

**THE STOCHASTIC UNIT COMMITMENT PROBLEM: A CHANCE
CONSTRAINED PROGRAMMING APPROACH CONSIDERING
EXTREME MULTIVARIATE TAIL PROBABILITIES**

by

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ABSTRACT

THE STOCHASTIC UNIT COMMITMENT PROBLEM: A CHANCE CONSTRAINED PROGRAMMING APPROACH CONSIDERING EXTREME MULTIVARIATE TAIL PROBABILITIES

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Reliable power production is critical to the profitability of electricity utilities. Power generators (units) need to be scheduled efficiently to meet electricity demand (load). This dissertation develops a solution method to schedule units for producing electricity while determining the estimated amount of surplus power each unit should produce taking into consideration the stochasticity of the load and its correlation structure. This scheduling problem is known as the unit commitment problem in the power industry. The solution method developed to solve this problem can handle the presence of wind power plants, which creates additional uncertainty.

In this problem it is assumed that the system under consideration is an isolated one such that it does not have access to an electricity market. In such a system the utility needs to specify the probability level the system should operate under. This is taken into consideration by solving a chance constrained program. Instead of using a set level of energy reserve, the chance constrained model determines the level probabilistically which is superior to using an arbitrary approximation.

In this dissertation, the Lagrangian relaxation technique is used to separate the master problem into its subproblems, where a subgradient method is employed in updating the Lagrange multipliers. To achieve this a computer program is developed that solves the optimization problem which includes a forward recursion dynamic program for the unit subproblems. A program developed externally is used to evaluate high dimensional multivariate normal probabilities. To solve the quadratic programs of the period subproblems an optimization software is employed.

The results obtained indicate that the load correlation is significant and cannot be ignored while determining a schedule for the pool of units a utility possesses. It is also concluded that it is very risky to choose an arbitrary level of energy reserve when solving the unit commitment problem. To verify the effectiveness of the optimum unit commitment schedules provided by the chance

constrained optimization algorithm and to determine the expected operation costs, Monte Carlo simulations are used where the simulation generates the realized load according to the assumed multivariate normal distribution with a specific correlation structure.

DESCRIPTORS

Chance Constrained Programming
Multivariate Probability Evaluation
Simulation

Lagrangian Relaxation
Scheduling
Unit Commitment

*To my wonderful parents,
Kamuran and Atila Öztürk*

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NOMENCLATURE

a_0, a_1, a_2	Constant parameters for quadratic fuel cost function
a_t	Total amount of power generated in time period t
A_j	$r \times n_j$ technology matrix
A_t	Event of $\sum_{k=1}^K P_{k,t} \geq l_t$
b	r dimensional right hand side vector
c, d	Constants used in the ALR formulation
$c_0, c_1, c_2, d_0, d_1, d_2$	Constants for univariate approximation
c_k	Fixed cost term for unit k
$C_i(d^{it})$	Cost function of d^{it}
C_k^c	Cold start cost for unit k
C_k^f	Fixed operation costs for unit k
$CF_{k,t}(P_{k,t}, \nu_{k,t})$	Cost of producing $P_{k,t}$ with active unit k at time t
d, s	Decision variable matrices of $I \times T \times K$ dimension
D^i	Set where $d^{i,T}$ is defined
$e(t)$	An error term that is a function of t
$f_j(x_j)$	Cost function of x_j
$f_{kl}(P_{k,t})$	A linear or convex piecewise quadratic function with L parts
\mathcal{F}	Family of events
$F(\cdot)$	Cumulative distribution function
g^i	Subgradient
I	Identity matrix
j	Index of decision variable x
J	Total number of decision variables

J^*	Objective function value of the optimal solution to the primal problem
J_{tk}	Unit matrix of size $t \times k$
K	The number of thermal generators available $k = 1, 2, \dots, K$
l_t	Residual demand for electricity in time period t
$L(\cdot)$	Ordinary Lagrangian function
$L_c(\cdot)$	Augmented Lagrangian function
n_j	Dimension of j^{th} decision variable
p	A probability value that describes the joint satisfaction probability of a given schedule
p_{lower}	Lower bound of p_{target}
p_{target}	Prescribed probability level by a utility with which to satisfy load
p_{upper}	Upper bound of p_{target}
\mathcal{P}	Probability measure
$P_i(t)$	Probability that the power generation is in state i at time period t
$P_{k,t}$	Decision variable representing the power produced by unit k at time period t
P_s	Estimate of the satisfaction probability from the simulation
$q(\lambda)$	Dual objective function value
$q^*(\lambda)$	Objective function value of the optimal solution to the Lagrangian dual problem
$Q_i(t)$	Probability that the system load will be greater than or equal

	to the total generated power in state i at time period t
r	Dimension of the right hand side vector of the LR problem
\mathfrak{R}^r	r dimensional set of real numbers
R	Correlation matrix
$R(t)$	System risk at time t
s_1	A constant used in the AR(1) model
s^i	Final step size at iteration i
S_c	Cold state start up cost
S_h	Hot state start up cost
$S_{kt}(\nu_{k,t})$	Cost of starting up unit k at time t
S^t	Set where $s^{I,t}$ is defined
t	Time
$t(p)$	A function of probability used in approximating the inverse of the normal distribution
$t_{coldstart}$	Amount of time a generator is considered to be in a hot state after it is turned off
t_{down}	Minimum number of hours required for a generator to stay down once it is off
t_{s_k}	Last time unit k was up
t_{up}	Minimum number of hours required for a generator to stay up once it is on
T	The planning horizon in terms of time periods $t = 1, 2, \dots, T$
w_t	Power obtained from wind power generators at hour t
X, Y	Random variables

X_j	Feasible set for x_j
z	z -value
z_1	z such that $F(z) = p_{lower}$ where F is the cumulative distribution function of a univariate normal
z_2	z such that $F(z) = p_{upper}$ where F is the cumulative distribution function of a univariate normal
z_{lower}	z such that $F(z) = p_{lower}$ where F is the cumulative distribution function of a t dimensional multivariate normal
z_{new}	z such that $F(z) = p_{target}$ where F is the cumulative distribution function of a t dimensional multivariate normal
z_{target}	z such that $F(z) = p_{target}$ where F is the cumulative distribution function of univariate normal
z_{upper}	z such that $F(z) = p_{upper}$ where F is the cumulative distribution function of a t dimensional multivariate normal
Z_i	Standard normal random variate
α	Probability level
α^i	Step size at iteration i
α_k	Thermal time constant for unit k
ϵ	A small constant used in calculating convergence of iterative algorithms
κ_k	Minimum uptime for unit k
λ	Lagrange multiplier
λ^\top	Transpose of λ
μ	Mean vector for load

$\nu_{k,t}$	$\begin{cases} 1 & \text{if unit } k \text{ is up at time period } t \\ 0 & \text{otherwise} \end{cases}$
ρ	Correlation coefficient of l_t and l_{t+1}
Σ	Covariance matrix for load
τ_k	Minimum downtime for unit k
$\Phi(\cdot)$	Cumulative distribution function of the standard normal distribution
Ω	Set of all possible outcomes

ABBREVIATIONS AND TERMINOLOGY

ALR	Augmented Lagrangian relaxation
APP	Auxiliary problem principle
AR	Auto regression
ARMA	Auto regressive moving average
CO_2	Carbon dioxide
CPU	Central processing unit
DP	Dynamic programming
GDP	Gross domestic product
ISR	Interconnected system risk
kWh	Kilowatt-hour
LP	Linear program
LR	Lagrangian relaxation
Max	Maximize
Min	Minimize
MIP	Mixed integer program
MW	Megawatt
MWH	Megawatt-hour
NO_x	Nitrogen oxides
NP	Nondeterministic polynomial
QP	Quadratic programming
SD	Standard deviation
SO_2	Sulfur dioxide
SR	Spinning Reserve

SSR Single system risk
UC Unit commitment
WEC Wind energy conversion

1.0 INTRODUCTION

Reliable power production is critical to the profitability of electricity utilities. Cost of power production is a significant part of the Gross Domestic Product (GDP) in the United States and efficient methods of production can help reduce this cost. Power is generated by transforming natural sources of energy into electricity, which is used to satisfy societal demands.

In general the power planning problem is based on three different sets of decisions which are dependent on the length of the planning time horizon. The first set consists of the long-term decisions (years) where the decision variables to be determined are the capacity, type, and number of power generators (units) to own. In the medium term (days, weeks), one needs to decide how to schedule (commit) the existing units for the planning horizon. Finally, in the short term (seconds to hours), the goal is to efficiently determine the amount of power each committed unit will produce to meet the real-time electricity demand. In general, the long-term problem is identified as the power expansion problem, the medium-term problem is identified as the unit commitment (UC) problem, and the short-run problem is called the economic dispatch (generator allocation) problem.

