$: two matched fixed agents from different groups switch from strategy j to strategy i. It is obvious that it requires more than two mistakes to leave the uniform convention $h_{i...i}$. **End of proof.**

The intuition for this result is as follows. Theorem 4 insures that only some uniform conventions can be the long-run outcome. Therefore, if all fixed agents share the same preferences (from the same population i), but coordinate on strategy $j \neq i$ in the uniform convention $h_{j...j}$, it is easy to see that only two mistakes are needed to leave the uniform convention $h_{j...j}$. More precisely, two matched fixed agents from distinct groups play strategy i by mistake and obtain the highest possible payoff. Since agents use the imitation rule when matched with strangers, any other fixed agent who can access this information will also play strategy i. Hence, the uniform convention $h_{i...i}$ is the most stochastically stable.

Example. 9 *Consider the Battle of the Sexes game from the previous section where the information structure is given by Figure 3. There are only two possible long-run outcomes: all four players coordinate on a unique strategy, i.e. either all of them go to a soccer match or all of them go to an opera. We have a unique long-run prediction only if both fixed agents are from the same population.*

Some information structures have a unique fixed agent. This fixed agents population determines the long-run outcomes: a uniform convention where all agents play the fixed agents preferred strategy.

Corollary 1 *Suppose that the information structure $\langle \nu, \tau \rangle$ is either a directed star, or a directed chain and the unique fixed agent is from population i. Then, there exists a sample size $s*$, such that for any $s \geq s*$, the perturbed process BI^{ϵ} converges with probability one to a uniform convention h_i.*

Proof. Note that any directed star or/and any directed chain have just one fixed agent.

The corollary now follows from Theorem 5. **End of proof.**

In this chapter we consider a population of heterogeneous agents who are randomly paired each period to play some coordination games. We show that the short-run predictions (mistake-free environment) depend on the information structure: connected fixed agents have to coordinate on the same strategy. Whereas in the long run, all agents coordinate on the same strategy. Moreover, the long-run prediction is unique when all fixed agents belong to the same population.

It is interesting to see that despite the divergence of preferences, agents can agree uniformly on one particular strategy in the long run. This coordination is obtained through the fixed agents highlighted by Glaeser, Sacerdote, and Scheinkman [18] in a study on criminal behavior.

The most important result of the chapter is the fact that sharing common preferences with a majority of others does not insure a favorable outcome. What matters most is not the number of agents in a population, but rather their quality. By spreading widely their information, fixed agents insure that their preferences are observed, directly or indirectly, by everyone and therefore influence the plays of others. This is the reason why an agents location (her population) is less important than her information. This also explains why a minority can sometimes impose its preferences.

3.0 THE SURVIVAL OF ALTRUISTIC ACTIONS IN PUBLIC GOOD GAMES

3.1 INTRODUCTION

One of the recurrent questions in economic literature concerns the existence and survival of Altruism. In a survey on individual and group selection theory, Bergstrom [5] points out that, in models with assortative matchings induced by a spatial structure, Altruism can only be sustained when the environment considered exhibits some particular characteristics. More precisely, agents interact locally so that the benefits of the game are only local. Furthermore, an agent chooses her strategy by imitating her most successful neighbors. These features were indeed used by Bergstrom and Stark [6], Eshel, Samuelson and Shaked [14] and Matros [28] in their pursue to prove that Altruism can survive. In particular, the type of population structure they considered is similar (a circle). It is interesting to note that this kind of structure belongs to a broader class of graphs called circulant graphs. This is why we decided to investigate the survival of Altruism in more general circulant graphs, which encompass those previously studied in the literature.

In this chapter, we do consider the imitation rule, but broaden the possible interactions among agents. More precisely, we extend the study of survival of altruism in a public good game similar to the one by Eshel, Samuelson and Shaked [14] to other circulant graphs. We adopt some assumptions which allow us to describe all possible short run outcomes and insure the survival of Altruism in the long run. We show in particular, that a condition on the number of links and the number of agents in the population is not enough to insure the survival of Altruism, contrarily to what Matros [28] found.

In their paper, Bergstrom and Stark [6]consider a population of farmers located on a road that loops around a lake. Each farmer plays a prisoners dilemma game with his two closest neighbors and chooses his strategy by imitating the most successful strategy within his neighborhood. It turns out that a stable outcome, one in which no one wants to revise her strategy, can be obtained only if cooperators (Altruists) are grouped in clusters of two or more.

Eshel, Samuelson and Shaked [14] consider an evolutionary version of a public good game which benefits are only local. In their paper, they study a finite population of agents placed on a circle. An agents neighbors are the nearest agent to her right, and the nearest agent to her left. They show that Altruism can survive within this network structure if Altruist are grouped together. Matros [28] generalized their paper by considering neighborhoods with a larger radius. In his paper, an agents neighbors are the k nearest agents to her right and the k nearest agents to her left. He showed that under some conditions between the number of links and the number of agents in the population, Altruism survives.

In this chapter, we consider a finite population of agents whose interactions are modeled by some undirected circulant Euler graph. Each period, agents need to decide whether to be Altruist or Egoist. If an agents decides to be an Altruist, she provides one unit of utility to all her neighbors and bears a certain cost for it. On the other hand, if she chooses to be an Egoist, she simply can enjoy the Altruism of her neighbors at no cost, provided that some of them are Altruists. We show that given some conditions on the number of common neighbors any two neighbors (and any two non-neighbors (strangers)) have, Altruism can survive.

The chapter is organized as follows: Section 3.2 introduces the model. Short run outcomes are presented in Section 3.3. Section 3.4 is devoted to the long run outcomes and Section 3.5 concludes.

3.2 THE MODEL

We first describe the graph structures we study in this chapter. We then describe the public good game. Finally, an evolutionary version of the game is specified.