The UC problem is solved for time horizons of one day to one week. Once system operators obtain this schedule they solve the economic dispatch problem on a rolling horizon basis using a time window of up to 15 minutes and change the outputs of the committed

units to reflect the revised demand estimates. Different time windows are used by different utilities, for instance, Pennsylvania-New Jersey-Maryland (PJM), New York and New England employ 5-minute intervals, California uses 10 minutes and the Electric Reliability Council of Texas uses 15 minutes. ^{[1]*}

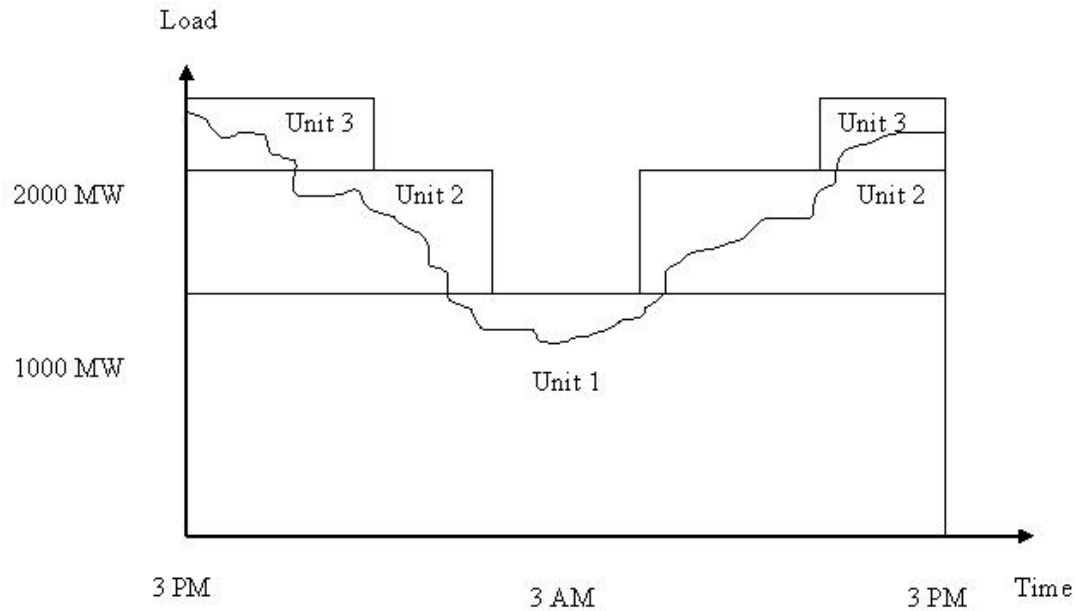


Figure 1.1 A Simple Scheduling Scenario

Electricity utilities typically possess numerous units and they need to commit these because electricity cannot be stored in a large-scale system and demand is a random variable-process fluctuating with the time of the day and the day of the week. Electricity demand, which is referred to as load in the power literature, varies with the temperature since both heating and cooling of indoor environments require electricity. The nonstorable property of electricity requires utilities to produce electricity at a rate that is close to the rate of consumption. In Figure 1.1 this is demonstrated for a simple scenario where there are 3

*Bracketed references placed superior to the line of text refer to the bibliography.

generators to meet the daily load. Here generator 1 is the least expensive and generator 3 is the most expensive resource. It is seen that generator 1 is always on to meet the base load and generators 2 and 3 are turned on when necessary. Since the load fluctuates, generators cannot produce the same amount of power at all times. In addition, power generators cannot be turned on instantly to satisfy the load; therefore generation schedules require advance planning. For all these reasons, the scheduling of power generators is a crucial economic decision for electricity utilities and is a difficult task. Neglecting the importance of this problem can have serious consequences. For example, if a utility over commits its generators using rule of thumb methods such as scheduling for load plus a percentage of the peak load, then the result is higher generation costs. On the other hand, under committing generators might result in blackouts in the event of a generator failure or an unexpected load spike.

The purpose of this dissertation is to introduce a solution method to schedule generators for producing electricity while determining the estimated amount of surplus power each generator should produce taking into consideration the stochasticity of load and its correlation structure. The solution method is designed to also handle the presence of wind power plants which creates additional uncertainty.

1.1 Electric Power System Characteristics

A power generation system of a electric utility consists of many different types of generators with various generating capacities and cost functions. The generation system can include hydroelectric plants, nuclear plants, thermal plants, wind power plants, etc. Thermal plants convert thermal potential to electricity and can consume coal, oil, gas, etc. Typically

the cost function of thermal generators consists of two parts. The first part is a fixed cost for employing the generators to produce power and the second part is dependent on the amount of fuel used which can take the form of a nonlinear function of the amount of fuel consumed. Usually a portion of these units are used all the time because they are the least cost alternatives. The peak portion of the load is satisfied using more expensive units and one needs a schedule to determine when to turn these units on since it may take couple hours for these generators to reach a level where they can operate at full capacity and there is also a start up cost associated with turning these generators on.

The formulation that appears in this dissertation is modeled for an isolated system, that is one that does not have access to an electricity market. Therefore, the system needs to meet the load using only its resources. This system can be a state where the power industry is not deregulated or an island where having a market is not feasible. This in turn requires that the load should be met with high probability. The method proposed in this dissertation takes this necessity into consideration by allowing the utility to specify the probability level (p_{target}) under which the system should operate.

There are two types of uncertainties present in the model that is considered here; uncertainties that arise from demand and from supply. The first one arises because of the stochastic character of demand for electricity. The load fluctuates with time, depending on the time of the day and the day of the week. The load also depends on the temperature. It has been recently observed that for a particularly large data set covering hourly load data for two consecutive summers for the Northeastern United States, the residual demand (l_t), after the temperature effect is removed, follows an AR(1) model. ^[2] Auto-regression (AR) is a parametric method that finds a statistical relationship in sets of data to utilize in fore-

casting future values of a variable. If load data can be modeled using an AR(1) model, this means that the current hour's load can be represented using the previous hour's load plus a random error as shown in (1-1). Here s_1 is a constant that can be estimated by fitting data and $e(t)$ is the error term.

$$l_t = s_1 l_{t-1} + e(t) \quad (1-1)$$

It follows from the AR(1) model that the distribution of the residual demand can be approximated by a multivariate normal distribution with a specific correlation structure. [3] A multivariate normal distribution is the generalization of bivariate normal distribution to more than 2 dimensions. The joint probability density of T random variables $l = (l_1, l_2, \dots, l_T)$ is normal with the mean vector $\mu = (\mu_1, \mu_2, \dots, \mu_T)$ and the symmetric positive definite covariance matrix $\Sigma = \sigma_{ij}$, where $\sigma_t = \sigma_{ij}$ when $i = j$, if the density has the form:

$$\Phi(l) = (2\pi)^{-\frac{T}{2}} |\Sigma|^{-\frac{1}{2}} e^{-\frac{(l-\mu)^T \Sigma^{-1} (l-\mu)}{2}} \quad (1-2)$$

The second type of uncertainty concerns supply. In the case where the power system includes wind power generators, the power generated by wind power plants varies over time depending on the velocity of the wind. When there are many wind power plants supplying power to the grid, the power supply from these can be approximated by a multivariate normal distribution. [4]

Both of these uncertainties are considered in this dissertation. In creating generator schedules, the proposed model does not simply assume forecasted or expected values of

these random variables but considers the correlation structure and the randomness of these variables. For simplicity, the stochasticity introduced by the failure of generating units is ignored since the time horizon is short and units are not expected to fail during this short period.

1.2 The Unit Commitment Problem

The UC problem is defined as determining the mix of generators and their estimated output level to meet the expected demand of electricity over a given time horizon (a day or a week), while satisfying constraints such as ramp rate limits, uptime and downtime constraints, reserve and energy requirements. Ramp rate limits reflect the time it takes to turn the generators on/off, while uptime and downtime constraints ensure that once a generator is up/down, it stays at that state for at least a given length of time. Reserve requirements ensure that the system has a minimum amount of surplus power at each time period in order to respond to load spikes or equipment breakdowns. Energy requirements ensure the system has enough supply to satisfy the load for each time period.

A large amount of research is dedicated to this problem to meet the needs of the power industry. When the UC problem is formulated for a large generating system, the resulting program is a large-scale nonlinear mixed integer program (MIP) with typically thousands of binary and continuous variables ^[5], and it is considered to be in the class of NP-hard problems. ^[6] (NP-hard problems is the class of problems which has no known polynomial time algorithms that can solve them for any given instance to optimality.) In addition, the solution times grow exponentially with system size, thus this requires efficient solution methods.

As mentioned previously, the reserve constraint is one of the constraints considered in the UC problem. Utilities are required to carry a reserve for many reasons such as load peaks, generator failures, scheduled outages, regulation, and local area protection. This reserve is defined as operating reserve. Operating reserve consists of two main parts: spinning reserve (SR) and nonspinning reserve. The additional electricity available (synchronized) to serve load immediately is defined as the SR. In other words, the difference between the total amount of electricity ready to serve the customers and the current demand for electricity is the SR. On the other hand, nonspinning reserve is not connected to the system but can serve the load within ten minutes. ^[7] When solving the UC problem researchers estimate the amount of SR and model it as a reserve constraint. In almost all applications of the UC problem, the SR requirement is taken to be deterministic. For example, in some applications it is taken to be 1.5 to 2 times the capacity of the largest generator or a percentage of the peak load. ^[8] Determining the appropriate amount of SR is similar to classical inventory problems faced in operations management. Companies keep large amounts of inventory to ensure that they can meet the demand independent of the problems that might arise in their production line. In a similar manner, utilities keep SR in case they have unexpected problems. However, keeping too much inventory or SR is costly and therefore techniques that can eliminate extra amounts of SR and guarantee service at a specified probability level p_{target} can be very useful. In this research, a method is proposed in which one can determine the SR so as to meet the load with a high probability over the entire time horizon (a day or a week) while a UC schedule is determined.