3.2.1 Graphs Structures

An undirected graph is a pair **G**$= (V, E)$ of sets such that $E \subseteq [V]^2$,where V are vertices and E are edges. The vertices represent the agents, whereas the edges represent the connections among agents. Two vertices i and j of G are adjacent or neighbors if $\{ij\}$ is an edge of **G**. Therefore, agent is neighbors are all agents who possess a (double-sided) link with agent i, as in Jackson and Wolinski [22]. Let G be the adjacency matrix of the graph **G**: $G= (g_{ij})_n$, where $g_{ij} = g_{ji} = 1$ if agents i and j are neighbors, $g_{ii} = 0$ for all $i = 1, ..., n$. Since all link are double-sided, $g_{ij} = g_{ji}$ for all $i, j = 1, ..., n$. It means that any matrix G is symmetric,

$$(1) \quad G^T = G$$

We consider neighborhoods where each agent has the same number, $2k$, of neighbors:

$$(2) \quad \sum_{j=1}^{n} g_{ij} = \sum_{i=1}^{n} g_{ij} = 2k$$

for all $i,\ j = 1, ..., n$. A graph which possesses this property is called $2k$-regular. Moreover, any $2k$-regular graph is an Euler graph.[1]Furthermore, since the graphs considered are $2k$-regular, it has to be the case that n is odd.

We assume that the graph is circulant: all agents must have similar type of neighbors:

$$(3) \quad g_{ij} = g_{i+1,j+1[mod\ n]}$$

A matrix which satisfies condition (3) is called circulant. This property means that each row is a cyclic shift of the previous row. Therefore, the i^{th} row is obtained from the first by a cyclic shift of $(i-1)$ steps,

$$g_{ij} = g_{1,j-i+1[mod\ n]}$$

This means that the adjacency matrix G is fully determined by its first row. Thus, we denote by $Circ\ (n,\ k;\ r)$ the circulant adjacency matrix G of size n, with $2k$ ones in each row, where $r = (r_1, ..., r_{2k}) \in \Re^{2k}$ and $r_1 = \{min\ j|g_{1j} = 1\}$, $r_2 = \{min\ j|j > r1\ and g_{1j} = 1\}$,..., $r_{2k} = \{max\ j|g_{1j} = 1\}$.The set r is referred to as the connection set.

[1]The degree of a vertex is defined as the number of links at this vertex. Since our graph is regular, the degree of each vertex is the same and is equal to $2k$. From Diestel [11], Theorem 1.8.1 : A connected graph is Eulerian if and only if every vertex has even degree (Euler 1736). It means that our graph is Eulerian: it admits a closed walk that traverses every edge of the graph exactly once. A closed walk is defined as a non-empty alternating sequence $v_0e_0v_1e_1...e_{k-1}v_k$ of vertices and edges such that $e_i = \{v_i, v_{i+1}\}$ for all $i < k$, and $v_0 = v_k$ (See Diestel [11]).

It follows that:

Proposition. 5 *Suppose that graph G is symmetric and circulant. Then* $g_{1j} = g_{i,((n+2)-j)[modn]}$ *for any* $j = 1, 2, ..., n$.

Note that our assumption (3) does not follow from properties (1) and (2).Finally, we consider only connected circulant graphs. From proposition 1 in Boesch and Tindell [10], it follows that:

Proposition. 6 *The circulant graph G represented by the matrix G is connected if and only if* $gcd\,(r_1 - 1, ..., r_{2k} - 1, n) = 1$

Proof. From proposition 1 in Boesch and Tindell [10]. **End of proof.**

Example. 10 *Consider the matrix G in Table 12.*[2] *It is not a circulant matrix, even though it satisfies properties (1) and (2).*

Note that all previous papers in the literature deal with undirected circulant Euler graphs.

Example. 11 *Bergstrom and Stark [6] and Eshel, Samuelson and Shaked [14] consider agents on a circle where each agent has two neighbors: one agent to her right and one agent to her left. This population structure can be described by the matrix in Table 13, which represents an undirected circulant Euler graph.*

Matros[28] also considers agents on a circle where each agent has $2k$ neighbors: k agents to her right and k agents to her left. This structure can be described by an undirected circulant Euler graph with adjacency matrix $Circ(n, k; r)$, where $r = (2, ..., k + 1, n - k + 1, ..., n)$.

The following example illustrates that there are other graph structures which satisfy properties (1) - (3).

Example. 12 *Let* $n = 21$ *and* $k = 3$. *Consider the matrix* $Circ\,(21, 3; r)$, *where* $r = (2, 10, 11, 12, 13, 21)$. *Figure 6 shows the corresponding undirected circulant Euler graph.*

Example. 13 *Let* $n = 8$ *and consider the matrix* $Circ\,(10, 2; r)$, *where* $r = (3, 5, 7, 9)$. *Figure 7 shows that the corresponding undirected circulant graph is NOT connected.*

[2]For clarity purposes, we do not write entries that are zeros and let empty spaces instead. This convention is adopted throughout the chapter.

3.2.2 Public Good Game (PGG).

We consider the following Public Good game. There are n agents. Each agent has $2k < n$, $k \geq 1$, neighbors with whom she exclusively interacts.

An agent can choose to be either an Altruist, or an Egoist. An altruist produces a public good which gives one unit of utility to all, $2k$, of her neighbors and incurs a net cost of $0 < c < 1$ to herself. An egoist produces nothing at no cost. Therefore, the payoff of an agent i is $\pi_i = N(A, i) - c$, if agent i is an Altruist, and $\pi_i = N(A, i)$, if agent i is an Egoist, where $N(A, i) \in \{0, ..., 2k\}$ is the number of is altruistic neighbors.

We can use our notation in order to rewrite the payoffs in the Public Good game. The payoff vector of all agents is $\pi = (\pi_1, ..., \pi_n)^T = Gx - cx$, where $x = (x_1, ..., x_n)^T$ and

$$x_i = 1, \text{ if agent } i \text{ is an Altruist, or}$$
$$x_i = 0, \text{ if agent } i \text{ is an Egoist}$$

An agent chooses her strategy by examining her payoffs and her neighbors payoffs. If her payoff is greater than the payoffs of her neighbors, she continues to play the same strategy. On the other hand, if the payoff of one or more of her neighbors is higher than hers, she adopts the strategy played by her most successful neighbor.