1.3 Stochastic and Correlated Load Structure

One important aspect to take into consideration is that most of the researchers modeling the UC problem employ single point estimates for the load. The proposed method differs in that it incorporates the stochasticity of the load into the model when determining the SR level and finding a UC schedule. Therefore, the model used in this dissertation requires stochastic programming techniques, where solution methods take into consideration the randomness in input parameters. The superiority of using stochastic programming over the expected value solution has been demonstrated over different types of problems. ^[9] For the UC problem this also holds because expected values of load do not accurately represent the realized values due to randomness in load.

There are two main differences between previous stochastic models used in solving the UC problem and the proposed approach. The first main difference arises from the assumptions made in formulating the UC problem. The proposed approach considers the random variables representing load to be continuous, which contrasts with previous approaches where the underlying continuous distribution is replaced by a discrete approximation and a scenario approach employing multi-stage stochastic programming techniques is used. In this dissertation chance constraints, a type of constraint the violation of which is dependent on a defined probability level, are added to the mathematical formulation of the UC problem in order to incorporate the randomness in the load variable into the model.

The second difference, mentioned before, is that the proposed method explicitly considers the correlation in load between time periods. It is shown that omitting correlation results in finding suboptimal solutions to the UC problem. This dissertation is the first work to recognize the value of this correlation structure and to analyze its effect on the objective

function value. To do this it takes into consideration the positive correlation matrix at each iteration of the algorithm and modifies the parameters of the stochastic constraint to improve the solution iteratively. To do this it is necessary to evaluate multivariate normal probabilities of a stochastic version of the load constraint at each iteration. This is a difficult task, especially if the time horizon is one week since this results in 168 hourly dimensions. For verification of the accuracy of the multivariate probability evaluation, Monte Carlo simulations are employed. The results obtained in this dissertation show the importance of modeling load as a random variable and of considering the correlation in load between time periods.

1.4 The Effect of Wind Energy on the UC problem

The UC problem is complicated even for conventional generation systems such as thermal and hydro. However, the problem becomes more complicated when intermittent energy sources such as wind power are incorporated into the power production mix. Traditional methods of power production, though economically sound, are becoming less desirable with the technological advances of sustainable power production. Oil based power production is becoming risky due to fluctuating oil prices, and although coal is available in large amounts, the pollution it generates is a public health concern. Both of these fuels also give rise to large amounts of CO_2 emissions, which contribute to global climate change. International efforts are currently attempting to reduce these emissions to a lower level. These efforts, combined with the public's growing concern with health and environmental issues, suggest that carbon based power production may need to be drastically limited in the future. This is why optimization methods for renewable energy technologies have become very important.

Using renewable power sources in place of traditional ones can reduce emissions as shown in Table 1.1. ^[10] This data shows what the reduction in the emissions could be per million kWh consumed, if wind energy conversion (WEC) is used instead of carbon based power production methods.

Table 1.1 Emission Reductions from Using WEC

Per Million kWh consumed
750-1000 tons of CO_2
7.5-10 tons of SO_2
3-5 tons of NO_x

Renewable energy technologies, such as photovoltaic, solar thermal, geothermal, biomass, tide and wind power are developing and some are beginning to compete with existing power production methods. WEC, in particular, is becoming cheaper than traditional methods in some areas of the world. Wind electricity costs have declined from 35 cents per kwhr to 5 to 7 cents per kwhr in the last 22 years. ^[10]

One of the characteristics of WEC is that the amount of electricity produced by a wind power plant fluctuates depending on the wind velocity. To incorporate wind power plant output into the power grid such that the system remains profitable, one must reconsider the UC schedule for the rest of the system. This has led to research in the intermittency effect of wind power on the UC schedule. Optimization methods can be used to mitigate the problem of intermittency and improve the profitability of the overall system. In other words, the variation in the generation of wind power can be considered when scheduling the thermal generators for the power system. In this dissertation the solution methodology developed is designed so that it can take wind power plants into account if they exist in

the power pool. It is assumed that the total wind power available from all wind power plants follows a normal distribution and this is considered when calculating the total power available. [4]

1.5 Contribution

This dissertation combines probabilistic analysis with optimization to solve a difficult but important problem. The proposed method represents the first effort to apply chance constrained programming to the UC problem. In this dissertation a stochastic optimization methodology is developed that is applicable to other large-scale systems. While the particular application in this dissertation is electric power generation, the approach should also be applicable to other large-scale stochastic systems, where the probability of satisfying a requirement has the utmost importance. This dissertation also represents an important synergy between the tools of probabilistic analysis and optimization used within industrial engineering with the analysis tools and power planning methods of electrical engineering.

As stated previously, the purpose of this dissertation is to (a) introduce a solution method to schedule generators for producing electricity, (b) estimate the amount of surplus power each generator should produce in the presence of wind power plants, (c) schedule generators taking into consideration the stochasticity of load and its correlation structure. The importance of the UC problem can be measured by the vast amount of literature that exists on this topic. The main motivation for these efforts is the possibility of obtaining savings from using a more efficient scheduling method to commit generators and determine the SR.

This dissertation not only builds upon the existing literature, but also introduces methods and models not used before to solve the UC problem. Major contributions of the work reported in this dissertation are the following:

- A method for scheduling thermal and wind power generators has been obtained to meet the load with a specified level of probability by using a chance constrained programming approach. The approach given here uses the mean, standard deviation and the correlation structure of the load data instead of using forecasted or approximated values of load for different time periods and provides a probability level for the load to be satisfied over all time periods. This is different than other stochastic programming approaches such as scenario based methods in that the proposed model considers the load random variable to be continuous instead of discrete, which better reflects reality. Previous research that has been conducted in this field has used stochastic programming and a recourse based approach, however the number of scenarios used is limited and increasing this number significantly increases the size of the scenario tree resulting in long computation times or no solution at all. ^[9] Furthermore these solutions are not guaranteed to meet demand with a high level of probability.
- An investigation is made to observe the effect of correlation on the schedules employing the method developed in this dissertation. This hasn't been investigated before in the UC context and the results of this research should prove very helpful in finding cost effective UC schedules. The proposed approach uses the correlation matrix for the load between time periods when calculating the value function at each iteration in the proposed algorithm. Considering these correlations improves the solution to the problem by reducing the allocated power. It is also shown that the objective function

value of the UC problem considering correlation is bounded above by the objective function value of the same problem if the correlation terms are omitted while solving the problem.

- Generalization of results by including wind power plants in the production pool is achieved in the solution methodology. Wind power plants are increasing in number and incorporating them into the existing UC problem solution methods is necessary for maintaining the profitability of utilities. This is accomplished in the proposed model by treating the power obtained from wind as a multivariate random variable and using the mean and standard deviation for different time periods similar to what is being done for the load.
- Finally, instead of using a deterministic level of SR, the proposed model determines the level probabilistically, which is superior to using an arbitrary approximation. Previous research in modeling the UC problem has usually considered the reserve level to be deterministic which does not match with reality. The proposed method uses chance constraints in order to ensure that the generators have sufficient SR such that the load is met at a specified probability level.

To achieve the aforementioned objectives a computer program is developed using the C programming language that solves the optimization problem that includes a forward recursion dynamic program. A program developed by Alan Genz in FORTRAN is used to evaluate high dimensional multivariate probabilities. ^[11] To solve the quadratic programs, CPLEX 7.0 interfaced with Visual C++ is employed. Finally, a computer program is developed using the C programming language to simulate the load and verify the results obtained

by the optimization algorithm. Using this simulation an expected cost analysis is also performed to evaluate the effect of satisfying demand with a high probability on the operating cost.

1.6 Overview of the Dissertation

The remainder of this dissertation is organized as follows:

Chapter 2 provides a survey of research that address the UC problem, wind power, SR, stochastic programming, and multivariate probability evaluation. For each of these areas, relevant works in the literature are discussed and analyzed. There is a tremendous amount of literature on the UC problem so only the methods that are used in this dissertation are analyzed in detail. Lagrangian relaxation (LR) is covered in more detail since this method has proven to be the most successful one in solving the UC problem. Works that address the intermittency issue of wind power are also presented.

Chapter 3 presents the deterministic problem and the corresponding mathematical formulation for the UC problem. Then the steps taken in developing the stochastic load constraint are provided. The effect of the correlated load structure is shown by proving that the objective function value to the correlated problem bounds from below the objective function value to the uncorrelated problem.

Chapter 4 states the chance constrained optimization problem and the stochastic formulation for the UC problem. Then it describes the method developed to solve the proposed problem. The method can be divided into two main parts. The first part describes the

procedure applied to solve the optimization problem, which includes solving a dynamic program, a quadratic program and the overall mixed integer program. The second part briefly explains the multivariate probability evaluation and elaborates on the iterative updating solution methodology developed that allows the method to reiterate and solve the optimization problem to converge to required probability level.

Chapter 5 lists six different problems from the literature that are used to test the applicability of the proposed method. The types of generators, their capacities, and their cost functions are shown. The data also contains the load profiles for all problem instances.