3.2.3 Evolutionary version of PGG.

We consider the following evolutionary version of the Public Good game. In each discrete time period, $t = 1, 2, ...$, a population of n agents plays the Public Good game. An agent i chooses a strategy $x_i^t \in \{0, 1\}$ at time t according to an imitation decision rule defined below. The play at time t is the vector $x^t = (x_1^t, ..., x_n^t)$.

Strategies are chosen as follows. An agent plays a strategy in period $t + 1$, which gives the highest payoff among her $2k$ neighbors and herself in the previous period t.

Assume that the sampling process begins in the period $t = 1$ from some arbitrary initial play x_0. Then we obtain a finite Markov chain on the finite state space $\{0, 1\}^n$ of states of the length n drawn from the strategy space $\{0, 1\}$ with an arbitrary initial play x^0. Given a play x^t at time t, the process moves to a state x^{t+1} in the next period, such a state is called a successor of x^t. We call this process unperturbed imitation dynamics with population size n and $2k$ neighbors, $Y^{n,k,0}$.

The unperturbed imitation dynamics process describes the short-run behavior of the model when agents behavior is mistake-free. Short-run predictions present some major interest due to fact that they arise rapidly, given the local interaction structure of the model, prevail for a long time(until a mistake is made), and depend on the initial state.

Now, suppose that agents use an imitation decision rule to choose a strategy with probability $1-\epsilon$ and make a mistake and choose a strategy at random with probability $\epsilon \geq 0$. The resulting perturbed imitation dynamics process $Y^{n,k,\epsilon}$ is an ergodic Markov process on the finite state space $\{0,1\}^n$. Thus, in the long run, the initial state is irrelevant.

3.3 SHORT RUN

We are able to characterize some of the absorbing states of the unperturbed imitation dynamics $Y^{n,k,0}$ - short-run outcomes with no further assumption. An absorbing state of the unperturbed process $Y^{n,k,0}$ is a state such that there is zero probability for the process $Y^{n,k,0}$ of moving from this state to any other state. The first result is straightforward and therefore, the proof is omitted.

Proposition. 7 *For any connected graph, the followings states are absorbing:*

A state where all agents are Altruists, $x = (1, ..., 1) \equiv \bar{1}$*,*

A state where all agents are Egoists, $x = (0, ..., 0) \equiv \bar{1}$*.*

The unperturbed imitation dynamics $Y^{n,k,0}$ can have other short-run outcomes. An absorbing set of the unperturbed imitation dynamics $Y^{n,k,0}$ is a minimal set of states such that there is zero probability for the unperturbed process $Y^{n,k,0}$ of moving from any state in the set to any state outside, and there is a positive probability for the unperturbed process $Y^{n,k,0}$ of moving from any state in the set to any other state in the set. (A singleton absorbing set is called an absorbing state.) Note that any absorbing state or set different from $\bar{1}$ (All agents are Altruists) and $\bar{0}$ (All agents are Egoists) has to contain both altruistic and egoistic agents. The following proposition shows that one of the Altruists has the highest payoff in such an absorbing state.

Proposition. 8 *Suppose that properties (1)-(3) hold, state* $x \notin \{\bar{1}, \bar{0}\}$*is absorbing, and* $\pi(x)$ *is the corresponding payoff vector. If*

$$\pi_i(x) = max\{\pi_1(x), ..., \pi_n(x)\},$$

then $x_i = 1$*, agent* i *is an Altruist.*

Proof. Consider an absorbing state $x \notin \{\bar{1}, \bar{0}\}$, and agent i such that

$$\pi_i(x) = max\{\pi_1(x), ..., \pi_n(x)\},$$

Since $0 < c < 1$, payoffs to an Altruist and an Egoist are always different. Therefore the highest payoff in the absorbing state can be obtained by either an Altruist or an Egoist and never both.

Suppose that agent i is an Egoist and has the highest payoff in the absorbing state. Then all her neighbors (all agents j such that $g_{ij} = 1$) have to be Egoists, $x_j = 0$, because they imitate the most successful agent - agent i. It means that $\pi_i(x) = \sum_{j=1}^{n} g_{ij}x_j = 0$, or all Egoists have the same (minimal) payoff of 0 in the absorbing state. Since we assume that $x \notin \{\bar{1}, \bar{0}\}$, there exist some Altruists in the absorbing state. From property (3) it follows that the graph is connected, or some Altruists have to have some Egoists as neighbors. This is not possible since the highest payoff of an Egoist is equal to zero. This is a contradiction, which means that agent i is an Altruist. **End of proof.**

The following result describes the minimal and the maximal number of Altruists in an absorbing state different from states $\bar{1}$ (All agents are Altruists) and $\bar{0}$ (All agents are Egoists). Denote by $\sharp(x, A)$ the number of Altruists in the state x.

Proposition. 9 *Suppose that properties (1) - (3) hold and state* $x \notin \{\bar{1}, \bar{0}\}$ *is absorbing. Then*

$$2k + 1 \leq \sharp(x, A) \leq n - 2.$$

Proof. Consider an absorbing state $x \notin \{\bar{1}, \bar{0}\}$. It follows from Proposition 2 that there exists an Altruist (agent) i, $x_i = 1$, such that

$$\pi_i(x) = max\{\pi_1(x), ..., \pi_n(x)\},$$

Then all $2k$ her neighbors (all agents j such that $g_{ij} = 1$) have to be Altruists, $x_j = 1$, because they imitate the most successful agent - agent i. It means that $\sharp(x, A) \geq 2k + 1$ and $\pi_i(x) = \sum_{j=1}^{n} g_{ij}x_j - cx_i = 2k - c$, with $i \neq j$.