Chapter 6 provides the solution of the problems listed in the previous chapter. The problems are solved under two different conditions. First zero correlation is assumed and the problems are solved using the methods described in this dissertation. Then the problems are solved again using positive correlations to observe the difference in the schedules and cost. Algorithm performance is also discussed in detail.

Chapter 7 provides a verification of the schedules obtained using Monte Carlo simulation. This simulation is also used to analyze the expected cost of the schedules obtained by the optimization algorithm. An analysis of stochastic SR determination is followed by a discussion of the reliability of the multivariate probability evaluation used.

Chapter 8 analyzes the results obtained and derives conclusions based on these results. The chapter concludes by a brief explanation of future research directions that are suggested by this dissertation.

2.0 LITERATURE REVIEW

In this chapter the earlier research that has been conducted related to the unit commitment problem in both the conventional and wind power areas will be presented. Section 2.1 will focus on the UC solution methods, especially Lagrangian relaxation and its variants. Section 2.2 summarizes the solution methods employed in solving the UC problem when the power pool includes wind power plants. Section 2.3 presents probabilistic SR determination methods and attempts of integrating this with the UC solution methods. Following this section, section 2.4 provides a synopsis of methods developed that considers the stochasticity of load when solving the UC problem. This section first describes recourse based models and then introduces joint chance constrained programming methods. Finally, section 2.5 surveys the methods used and developed in evaluating high dimensional multivariate normal distributions.

2.1 Solution Methods for the UC Problem

A number of diverse optimization methods has been employed to solve the UC problem. The models used for the UC problem differ in details, but the main objective of all these models is finding an optimal or near optimal schedule by solving the MIP which includes both binary and continuous decision variables. The model used in this dissertation appears in Chapter 3. Initially in practice, most of the utilities used heuristics such as priority lists

(merit order scheduling). This method creates a list of units to be turned on in a given order with increasing load. The order is determined according to increasing/decreasing operating costs. ^[12] In the last 30 years more sophisticated and efficient methods have been developed. Some of the traditional optimization methods used are exhaustive enumeration, dynamic programming (DP), branch and bound, Benders decomposition, separable programming, network flow programming, LR, augmented Lagrangian relaxation (ALR), risk analysis, and decision analysis. Metaheuristic methods such as genetic algorithms, simulated annealing, tabu search or expert systems have also been employed. Stochastic programming has also been considered and tests have been conducted to show the merit of this method. ^[13]

2.1.1 Lagrangian Relaxation

LR is a common method used for solving MIPs. This method is especially useful when there is a set of constraints that when relaxed makes the overall problem simpler to solve. Out of all the different methods employed to solve the UC problem, LR is the most widely used method because of its success in solving large-scale problems. Therefore, the focus in the UC research has been to find methods that can make this method more efficient. In order to understand LR more clearly, consider the general mathematical formulation given in equations (2-1)-(2-3). Here x_j is the n_j dimensional decision variable vector, b is the r dimensional right hand side vector, $f_j(x_j)$ is the objective function and n_j dimensional X_j 's are the sets over which the x_j 's are defined. Note that A_j is a $r \times n_j$ technology matrix.

LR1

$$\text{Min } \sum_{j=1}^J f_j(x_j) \quad (2-1)$$

$$\text{subject to } \quad x_j \in X_j, j = 1, 2, \dots, J \quad (2-2)$$

$$\sum_{j=1}^J A_j x_j = b \quad (2-3)$$

If constraint (2-3) is relaxed in problem **LR1**, the following equivalent problem, **LR2**, will be obtained. In this formulation the original constraint (2-3) is relaxed and included in the objective and λ is the corresponding Lagrange multiplier vector .^[14] Due to relaxation the resulting mathematical program is easier to handle.

LR2

$$\text{Max } q(\lambda) \quad (2-4)$$

$$\text{subject to } \quad \lambda \in \mathfrak{R}^r \quad (2-5)$$

where

$$q(\lambda) = \sum_{j=1}^J \min_{x_j \in X_j} \{f_j(x_j) + \lambda^\top A_j x_j\} - \lambda^\top b$$

Historically, two methods have dominated as solution methods to help solve problems that utilize the LR technique: column generation and subgradient optimization. Column generation adjusts the Lagrange multipliers by solving a subproblem and a master problem iteratively. Subgradient optimization modifies the Lagrange multipliers by determining a

direction, the subgradient, that increases the objective function value of the dual problem. In solving the UC problem the latter method is preferred because the calculations of a given iteration are straightforward and fast. ^[15]

The first application of LR to the UC problem can be found in the paper by Muckstadt and Koenig. ^[16] The authors here decompose the problem into different unit subproblems and use branch and bound to solve a small UC problem. Merlin and Sandrin ^[17] employ LR and use a modified version of the subgradient method to update the Lagrange multipliers and solve a UC problem for 172 units in less than 2 minutes. In this article the authors also suggest a different approach for the determination of SR levels but do not implement this to obtain any results. They define the Lagrange multiplier corresponding to the reserve constraint as the marginal utility for the SR (product of cost of failure and the probability of not satisfying demand) and suggest that it is possible to give a SR level using chance constraints. This suggestion is implemented in this dissertation by using chance constraints. Bard also uses a similar approach but includes ramping constraints in the problem and instead of branch and bound uses DP. ^[18] Zhuang and Galiana ^[19] also employ a similar approach and solve the UC problem using a new heuristic to find a solution.

One problem LR based methods face is that the final solution obtained is seldom feasible, which requires using a follow up method to obtain a feasible solution. Moreover, when the UC problem needs to be solved for a time period that extends to one month this method takes a long time to converge to the optimal solution and in some cases the algorithm keeps oscillating around the optimal point without any convergence. Aoki et al. ^[20] present an algorithm that uses a variable metric method that overcomes this problem. The variable metric method in general is a special conjugate gradient method and can be interpreted as

an extension of Newton's method, which identifies the maximum of a function by searching for a zero of the gradient. In each step of Newton's method, the objective function is approximated by a second order Taylor polynomial. However, the variable metric method does not require the second order partial derivatives needed for the Taylor polynomial. Instead, it approximates these values using information from previous iterations. However, when the objective function is not strongly convex, this method is not stable and cannot reach a good solution. Here a function $f(\cdot)$ is strongly convex if it satisfies the following property:

$$(\nabla f(x) - \nabla f(y))'(x - y) \geq \beta \|x - y\|^2 \quad (2-6)$$

, where β is a constant multiplier which takes on values in the interval $[0, 1]$. Therefore, to overcome this difficulty, Batut and Renaud ^[21] utilize augmented Lagrangian relaxation (ALR). To understand how ALR works, consider the following initial problem with equation (2-10) being the linking constraint which links the decision variable of different time periods and units to each other. The decision variables in this problem are d and s and $C_i(d^{i,t})$ is the cost associated with variable d .

$$\min_{d^{I,T}, s^{I,T}} \sum_{i \in I} C_i(d^{i,t}) \quad (2-7)$$

$$d^{i,T} \in D^i \quad \forall i \in I \quad (2-8)$$

$$s^{I,t} \in S^t \quad \forall t \in T \quad (2-9)$$

$$d^{i,t} = s^{i,t} \quad \forall i \in I, \forall t \in T \quad (2-10)$$

ALR introduces a quadratic penalty term to the objective function and one obtains the dual Lagrangian function $L_c(d^{I,T}, s^{I,T}, \lambda^{I,T})$ where $\lambda^{I,T}$ is the Lagrange multiplier and c is a constant:

$$L_c(d^{I,T}, s^{I,T}, \lambda^{I,T}) = L(d^{I,T}, s^{I,T}, \lambda^{I,T}) + \sum_{i \in I} \sum_{t \in T} \frac{c}{2} |s^{i,t} - d^{i,t}|^2 \quad (2-11)$$

where $L(d^{I,T}, s^{I,T}, \lambda^{I,T})$ is the ordinary Lagrangian:

$$L(d^{I,T}, s^{I,T}, \lambda^{I,T}) = \sum_{i \in I} C_i(d^{i,t}) + \sum_{i \in I} \sum_{t \in T} \lambda^{i,t} (s^{i,t} - d^{i,t}). \quad (2-12)$$

A consequence of adding this term is that the objective function now becomes inseparable since $|s^{i,t} - d^{i,t}|^2$ introduces terms that cannot be separated, which complicates the problem. In order to overcome this problem the Auxiliary Problem Principle (APP) is used to linearize the terms. There are a couple of advantages to this method: first, the objective function becomes differentiable and second, the solution of the dual yields a primal solution. To employ the APP, the algorithm divides the problem into two subproblems. Starting from an initial solution $(s_0^{i,t}, d_0^{i,t})$, in each iteration, k subsequent programs are solved where previous values of $s_{k-1}^{i,t}, d_{k-1}^{i,t}$ are constant. Here (2-13) gives the next set of $d^{i,t}$ values and (2-14) gives the next set of $s^{i,t}$ values and (2-15) gives the next set of λ 's. Note that c and d are constants.

$$\min_{d^{i,T} \in D^i} C_i(d^{i,t}) + \frac{d}{2} \sum_{t \in T} (d^{i,t} - d_{k-1}^{i,t})^2 - \sum_{t \in T} (\lambda_{i,t}^{k-1} + c(s_{k-1}^{i,t} - d_{k-1}^{i,t})) d^{i,t} \quad \forall i \in I \quad (2-13)$$

$$\min_{s^{I,t}} \frac{d}{2} \sum_{i \in I} (\lambda_{i,t}^{k-1} + c(s_{k-1}^{i,t} - d_{k-1}^{i,t})) s^{i,t} \quad \forall t \in T \quad (2-14)$$

$$\lambda_{I,T}^k = \lambda_{I,T}^{k-1} + c(s^{I,T} - d^{I,T}) \quad (2-15)$$

A recent comparison between ALR and LR techniques for the UC problem is made by Beltran and Heredia. ^[22] They show that LR yields infeasible primal solutions and differentiability is not ensured but the problem is separable. Whereas, ALR yields feasible primal solutions and differentiability is ensured but the objective function is not separable. However, the use of the APP with ALR guarantees convergence in the convex case, which makes the latter method preferable. In this dissertation LR is preferred since the main motivation is not to improve the optimization method used but to test the applicability of chance constrained programming and understand the value of considering correlation of load between time periods.

2.1.2 Techniques to Ensure LR Feasibility

Since it is possible for LR to terminate with an infeasible solution, it is sometimes necessary to derive a feasible solution from the resulting solution. Typically heuristic methods are used for this. This is demonstrated in the work by Li et al. ^[23] First, the UC problem is solved using LR iteratively. If the solution is not feasible, because all of the demand is not satisfied in 1 or more time periods, then it is followed by a heuristic method they call sequential unit commitment to find a feasible solution. If the feasible solution over commits

the generators, then a unit decommitment method is used to decommit generators. Decommitment is used to improve a solution obtained by LR. Basically, what decommitment accomplishes is that it decommits any generators that can be removed from the solution set without making the objective function worse. In a similar work, Tseng et al. ^[6] employ decommitment to solve the UC problem and provides improved schedules.

2.1.3 Other Techniques for Solving the UC Problem

Feasibility of the application of metaheuristic methods has also been shown in solving the UC problem. One example of this is a paper by Dasgupta and McGregor. ^[24] In this paper, they solve a problem with ten thermal generators and obtain “quite good” results. Other methods such as genetic algorithms have also been used as an auxiliary method to LR. Orero and Irving ^[25] employ this method and solve a ten-unit system to demonstrate the utilization of this method. The main difficulty with this method is the lack of a solid theoretical basis for the genetic algorithm’s control parameter settings across a wide range of problem domains. ^{[25],[26]} Similarly the feasibility of evolutionary programs as an alternative improvement method is also demonstrated by Duo et al. ^[27] Other improvements are also made such as including additional constraints on voltage and transmission to the problem to obtain more realistic results. For example, Shaw, Xia et al. and Wang et al. solve a UC problem in the presence of these constraints. ^[28,29,30]

2.2 The UC Problem in the Presence of Wind Power

As shown, the UC problem is a complicated problem when only thermal generators are considered. This problem gets more complicated when other power production methods are

included in the problem. For example, wind power plants complicate the problem because the output of wind power plants at any given time is a random variable. Since initial appearance of wind power is in small isolated systems, the first attempts to consider wind power's intermittency effect is investigated in these systems. Bakirtzis and Dokopoulos^[31] consider a small mixed system that consisted of diesel engines, solar and wind power plants with storage and obtain a UC schedule using DP. For both the power output of the wind power plant and the load they use forecasted values. In a similar work Contaxis and Kabouris^[32] estimate power output of a wind power plant and the load by using autoregressive moving average (ARMA) models and use DP to solve the resulting problem.

Including wind power into the power pool also influences the magnitude of the SR. This is investigated by Gupta et al.^[33] where they iteratively look at the cost savings by adding wind power to the conventional power pool. They do this for every hour but do not directly integrate wind power into the UC problem and treat it as non-dispatchable. They conclude that since the wind energy cannot have any reserve there must be extra SR allocated by other generators.

A different method of incorporating wind power into the UC problem was published by Dokopoulos et al.^[34] The authors of the paper first find a UC schedule without considering wind power and then substitute wind power in solving the economic dispatch. They solve the UC problem by DP and simulate the whole system using Monte Carlo simulation. They observe that the fuel cost decreases when the total available wind power increases.

Expert systems have also been employed in solving the UC problem including wind power generation. Padhy et al.^[35] develop a solution method based on modeling the load by using fuzzy neural networks as an alternative to DP. They conclude that their method can solve

the UC problem including wind power generation. A recent work that is done in this area is supported by the European Union CARE Project. The authors in this work employ genetic algorithms in order to find a UC schedule for a system composed of thermal and wind power generators. For the SR determination they choose a percentage of the load instead of using a probabilistic criterion. They demonstrate the feasibility of using genetic algorithms in solving this problem. [36]

The wind power planning literature reviewed in this section differs from the proposed work in three aspects. First the proposed work models load and power from wind power plants as a continuous random variable instead of using single point estimates. Second it considers probabilistic reserve determination and third it considers the correlation structure of load between time periods which has not been considered before.

2.3 Spinning Reserve

In general SR requirements have been determined by rule of thumb methods such as a percentage of the peak load or 1 to 2 times of the largest generator capacity, etc. [37, 38, 8] However, these do not take into consideration the fact that load is stochastic. Not accounting for the stochasticity of the load may result in blackouts due to insufficient reserves but more often than that in loss of profit due to excess reserves or startup costs associated with attempts to fix schedules at the economic dispatch stage. One can use the inventory analogy mentioned in section 1.2 to understand the value of using a more accurate way of determining the SR.

In the literature, one of the first attempts to utilize probabilistic SR determination that considers reliability of generators is made by Billinton and Chowdhury. ^[37] They define the system risk at time t to be $R(t)$:

$$R(t) = \sum_i P_i(t)Q_i(t) \quad (2-16)$$

Here $P_i(t)$ is the probability that the generation system is in state i at time t and $Q_i(t)$ is the probability that the system load will be equal to or greater than the generation in state i at time t . The generators can be in one of three states: up, derated (reduced capacity), or failed. Additional states are also considered such as cold reserve (state where the generator has to go through a complete start up), hot reserve (state where the generator has to go through a partial start up process which takes a shorter time), etc. With this formulation it is possible to incorporate the reliability of the generators into the SR calculation. Then they proceed to define two different risks: single system risk (SSR) and interconnected system risk (ISR). A single system is required to meet its own SSR, meaning that given a constant SSR the system's reliability has to be better than this threshold. It also has to beat the predefined ISR, meaning that when considered as a pool with other systems the overall reliability has to beat the ISR value. The next step taken is to prepare tables of different load possibilities with the required number of generators and capacity for a given risk level. The main deficiency of this method is that it is very tedious and general. It doesn't take into consideration many other variables in the UC problem but only looks at the reliability issue and comes up with a production level. Chowdhury and Billinton completed an extension to this work where the systems can purchase power from the power market. ^[39]

Qinghua et al. ^[40] employ a different approach in determining SR. They formulate the UC problem in a different fashion and minimize the SR instead of the operating cost with respect to the usual UC constraints and a reliability constraint, which ensures that the system meets the required reliability level. To do this they use the system risk level calculations derived in the work by Billinton and Chowdhury. ^[37] They add a deterministic constraint to the program to satisfy this reliability level. The main advantage of this work is that it attempts to integrate the reliability constraints into the UC problem. However, using the objective of minimizing SR cannot be realistic, since the production cost and the total cost of turning generators on/off are more important than a low level of SR.

In a different attempt to incorporate the UC problem and probabilistic SR determination, Wu and Gooi ^[41] solve the UC problem using LR and at each iteration compare the SR level with the pre-calculated reliability levels of the SR and if the SR does not satisfy the reliability level then they reiterate. They demonstrate their algorithm's use for a 36-unit system and claim that it actually reduces the total system cost. Gooi et al. ^[42] accomplish a similar task in their work. These works differ from the proposed work because first they do not consider the correlation of load between time periods and more importantly they consider load to be deterministic and look at deficiencies only due to reliability problems of the generators.

All the papers mentioned in this section formulate the problem so that the reliability of the generators can be incorporated into the UC problem. There is also research that has been completed to incorporate the randomness of the load into the UC formulations. In order to improve the rules currently used for determining SR, Bobo et al. ^[8] look at a different policy from the one used by the Electric Reliability Council of Texas to determine the SR. They define SR to be different from its general definition. They call the difference between the

maximum amount a generator can produce minus the load to be the SR. They define responsive reserve to be that portion of the SR deployed automatically within the first 60 seconds following a frequency disturbance. For example, if one has only one generator to meet load, then SR would be the capacity of that generator minus the current load. The responsive reserve is predefined and is equal to $Min(SR, \text{predefined value such as } 20\% \text{ of current load})$. Instead of the utility's ongoing policy where each generator contributed 20% of its capacity to responsive reserve, they look at varying percentages of the generators' capacities. They observe that using a responsive reserve both decreases the fuel costs and increases the system reliability.