Since we assume that state $x \notin \{\bar{1}, \bar{0}\}$, there exists an Egoist, agent l, in the absorbing state x. This Egoist must have another Egoist in her neighborhood from Proposition 2, because otherwise

$$\pi_l(x) = 2k > 2k - c = \pi_i(x) = max\{\pi_1(x), ..., \pi_n(x)\}.$$

Therefore, $\sharp(x, A) \geq n - 2$. **End of proof.**

The following example describes an absorbing state with exactly $2k + 1$ Altruists.

Example. 14 *Let $n = 35$ and $k = 2$. Consider the graph in Figure 8 with adjacency matrix $Circ(35, 2; r)$, where $r = (2, 11, 26, 35)$. The state where agent 1 and all her neighbors, agents 2, 11, 26, and 35, are Altruists is absorbing.*

We cannot say more in general about short-run outcomes without specifying more assumptions. Suppose that the graph structure satisfies the following local property: any two neighbors must have at least $k - 1$ common neighbors and any two strangers can have at most k common neighbors. (5) If $g_{ij} = 1$, then $\sum_{l=1}^{n} g_{il}g_{jl} \geq k - 1$. (6) If $g_{ij} = 0$, then $\sum_{l=1}^{n} g_{il}g_{jl} \leq k$.

If the local property holds, we can describe all short run outcomes.

Conjecture 1 *Suppose that properties (1) - (3) and (5) - (6) hold. Then the followings are absorbing sets:*

(I) A state where all agents are Altruists,

(II) A state where all agents are Egoists,

(III) A state where two agents i and j are Egoists, $g_{ij} = 1$ such that

$$\sum_{l=1}^{n} g_{il}g_{jl} = max_h\left(\sum_{l=1}^{n} g_{il}g_{hl}\right),$$

and all other agents are Altruists,

(IV) A set of three states. In state 1, there is one agent i who is Egoist and all other agents are Altruists. In state 2, agent i and her whole neighborhood, $2k + 1$ agents, are

Egoists and all other agents are Altruists. In state 3, agent i and two of her neighbors, are Egoists and all other agents are Altruists.

(V) A set or state which is a combination of (III) and (IV).

The following proposition concerns some particular symmetric circulant Euler graphs.

Proposition. 10 *Suppose that the graph structure is* $Circ(n, k; r)$ *with* $r = (2, ..., k-1, j, j+1, ..., j+k, n-k, ..., n)$ *and* $j = \left\lceil \frac{n+1-k}{2} \right\rceil + 1$.[3] *Then the followings are absorbing sets:*

(I) A state where all agents are Altruists,

(II) A state where all agents are Egoists,

(III) A state where two agents i and j are Egoists, $g_{ij} = 1$ *such that*

$$\sum_{l=1}^{n} g_{il}g_{jl} = max_h \left(\sum_{l=1}^{n} g_{il}g_{hl}\right),$$

and all other agents are Altruists,

(IV) A set of three states. In state 1, there is one agent i who is Egoist and all other agents are Altruists. In state 2, agent i and her whole neighborhood, 2k + 1 agents, are Egoists and all other agents are Altruists. In state 3, agent i and two of her neighbors, are Egoists and all other agents are Altruists.

(V) A set or state which is a combination of (III) and (IV).

Proof. It is obvious that the states where all agents are Egoists or all agents are Altruists are absorbing.

To find the remaining absorbing classes consider what happens to a cluster of Altruists. Note that any cluster of Altruists consisting of $1, 2, ..., k+1$ agents will immediately disappear. So, Altruism can survive in groups of the length $k+2$ or more. Consider what happens to a cluster of Egoists. Any cluster consisting of three or more Egoists will shrink in the next period. It will shrink until the cluster of Egoists becomes of two or one. The cluster of two Egoists will not change. However, if there is only one Egoist among her $2k$ altruistic neighbors, the whole neighborhood all $2k+1$ agents - will become Egoists in the next period. Then this cluster of Egoists consisting of $2k+1$ agents shrinks to the cluster of three Egoists, then one. This cycle will be repeated again. **End of proof**

[3] $\lceil x \rceil$ denotes the integer part of x.

The above proposition is part of the conjecture regarding the short-run outcomes, and represents only some of the graphs which satisfies property 1 through 5. The decomposition of neighborhoods into more than two bocks is possible and is considered in the conjecture.

Note that the local property (5)-(6) holds in Bergstrom and Stark [6], Eshel, Samuelson and Shaked [14], and Matros [28] where agents are located on a circle. The following corollary describes all absorbing sets for a circle.

Corollary 2 *Suppose that the graph structure is* $Circ(n, k; r)$ *with* $r = (2, ..., k+1, n-k+1, ..., n)$. *Then the followings are absorbing sets:*

(I) A state where all agents are Altruists,

(II) A state where all agents are Egoists,

(III) A state where two agents i *and* $i+1$ *are Egoists, and all other agents are Altruists,*

(IV) A set of three states. In state 1, there is one agent i *who is Egoist and all other agents are Altruists. In state 2, agent* i *and her whole neighborhood,* $2k+1$ *agents, are Egoists and all other agents are Altruists. In state 3, agent* i *and two of her neighbors, are Egoists and all other agents are Altruists.*

(V) A set or state which is a combination of (III) and (IV).

Proof. Note that properties (1)-(3) and (5)-(6) hold for a circle $Circ(n, k; r)$ with $r = (2, ..., k+1, n-k+1, ..., n)$. The Corollary follows from Conjecture 1. **End of Proof.**

3.4 LONG RUN

In order to find the states or sets of states that are stochastically stable, we need to find the resistance of the rooted trees among all states. It is easy to see that for any kind of network structure, only one mistake is needed to go from the absorbing state where all agents are Altruists to another absorbing state or set.