Li and Chen ^[43] employ a fuzzy neural network to determine the level of SR by looking at historical data on SR levels to detect a relationship and then they continue to train their system. Similarly Guan et al. ^[44] model the reserve constraints as fuzzy constraints, which are similar to chance constraints, and replace these with their deterministic equivalents to solve the problem. Here a fuzzy constraint is a soft constraint and satisfying these constraints is not strictly required when finding a solution to the problem.

2.4 Stochastic Programming

For simplicity in modeling large-scale integer programs stochasticity is ignored and modelers assume that the parameters of the problem that is being modeled are exactly known or can be approximated (or forecasted) with a very small error margin. This is done because it reduces the solution time and results in a model that is easier to solve. However in most cases these parameters have an underlying probability distribution and should be modeled as random variables. This clearly complicates the problem but, on the other hand, makes

the results obtained more realistic and can result in improvement of the objective function value. There are a number of different ways of incorporating this randomness into the overall model. In this section two of these are considered and their application to solving the UC problem is discussed.

2.4.1 Recourse Based Models

Recourse based models assume a corrective set of decisions can be made once the outcomes of random variables are observed. Depending on the type of the problem this can be a multi-stage program or a two-stage program. In a two-stage problem a set of recourse decisions are taken only once. On the other hand for a multi-stage problem a new set of recourse decisions can be taken at each stage of the problem.

For the two-stage program one can divide the set of decision variables into two. The first set contains the variables that need to be set in the beginning of the time horizon. The second set of variables can be decided on after the randomness for the second stage is revealed. The distribution for the random variable in the program is approximated by a discrete distribution. Then each possible outcome is assigned a weight, that reflects the probability of its occurrence. When the overall problem is solved the solution is made up of two sets of decision variables: first stage variables and second stage variables. Second stage variables consist of a different decision variable for each possible outcome modeled.

Similarly one can build a scenario tree estimating the possible outcomes for the corresponding random variable for a multi-stage program. Solving the overall problem with this scenario tree will not only provide the values for the first-stage variables but also the

corresponding decision variables for each stage. These multi-stage variables take into consideration not only the outcome of the random variable just observed but also the other possibilities in the remaining stages.

One of the first attempts to solve the UC problem using stochastic optimization has been made by Carpentier et al. ^[45] Until this work, most UC solution methods have treated load as deterministic. However, in reality the load forecast is only approximate. In their work, possible future outcomes in load are represented by a scenario tree, an example of which is shown in Figure 2.1.

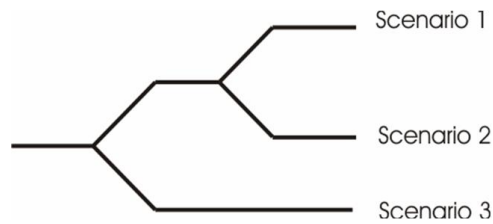


Figure 2.1 Scenario Tree for the Multi-stage Stochastic Program

This method assumes that only a finite number of scenarios are possible. Since it is impossible to use all of the finitely many possible scenarios, only a few of these are used to avoid combinatorial explosion of the computations. A probability of occurrence is attached to each scenario, where all the probabilities added up to one. The generator subproblems are solved using techniques similar to the ones used in the work by Batut and Renaud. ^[21] Solving the UC problem they obtain a three-dimensional array representing the status of each unit at each time period under each scenario. Carpentier et al. test their methods using one hundred scenarios and a time horizon of 48 hours. They validate these results using simulation.

One of the first attempts to combine the UC problem and the economic dispatch problem is made by Birge and Takriti. ^[46] In their work they handle the stochasticity of load by using multi-stage stochastic programming techniques. They approximate the nonlinear subproblems to their linear equivalents and solve these to attain a lower bound on the solution. Then they employ heuristic rules to derive a feasible solution. To combine these two different time horizon problems, they utilize match-up scheduling. To do this they create a pre-schedule offline and attach a weight to different possible scenarios. If, after obtaining outcomes, the observed scenario weights differ significantly from the original forecast, then match-up begins. For example, if a scenario that is not considered occurs, then match-up is required. First, they define a match-up point and a subproblem. Then operators take some corrective actions such as turning generators on and off or changing the amount of power obtained from a generator so that the operations can return to the normal pre-schedule. The general schedule is also updated if necessary. They claim this method can come as close as desired to an optimal solution. They demonstrate the application of their method for a system with six hydro units and more than 100 thermal generators but use only ten scenarios. ^[46]

Takriti and Birge's method is criticized by Dentcheva and Romisch ^[47] to be inefficient because even for a small scenario tree the method creates numerous variables. For instance, Birge and Takriti ^[46] use only ten scenarios to demonstrate the application of their algorithm. Dentcheva and Romisch model the problem instead as a two-stage stochastic program. In their model Dentcheva and Romisch separate the binary variables that indicate a generator's state into two stages. This enables the model to take compensatory actions in the second-stage and turn on generators or increase the output levels of the already working generators

to meet the load. This means that for each time period there are two decision variables for both the status and the power output of the generator. However, they do not provide any numerical or performance results. Caroe and Schultz ^[48] have a similar critique of the Birge and Takriti approach. The authors take advantage of two-stage stochastic programming and combine LR with branch and bound to solve the UC problem. In Caroe and Schultz's paper two-stage stochastic programming works in the following manner. They define the amount of power that will be generated by thermal generators in all time periods as first-stage variables and the amount of power that will be obtained using gas turbines, which take less time to turn on and off, as the second-stage variables. These results are used in committing generators. Naturally, they take into consideration the fact that the gas turbines are more expensive to utilize. Similar to Birge and Takriti ^[46], Nowak and Romisch ^[49] employ multi-stage stochastic programming but solve the subproblems by using ALR. They claim that for their solution the duality gap is not dependent on the number of scenarios used.

2.4.2 Joint Chance Constrained Programming

The presence of random variables in the UC problem requires methods that should take into consideration stochasticity in the problem. There are different possible approaches one can take to model random variables in a mathematical program. One of these methods is joint chance constrained programming where a defined probability level is stipulated for the violation of a set of constraints. ^[50] This chance constraint can then be converted a deterministic form and the resulting program can be solved. ^[51] Especially when the random variables follow a multivariate normal distribution appropriate solution methods have been developed. ^[52]

This approach has been used before in power planning to model random variables for a wide range of problems. For example Sharaf and Berg ^[53] make use of this method to solve a power transmission problem to determine the optimal transmission network in which the load and fuel prices are modeled as random variables. In the power capacity expansion problem a similar approach is taken in modeling load by Shiina ^[54] and Anders ^[55]. Anders looks at the problem where the load is approximated by a multivariate normal distribution and is not correlated between time periods. Similarly Shiina looks at the same problem but also considers the correlations and develops a method where the cumulative distribution function integral is approximated by a Gauss quadrature formula which uses an approximate discrete sum to the continuous integral. Some aspects of Shiina’s approach are similar to the proposed work with a couple of significant differences. First of all, the UC problem is different from the power expansion problem in both the description and solution methods since the UC problem is a MIP while the power expansion problem is modeled as a LP by Shiina. Second, the time horizon Shiina looks at is four time periods where each period is three months long. The proposed work looks at time periods as long as four days where the use of the same approach will not work since this creates 96 nested sums in approximating the chance constraint.

2.5 Multivariate Probability Evaluation

Calculating multivariate probabilities for multivariate normal distribution is often a difficult problem. There are reliable methods to calculate probabilities for dimensions up to seven. A paper by Schervish ^[56] demonstrates such a method that employs a locally adaptive numerical integration strategy. The locally adaptive technique is an integration technique

that splits the original integration region and integrates again over the region with a higher error to reduce the estimation error. Royen ^[57] and Iyengar ^[58] also tackle the problem of calculating multivariate probabilities of regions using different methods.

For the UC problem solved in this dissertation the satisfaction probability of the load constraint needs to be calculated at each iteration. If the time period is 1 or 2 hours then these probabilities can be calculated in the following manner. To introduce the problem start by defining a_t which corresponds to the total sum of power generated using generators 1 through N at time period t :

$$a_t = \sum_{k=1}^N P_{k,t} \quad (2-17)$$

Look at a simple case where there is no correlation between time periods. If $l \sim N_T(0, I)$, that is l follows a T -variate normal distribution with mean 0 and covariance vector I then one can make the following observations for the simple cases where $T = 1$ and $T = 2$.

$$T = 1 \quad \rightarrow \quad \Phi(a_1) = 1 - \alpha \quad \Rightarrow \quad a_1 = \Phi^{-1}(1 - \alpha) \quad (2-18)$$

$$T = 2 \quad \rightarrow \quad \Phi(a_1)\Phi(a_2) = 1 - \alpha \quad \Rightarrow \quad a_2 = \Phi^{-1}\left[\frac{(1 - \alpha)}{\Phi(a_1)}\right] \quad (2-19)$$

If, in fact, there is correlation one needs to make the following changes. Assume in the simplest case, where $X, Y \sim N(0, 0, 1, 1, \rho)$ and $\rho \geq 0$, to find $P[X \geq a_1 \ \& \ Y \geq a_2]$ the following can be used. Note that Z_i is the standard normal random variate and the Z_i 's are independent:

$$X = \sqrt{\rho}Z_1 + \sqrt{1-\rho}Z_2, \quad (2-20)$$

$$Y = \sqrt{\rho}Z_1 + \sqrt{1-\rho}Z_3. \quad (2-21)$$

Where $E[X] = E[Y] = 0$ and $Var[X] = Var[Y] = 1$, $Cov(X, Y) = \rho$. This is true because the sum of two independent, identically distributed variables that follow a normal distribution also follows a normal distribution. The covariance of X and Y will equal ρ since the Z_i 's are independent, and

$$\begin{aligned} cov(X, Y) &= \rho \ cov(Z_1, Z_1) + \sqrt{\rho(1-\rho)} \ cov(Z_1, Z_3) + \\ &\sqrt{\rho(1-\rho)} \ cov(Z_1, Z_2) + (1-\rho) \ cov(Z_2, Z_3) = \rho. \end{aligned} \quad (2-22)$$

If one replaces X and Y in $P[X \geq a_1 \ \& \ Y \geq a_2]$ with their equivalent in (2-20) and (2-21) then the following is obtained:

$$P[X \geq a_1 \ \& \ Y \geq a_2] = P[\sqrt{\rho}Z_1 + \sqrt{1-\rho}Z_2 \geq a_1 \ \& \ \sqrt{\rho}Z_1 + \sqrt{1-\rho}Z_3 \geq a_2]. \quad (2-23)$$

Conditioning on $Z_1 = z$ one can write the following:

$$P[X \geq a_1 \ \& \ Y \geq a_2] = \int_{-\infty}^{\infty} P[\sqrt{1-\rho}Z_2 \geq a_1 - \sqrt{\rho}z \ \& \ \sqrt{1-\rho}Z_3 \geq a_2 - \sqrt{\rho}z]\Phi(z)dz. \quad (2-24)$$

Since Z_2 & Z_3 are independent, one can separate the probabilities and write the following expression:

$$P[X \geq a_1 \ \& \ Y \geq a_2] = \int_{-\infty}^{\infty} P[Z_2 \geq \frac{a_1 - \sqrt{\rho}z}{\sqrt{1-\rho}}] P[Z_3 \geq \frac{a_2 - \sqrt{\rho}z}{\sqrt{1-\rho}}] \Phi(z)dz. \quad (2-25)$$

Then, using the properties of the normal distribution, one can write the following equivalent statement:

$$P[X \geq a_1 \ \& \ Y \geq a_2] = \int_{-\infty}^{\infty} (1 - \Phi(\frac{a_1 - \sqrt{\rho}z}{\sqrt{1-\rho}}))(1 - \Phi(\frac{a_2 - \sqrt{\rho}z}{\sqrt{1-\rho}}))\Phi(z)dz. \quad (2-26)$$

From here it is possible to calculate the value of the integral using an approximation to the normal integral. However, for time periods longer than two units, a different method needs to be used. Under the assumption of a stationary AR(1) distribution for the load process the multi-dimensional correlation matrix is as follows:

$$R = \begin{pmatrix} 1 & \rho & \rho^2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & \dots & \rho^{T-2} \\ \rho^2 & \rho & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ \cdot & & & & \cdot \\ \rho^{T-1} & \rho^{T-2} & \dots & \dots & \dots & 1 \end{pmatrix}$$

, where ρ is the correlation coefficient of l_t and l_{t-1} . Some researchers suggest replacing the entries of matrix R with $\bar{\rho}$ where $\bar{\rho} = \frac{1+\rho+\dots+\rho^T}{T+1}$ and check if this gives acceptable results. This method is used in Iyengar's work and satisfactory results are obtained for certain types of problems. ^[58] An example \bar{R} matrix is shown below. \bar{R} is preferred to R due to the computational advantage in evaluating the resulting multiple integral.

$$\bar{R} = \begin{pmatrix} 1 & \bar{\rho} & \bar{\rho} & \dots & \bar{\rho} \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ \bar{\rho} & \bar{\rho} & \dots & \dots & 1 \end{pmatrix}$$

A different approach to multivariate probability calculation in convex polyhedra is used by Iyengar ^[59] where he employs importance sampling, which is a variance reduction technique that chooses a good distribution to simulate a normal distribution. In his paper Iyengar provides an approximation to the multivariate probability by using rotation, which is a plot interpolation technique for three dimensional data.

A survey of different methods for evaluating multivariate probabilities can be found in the paper by Deak. ^[60] More recently, a method developed by Genz ^[11] provides an algorithm

that calculates multivariate normal tail probabilities with up to 500 dimensions with a small error margin. In his paper, Genz uses three transformations so that the transformed form can be evaluated using a subregion adaptive numerical integration algorithm which is a variation of locally adaptive numerical integration. The first transformation performed is a Cholesky decomposition transformation. Cholesky decomposition is the transformation of a symmetric matrix to a product of a lower triangular matrix and its transpose. The second transformation inverts the random variables corresponding to each integrand using the properties of the normal distribution. And finally the last action is to transform the limits of the integral. After this, the transformed integral becomes suitable to be evaluated by a subregion adaptive numerical integration algorithm. Genz compares his algorithm's performance to existing methods and demonstrates its superiority to existing methods. In the algorithm developed for this dissertation, Genz's code is used to evaluate multivariate probabilities.

In this chapter relevant work from the literature is discussed. There are many differences between the methods used in this dissertation and the previous approaches applied to the UC problem. This dissertation solves the UC problem, where there is a mix of generators providing power including conventional thermal generators and wind power generators. The proposed model accounts for the randomness of the power produced by the wind power generators as well as the load. Although some work has been done in the wind UC field, none of the papers that cover UC scheduling consider wind power as a random variable. This dissertation considers the correlation in load between time periods and analyzes these effects. It also models the demand using chance constraints and a continuous probability distribution instead of a discrete approximation as used in recourse based models.

3.0 A STOCHASTIC FORMULATION OF THE UC PROBLEM

As summarized in Chapter 2, for most of the solution methods developed for the UC problem the electricity demand is modeled as deterministic. However, this approach can result in schedules that are unrealistic. In the previous chapter, recourse based models are introduced and the advantages and disadvantages of this approach are discussed. In this chapter, a novel approach in modeling the stochastic UC problem is introduced where a chance constraint is used in modeling the load constraint. The following section restates the UC problem. Section 3.2 presents a mathematical programming formulation of the problem. In section 3.3 a stochastic load constraint is developed to be used in solving the chance constrained program. Finally, section 3.4 discusses the effect of correlation on the optimum value of the objective function.

3.1 Problem Statement

The model that will be used in this dissertation considers a power generation system consisting of two classes of generators: thermal generators and wind power generators. It is assumed that neither the wind power generators nor the thermal generators fail in the short time horizon considered (at most four days). The model that is presented does not explicitly consider the power obtained from the wind power generators at hour t , w_t . Since both w_t and residual load, l_t , can be approximated using a normal distribution, then $(l_t - w_t)$

can also be approximated by a normal distribution. Therefore using this residual load in the methodology will implicitly take into consideration wind power. The notation used throughout this section can be found in the nomenclature section.

In solving the UC problem one seeks answers to the following questions:

- From the pool of generators that the utility owns, which ones should be turned on?
- How much power should the generators that are on produce?
- Once a generator is turned on how long should it stay up? And once it is down how long should it stay down?

As stated earlier, the assumption is that the system needs to operate on its own without any access to secondary power sources; therefore the utility needs to decide what probability level it would desire to operate under. Often, the utility must satisfy demand with a high level such as 0.9999.

The objective of this problem is to satisfy the load with a specific probability level while minimizing the operating cost. This objective is subject to several constraints. The first set of constraints is capacity constraints which ensure that the generators operate within their minimum and maximum generation levels. The second set of constraints are uptime and downtime constraints that ensure that once a generator is up/down, it stays at that state for at least a given length of time. Finally, the last set of constraints ensures that the demand for electricity is satisfied for all time periods. For simplicity, ramp rate constraints are not included in the model but can be without any changes in the rest of the model. In the next section the deterministic formulation for the UC problem is presented. This is followed by the presentation of the model used in this dissertation.

3.2 Deterministic Model Formulation

The basic deterministic formulation, model **UC-D**, is given in equations (3-1)-(3-7).

UC-D

$$\text{Min} \sum_{k=1}^K \sum_{t=1}^T \{CF_{k,t}(P_{k,t}, \nu_{k,t}) + S_{kt}(\nu_{k,t})\} \quad (3-1)$$

subject to

$$\nu_{k,t} P_{k,\min} \leq P_{k,t} \leq \nu_{k,t} P_{k,\max} \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T \quad (3-2)$$

$$\nu_{k,t} - \nu_{k,t-1} \leq \nu_{k,\kappa}, \quad \kappa = t + 1, \dots, \min\{t + \kappa_k - 1, T\}, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T \quad (3-3)$$

$$\nu_{k,t-1} - \nu_{k,t} \leq 1 - \nu_{k,\tau}, \quad \tau = t + 1, \dots, \min\{t + \tau_k - 1, T\}, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T \quad (3-4)$$

$$\sum_{k=1}^K P_{k,t} \geq l_t \quad t = 1, 2, \dots, T \quad (3-5)$$

$$\nu_{k,t} \in \{0, 1\}, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T \quad (3-6)$$

$$P_{k,t} \geq 0, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T \quad (3-7)$$

The objective function (3-1) consists of two terms. The first term considers the fuel costs and consists of two parts:

$$CF_{k,t}(P_{k,t}, \nu_{k,t}) = \max_{l=1,\dots,L} f_{kl}(P_{k,t}) + \nu_{k,t}c_k \quad (3-8)$$

The first part is a linear or convex piecewise quadratic function with L parts, based on the power output of the unit during a time interval, $f_{kl}(P_{k,t})$, and the second part is a fixed cost term, c_k .