Conjecture 2 *Suppose that properties (1)-(3) and (5)-(6) hold and* $k \geq 2$. *Then*

1. If $n > 4(k+1)(k+2)$, *the limiting distribution of the imitation dynamics process* $Y^{n,k,\epsilon}$ *puts a positive probability on all absorbing sets except for the absorbing state where all* *agents are Egoists.*

2. If $n < 4(k+1)(k+2)$, the limiting distribution of the imitation dynamics process $Y^{n,k,\epsilon}$ contains only the absorbing state where all agents are Egoists.

3. If $n = 4(k+1)(k+2)$, the limiting distribution of the imitation dynamics process $Y^{n,k,\epsilon}$ puts a positive probability on all absorbing sets.

The following proposition describes all stochastically stable sets for some particular symmetric circulant Euler graphs:

Proposition. 11 *Suppose that the graph structure is $Circ(n,k;r)$ with $r = (2,...,k-1,j,j+1,...,j+k,n-k,...,n)$ and $j = \left\lceil \frac{n+1-k}{2} \right\rceil + 1$ Then*

1. If $4(k+1)(k+2)$, the limiting distribution of the imitation dynamics process $Y^{n,k,\epsilon}$ puts a positive probability on all absorbing sets except for the absorbing state where all agents are Egoists.

2. If $n < 4(k+1)(k+2)$, the limiting distribution of the imitation dynamics process $Y^{n,k,\epsilon}$ contains only the absorbing state where all agents are Egoists.

3. If $n = 4(k+1)(k+2)$, the limiting distribution of the imitation dynamics process $Y^{n,k,\epsilon}$ puts a positive probability on all absorbing sets.

Proof. From Theorem 1 follows that any absorbing class can contain $n, n-2, n-3, n-4, ..., k+3, k+2$, or 0 Altruists. Note that it is enough to make just one mistake, ϵ, for moving from the absorbing class (iii) to the absorbing class (iv) from Theorem 1 and vice versa. It means that these absorbing classes have the same stochastic potential. It also means that the absorbing class (v) will have the same stochastic potential as absorbing classes (iii) and (iv).

One error, ϵ, is enough for moving from the absorbing state (recurrent class (ii)) to an absorbing state with $n-1$ Altruists and vice versa (absorbing class (iv)).

$(k+2)$ errors are required for moving from the absorbing state with all Egoists (from the absorbing class (i)) to the absorbing class with only 2 Egoists for n even (to absorbing class (iii)), or to sets of blinkers with 1, $2k+1$, or 3 Egoists (to the absorbing class (iv)). These $(k+2)$ errors must create a cluster consisting of $k+2$ Altruists.

What is the smallest number of mistakes which is necessary to make for moving from the absorbing class with at least $(k+2)$ Altruists to the recurrent class with all Egoists? Theorem 1 shows that the cluster of Altruists is at least of the length $(k+2)$ and the cluster of Egoists is at most of the length $(2k+1)$ in any absorbing class. There must be at least one mistake per cluster of Altruists for moving to the absorbing state where all are Egoists. After such a mistake every cluster must consist of at most $(k+1)$ Altruists in order to disappear in the next period. It is possible for a cluster of the maximal length of $(2k+3)$. That cluster must be between two clusters of Egoists and each of them consists of at most $(2k+1)$ agents. Hence, at least

$$\frac{n}{(2k+3)+(2k+1)} = \frac{n}{4(k+1)}$$

mistakes are necessary to move from the absorbing class (iii) or (iv) into the absorbing state with all Egoists, absorbing state (i). The statement of the theorem follows immediately. **End of proof.**

The above proposition is part of the long-run conjecture, and represents only some of the graphs which satisfies property 1 through 5. The decomposition of neighborhoods into more than two bocks is possible and is considered in the conjecture.

As we already emphasized, the local property (5) - (6) holds in Bergstrom and Stark [6], Eshel, Samuelson and Shaked [14], and Matros [28]. The following corollary describes all stochastically stable sets for a circle .

Corollary 3 *Suppose that the graph structure is* $Circ\,(n,k;r)$ *with*
$r=(2,...,k+1,n-k+1,...,n)$ *and* $k\geq 2.$ *Then*
1. If $4\,(k+1)\,(k+2)$*, the limiting distribution of the imitation dynamics process* $Y^{n,k,\epsilon}$ *puts a positive probability on all absorbing sets except for the absorbing state where all agents are Egoists.*
2. If $n<4\,(k+1)\,(k+2)$*, the limiting distribution of the imitation dynamics process* $Y^{n,k,\epsilon}$ *contains only the absorbing state where all agents are Egoists.*
3. If $n=4\,(k+1)\,(k+2)$*, the limiting distribution of the imitation dynamics process* $Y^{n,k,\epsilon}$ *puts a positive probability on all absorbing sets.*

Proof. Since properties (1)-(3) and (5)-(6) hold for a circle $Circ\,(n,k;r)$ with $r=(2,...,k+1,n-k+1,...,n)$, the Corollary follows from Conjecture 2. **End of proof.**

The next examples show that the local property (5)-(6) must hold for the survival of Altruism.

Example. 15 *Let* $n=91$ (>48) *and* $k=2$. *Consider the graph in Figure 9 with adjacency matrix* $Circ\,(91,2;r)$, *where* $r=(2,39,54,91)$. *The state where all agents are Egoist is the long-run outcome, because the local property is not satisfied.*

Example. 16 *Let* $n=91$ (>80) *and* $k=3$. *Consider the graph in Figure 10 with adjacency matrix* $Circ\,(91,3;r)$, *where* $r=(2,45,46,47,48,91)$. *The state where all agents are Egoist is NOT the long-run outcome, and Altruism DOES survive in the long run.*

Example. 17 *Let* $n=21$ (<80) *and* $k=3$. *Consider the graph given by Figure 11 with a non-circulant adjacency matrix The state where all agents are Egoist is the long-run outcome.*

3.5 CONCLUSION AND EXTENSIONS

In this chapter, we focused our attention to symmetric regular circulant population structures (graphs), and studied the outcomes of a public good game which benefits are only local. We demonstrate that contrarily to what has been previously shown in the literature, a condition between the number of links and the number of agents is not enough to insure the survival of Altruism. The number of common neighbors of any two adjacent vertices (and any two non-adjacent vertices) does play a role in the outcomes selection.

Further work could be done by considering more general graphs and determine which properties may help Altruism to survive. Among the possible properties one could explore, Eulerian and Hamiltonian cycles[4] may be of particular interest.

[4] A graph is said to be Hamiltonian if it admits a Hamilton tour. This concept is similar to the Euler tour, but the closed walk needs to contain every vertex (not edge) exactly once. (See Diestel [11], Chapter 10)

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APPENDIX A

PROOF THEOREM 2

Consider first the number of mistakes needed for a source or disconnected locality L^x that follows a local Risk Dominant convention, to finally adopt a local Pareto Efficient convention. With positive probability, given assumption 2 on the population structure, an agent from L^x, to whom I refer as agent i, can be matched with a Stranger, denoted agent j, and both make the mistake of playing x_1 instead of x_2, thus getting a payoff of a. Note that agent j may or may not belong to L^x. This gives agent i an aspiration level of at least $[d + (a - d)\, m]$ which is greater than the aspiration level of any of her Neighbors, which equals d. The aspiration level of agent j is also above her Neighbors. Therefore, if agent i and agent j are matched for the next m periods, which can happen with positive probability, they sample some strategy-payoff pairs from their own history, and play the strategy that gave them the highest average payoff, which is strategy x_1. The reason why they always sample from their history is because their aspiration level keeps going up each time they are matched, therefore always being above their Neighbors.Following a similar reasoning adopted for the proof of theorem 3, there is a positive probability that agent i and one agent from R_i, agent k, within L^x are matched for the next s periods[1] and that the last s plays of agent k are constituted by the same strategy x_1. Therefore, agent i and agent k have a positive probability to have a memory that contains only the strategy x_1. Repeating this reasoning with each agent in L^x, one can see that a source or disconnected locality can go from a local Risk Dominant convention to a local Pareto Efficient one in only two mistakes.

This result gives rise to the following comment. If one considers the case where all source and disconnected localities have adopted the same local convention, in particular the risk dominant one, two mistakes are needed to convert the first source locality to a local Pareto Efficient convention. Nonetheless, only one mistake after that will be needed to convert any other source locality or disconnected ones. To see why, use assumption 2 on the population and consider the case where one agent from a source locality that is already fixed in the local Pareto efficient convention, is paired with a Stranger from another locality. Only one mistake is needed, from the agent whose history is composed by x_2 only to convert her locality to the local Pareto Efficient convention.

Therefore, let Z be the number of localities and λ the proportion of localities that are either source or disconnected. Following Ellison [8], the radius of a state containing $\lambda + Z$ localities following the local Risk Dominant convention is equal to 1 when $\lambda > 0$, and is equal to 2 for $\lambda = 0$.

Consider now the case where a source or disconnected locality L^x that follows a local Pareto Efficient convention, switches to a local Risk Dominant convention. Let $\lceil x \rceil$ be x if x is an integer, and be the $\{(integer part of x) + 1\}$ if x is not an integer. There is a positive probability that an agent within L^x, denoted agent i, is matched with a Neighbor, agent j, for the next s periods. With positive probability, agent i may make the mistake of playing x_2 instead of $x_1$during $\frac{a-c}{a-b-c+d}s$ of the s matches. Therefore, the last s plays of agent i are constituted by the same strategy x_2 for $\frac{a-c}{a-b-c+d}s$ times. There is also a positive probability that an agent from R_i, agent k, is matched with agent i for the next s periods, and that agent k samples a string of plays that is composed of $\frac{a-c}{a-b-c+d}s$ times with strategy x_2 in agent is history. This means that agent k, at the end of these s matchings, will have an

[1] Note that if agent i is the only agent in the locality L^x, the number of mistake to go from a Risk Dominant to a Pareto Efficient local convention is just one.

Table 1: Transition from One Absorbing State to the Next

Row 1	$0-$ *PE*	$\lambda Z-$ *RD*
Row 2	$1-$ *PE*	$(\lambda Z-1)-$ *RD*
$\vdots$	$\vdots$	$\vdots$
Row $n+1$	$n-$ *PE*	$(\lambda Z-n)-$ *RD*
$\vdots$	$\vdots$	$\vdots$
Row Z	$(\lambda Z-1)-$ *PE*	$1-$ *RD*
Row $Z+1$	$\lambda Z-$ *PE*	$0-$ *RD*

history of plays that contains the same string, of length $\frac{a-c}{a-b-c+d}s$ of strategy x_2, as agent i. Therefore, with positive probability, agent i and agent k can be matched again for m more periods and always sample $\frac{a-c}{a-b-c+d}s$ plays of the strategy x_2, leading them to coordinate on the equilibrium (x_2, x_2), and face a history of plays composed uniquely by x_2. With positive probability, every agent in L^x can have her history of plays composed by x_2 only. Therefore, one can see that a source or disconnected locality can go from a Pareto Efficient convention to a Risk Dominant one in only $\frac{a-c}{a-b-c+d}s$ mistakes.

In the case where one goes from local Pareto Efficiency to local Risk Dominance, $\frac{a-c}{a-b-c+d}s$ mistakes are needed for each locality that is either source or disconnected, in order to switch. Therefore the coradius of a state containing $(1-\lambda)Z$ localities following the local Pareto Efficient convention, is equal to $\frac{a-c}{a-b-c+d}s$ for $\lambda \leq 1$.

Consider Table 1:

Remember that λZ is the number of localities that are either source or disconnected. The first row of the table corresponds to the case where every source or disconnected locality follows the local Risk dominant convention, whereas the last row corresponds to the situation where they all follow the local Pareto Efficient convention. This table is enough to characterize all the absorbing states of the process according to theorem 3.

In order to go from a state characterized by row n to a state characterized by row $n+1$, only 1 mistake is needed for $n \geq 2$, but 2 mistakes are needed when one goes from row 1 to row 2. On the other hand, to go from row $n+1$ to row n, $\frac{a-c}{a-b-c+d}s$ mistakes are needed, for $n \geq 1$.

The total number of mistakes of all trees rooted at the state described in row 1 is: $2+(\lambda Z-1)=1+\lambda Z$. For any other state described in row $n \geq 2$, the total number of mistakes rooted at the state described in row n+1 is: $(\lambda Z-n)+n\frac{a-c}{a-b-c+d}s$. The state that has minimum stochastic potential is therefore the state described in row 1, as long as $\left\lceil \frac{a-c}{a-b-c+d}s \right\rceil > 2$,which is equivalent to the condition that $s > \frac{a-c}{a-b-c+d}$. **End of proof.**

APPENDIX B

LIST OF TABLES

Table 2: Payoff Matrix - Symmetric Coordination Game

	x_1	x_2
x_1	a, a	b, c
x_2	c, b	d, d

Table 3: Payoff Matrix

	x_1	x_2
x_1	$10, 10$	$0, 9$
x_2	$9, 0$	$7, 7$

Table 4: History in period $t + 3$

Period	Agent 1	Agent 2	Agent 3	Agent 4
t	$(x_1, 10, St)$	$(x_1, 10, St)$	$(x_1, 10, St)$	$(x_1, 10, Ng)$
$t+1$	$(x_2, 7, Ng)$	$(x_2, 7, Ng)$	$(x_1, 0, St)$	$(x_2, 9, St)$
$t+2$	$(x_1, 10, St)$	$(x_2, 9, Ng)$	$(x_1, 10, Ng)$	$(x_1, 0, St)$
$t+3$	$(x_1, 0, Ng)$	$(x_2, 9, Ng)$	$(x_1, 10, St)$	$(x_1, 10, St)$

Table 5: History in period $t + 4$

Period	Agent 1	Agent 2	Agent 3	Agent 4
$t+1$	$(x_2, 7, Ng)$	$(x_2, 7, Ng)$	$(x_1, 0, St)$	$(x_2, 9, St)$
$t+2$	$(x_1, 10, St)$	$(x_2, 9, Ng)$	$(x_1, 10, Ng)$	$(x_1, 0, St)$
$t+3$	$(x_1, 0, Ng)$	$(x_2, 9, Ng)$	$(x_1, 10, St)$	$(x_1, 10, St)$
$t+4$	$(x_2, 7, Ng)$	$(x_2, 7, Ng)$	$(x_1, 10, St)$	$(x_1, 10, St)$

Table 6: Payoff Matrix Heterogeneous Population

$$\mathbf{A}^k = \begin{pmatrix} a^k_{11} & a^k_{12} & \dots & a^k_{1k} & \dots & a^k_{1K} \\ a^k_{21} & a^k_{22} & \dots & a^k_{2k} & \dots & a^k_{2K} \\ \vdots & \vdots & & \vdots & & \vdots \\ a^k_{k1} & a^k_{k2} & \dots & a^k_{kk} & \dots & a^k_{kK} \\ \vdots & \vdots & & \vdots & & \vdots \\ a^k_{K1} & a^k_{K2} & \dots & a^k_{Kk} & \dots & a^k_{KK} \end{pmatrix}$$

Table 7: Payoff Matrix Men Population

$$\mathbf{A}^m = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

Table 8: Payoff Matrix Women Population

$$\mathbf{A}^w = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Table 9: Payoff Matrix Man versus Man

	S	**O**
S	2, 2	0, 0
O	0, 0	1, 1

Table 10: Payoff Matrix Woman versus Woman

	S	**O**
S	1, 1	0, 0
O	0, 0	2, 2

Table 11: Payoff Matrix Man versus Woman

	F	**M**
F	2, 1	0, 0
M	0, 0	1, 2

Table 12: Properties (1) and (2) are satisfied, but the matrix is not circulant

$$\begin{pmatrix} \mathbf{0} & \mathbf{1} & & & & & & & & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & & & & \\ & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & & & \\ & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & & \\ & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & \\ & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & \\ & & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & \\ & & & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & \\ & & & & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & & & & & & & & \mathbf{1} & \mathbf{0} \end{pmatrix}$$

Table 13: $Circ\,(n, 2; r)$, where $r = (2, n)$

$$\begin{pmatrix} \mathbf{0} & \mathbf{1} & & & & & & & & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & & & & \\ & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & & & \\ & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & & \\ & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & & \\ & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & & \\ & & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & & \\ & & & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} & \\ & & & & & & & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & & & & & & & & \mathbf{1} & \mathbf{0} \end{pmatrix}$$

APPENDIX C

LIST OF FIGURES

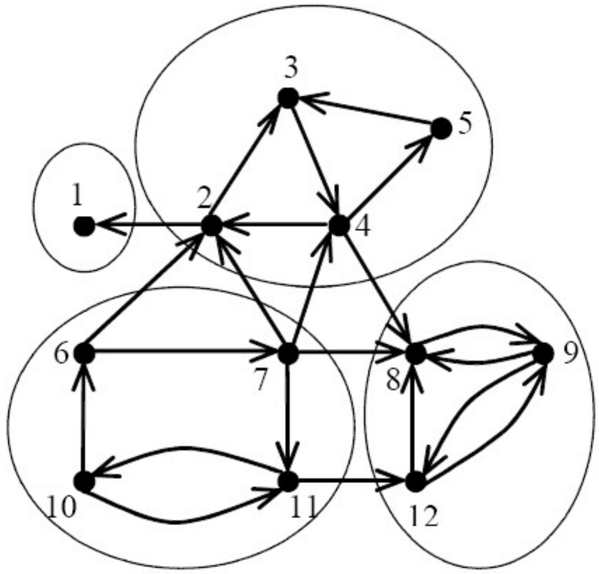

Figure 1: Sink and Source localities (Strongly Connected Components)

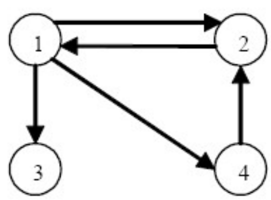

Figure 2: Information Structure

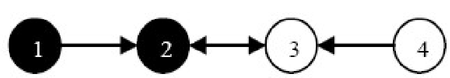

Figure 3: Information Structure Example 5

Figure 4: Information Structure Example 6

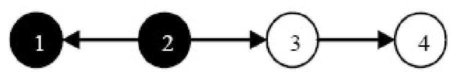

Figure 5: Information Structure Example 8

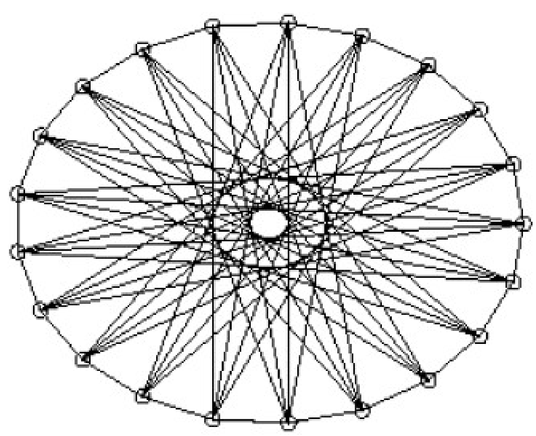

Figure 6: Connected circulant Euler graph

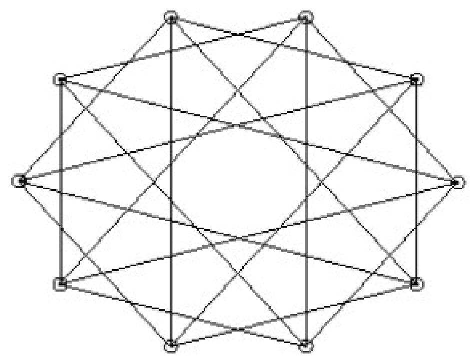

Figure 7: Disconnected circulant Euler graph

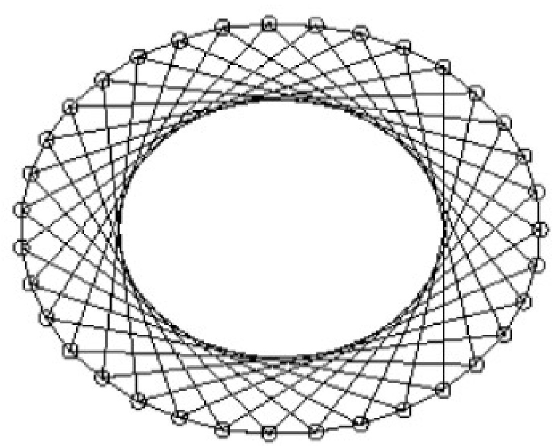

Figure 8: Circulant graph with 35 nodes

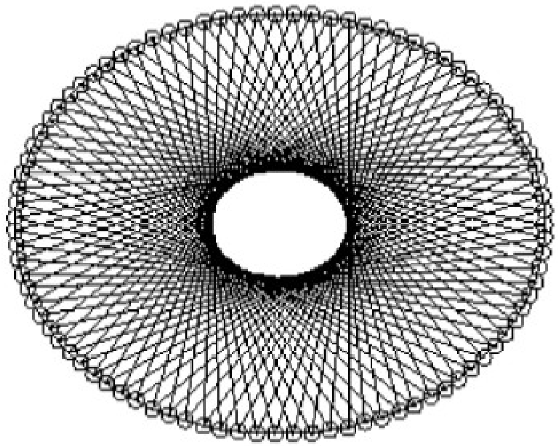

Figure 9: Circulant graph with 91 nodes - Local property is not satisfied

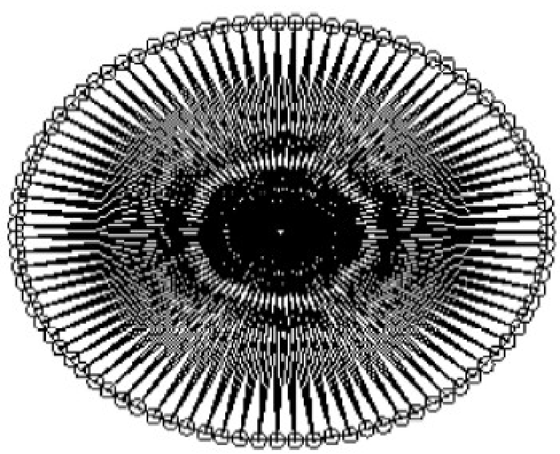

Figure 10: Circulant graph with 91 nodes - Local property is satisfied

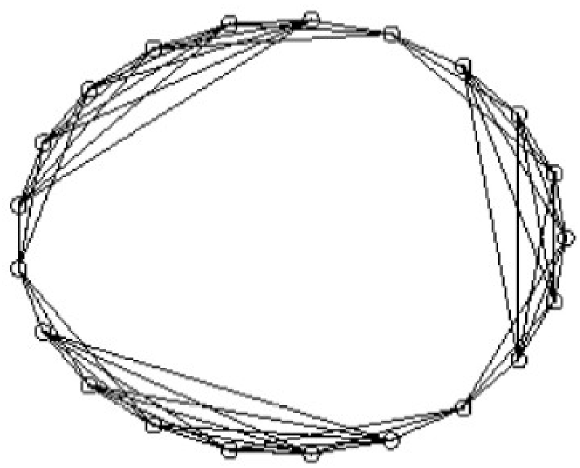

Figure 11: Non-Circulant graph with 21 nodes - Local property is not satisfied