The second term of the objective function considers the start up cost, where the cost is time dependent as shown in (3-9).

$$S_{kt}(\nu_{k,t}) = (C_k^f + C_k^c(1 - e^{-\frac{-(t-t_{s_k})}{\alpha_k}}))\max\{\nu_{k,t} - \nu_{k,t-1}, 0\}, \quad t = 2, \dots, T \quad (3-9)$$

In (3-9) C_k^f represents fixed costs, C_k^c represents cold start costs, α_k is the thermal time constant for the unit k , and $t - t_{s_k}$ is the downtime of unit k , between the current time period t and the last time period the unit was up, t_{s_k} , where $s_k = \max\{s \in \mathbf{N} : \nu_{k,t-j} = \nu_{k,t-1}, j = 2, \dots, s\}$. Shutdown costs, though not included in the formulation, can be included without any change to the rest of the formulation.

The problem constraints are listed in equations (3-2) through (3-5). Constraint (3-2) ensures that the power generated matches the minimum and maximum capacity requirements of the corresponding generators for all time periods. Constraints (3-3) and (3-4) are the uptime/downtime constraints that force the generators to stay up for a specified amount of time, κ_k , once they are turned on and stay down for a specified time period, τ_k , once they are shut down. The last set of constraints (3-5) are the linking constraints that link

the decision variables of different generators and time periods. These constraints ensure that the estimated load is satisfied in all time periods. They cause difficulties in solving the problem because adding them to the constraint set makes the problem inseparable thus requiring more sophisticated techniques for finding a solution.

3.3 Development of the Stochastic Load Constraint

To convert the deterministic model presented in Section 3.2 to a stochastic one the load constraint is modified. Let l_t be equivalent to the residual load after the temperature effect is removed and be a random variable. Note that replacing l_t with $l_t - w_t$ will give similar results. As stated in the work by Valenzuela et al. [2], one can approximate the load as a multivariate normal distribution with a specific correlation structure: $l \sim N(\mu, \Sigma)$ with mean vector μ and covariance matrix Σ . Then one can replace constraint set (3-5) by the single probabilistic constraint (3-10).

$$P\left[\sum_{k=1}^K P_{k,t} \geq l_t \text{ for } t = 1, 2, \dots, T\right] \geq 1 - \alpha \quad (3-10)$$

The purpose in doing this is to give a joint probability level on satisfying the linking constraint over all time periods. Note that the distribution for the load is T-variate. This constraint states that the probability that the load is satisfied in all time periods is at least $p_{target} = (1 - \alpha)$. For example if p_{target} is equal to 0.90 then out of 10 possible realizations in at most one of them the load in at least one time period won't be met. Replacing $\sum_{k=1}^K P_{k,t} \geq l_t$ by A_t the following equivalent constraint is obtained:

$$P[A_t \text{ for } t = 1, 2, \dots, T] \geq 1 - \alpha. \quad (3-11)$$

It is not possible to invert this probabilistic constraint to a deterministic form therefore another constraint is necessary to solve the stochastic optimization problem. So instead the following constraint $P[A_t^c] \leq \frac{\alpha}{T} \quad t = 1, 2, \dots, T$ is used in solving the problem and it is shown that the latter probabilistic constraint is more conservative than the former one. The following proposition states this and gives a formal proof.

Proposition 1 *The feasible region represented by $P[A_t^c] \leq \frac{\alpha}{T} \quad t = 1, 2, \dots, T$ is a subset of the region represented by the constraint $P[A_t \text{ for } t = 1, 2, \dots, T] \geq 1 - \alpha$.*

Proof: Constraint (3-10) is equivalent to the following constraint:

$$P\left[\bigcap_{t=1}^T \left[\sum_{k=1}^K P_{k,t} \geq l_t\right]\right] \geq 1 - \alpha. \quad (3-12)$$

The term in inner brackets can be replaced with A_t . Using DeMorgan's law, which states that the complement of a conjunction is the disjunction of the complements or vice versa, and applying the complement rule one obtains the following equivalent constraints:

$$P\left[\bigcap_{t=1}^T A_t\right] = 1 - P\left[\bigcup_{t=1}^T A_t^c\right] \geq 1 - \alpha \quad (3-13)$$

$$P\left[\bigcup_{t=1}^T A_t^c\right] \leq \alpha. \quad (3-14)$$

Consider the first expression stated in the proposition which is restated below:

$$P[A_t^c] \leq \frac{\alpha}{T} \quad t = 1, 2, \dots, T. \quad (3-15)$$

Then (3-15) implies that the following constraint must hold:

$$\sum_{t=1}^T P[A_t^c] \leq \alpha. \quad (3-16)$$

Here, if Boole's inequality, which states the following, is used,

$$P\left[\bigcup_{t=1}^T A_t^c\right] \leq \sum_{t=1}^T P[A_t^c] \quad (3-17)$$

one can conclude that the feasible region represented by (3-15) is a subset of that represented by (3-14). Therefore, if the former one is satisfied, the latter one is also satisfied. \square

Note that the right hand side value of (3-15) does not have to be the same for each time period, however for simplicity this special case will be considered in the remainder of this dissertation. Different values for different time periods can be used which can possibly improve the solutions obtained by obtaining a lower SR level. Now, using (3-15) and replacing A_t^c one can write the following:

$$P\left[\sum_{k=1}^K P_{k,t} < l_t\right] \leq \frac{\alpha}{T} \quad \text{for } t = 1, 2, \dots, T. \quad (3-18)$$

Using the properties of the Normal distribution one obtains the following expressions by inverting the above expression. Here $\mu = (\mu_1, \mu_2, \dots, \mu_T)$ is the T dimensional mean vector and σ_t is the standard deviation corresponding to time period t .

$$1 - \Phi\left[\frac{\sum_{k=1}^K P_{k,t} - \mu_t}{\sigma_t}\right] \leq \frac{\alpha}{T} \quad \text{for } t = 1, 2, \dots, T \quad (3-19)$$

$$\Phi\left[\frac{\sum_{k=1}^K P_{k,t} - \mu_t}{\sigma_t}\right] \geq 1 - \frac{\alpha}{T} \quad \text{for } t = 1, 2, \dots, T \quad (3-20)$$

Finally, the resulting constraint is shown in (3-21) where $Z_{(1-\frac{\alpha}{T})}$ is the z-value of the corresponding probability value of $(1 - \frac{\alpha}{T})$ for the standard normal distribution:

$$\sum_{k=1}^K P_{k,t} \geq \mu_t + Z_{(1-\frac{\alpha}{T})}\sigma_t \quad \text{for } t = 1, 2, \dots, T. \quad (3-21)$$

For the purposes of the algorithm used in this dissertation constraint (3-22) is used instead of (3-21) where z is varied until the left hand side of (3-10) is within the ϵ neighborhood of $(1 - \alpha)$.

$$\sum_{k=1}^K P_{k,t} \geq \mu_t + z\sigma_t \quad \text{for } t = 1, 2, \dots, T. \quad (3-22)$$

Remark: Note that as α approaches 0 the z-value in (3-22) approaches ∞ . This happens since the second stochastic constraint derived (3-15) is a conservative one and is not equivalent to the original stochastic constraint (3-10). In the algorithm, however, this will not cause any problems since at each iteration the original stochastic constraint will be evaluated and the z-value will be updated to make the probability converge to $(1 - \alpha)$. \square

Now the problem is solved using constraint (3-22) instead of (3-5). Note that using this new constraint ensures separability of the original probabilistic constraint (3-10) over different time periods. Also using this constraint shrinks the feasible region. This is true because the new set of constraints is tighter than the previous ones for the minimization problem that is being solved. Once the optimum values of the decision variables $P_{k,t}$ are obtained using constraint (3-22), one can calculate the multivariate probability on the left hand side of (3-10) with this set of values. The $P_{k,t}$ values will be larger than the corresponding values when the probability constraint (3-10) is used. If this value is a lot higher than $(1 - \alpha)$ one

can then replace the z -value in (3-22) by a smaller value and iterate the algorithm until the inequality (3-10) is satisfied as an equality. The resulting stochastic formulation is shown on the next page. The only difference between the stochastic UC formulation **UC-S** and problem **UC-D** is due to the load constraint. Now the load is considered to be a random variable instead of an estimated value.

UC-S

$$\text{Min} \sum_{k=1}^K \sum_{t=1}^T \{CF_{k,t}(P_{k,t}, \nu_{k,t}) + S_{kt}(\nu_{k,t})\}$$

subject to

$$\nu_{k,t} P_{k,\min} \leq P_{k,t} \leq \nu_{k,t} P_{k,\max} \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T$$

$$\nu_{k,t} - \nu_{k,t-1} \leq \nu_{k,\kappa}, \quad \kappa = t + 1, \dots, \min\{t + \kappa - 1, T\}, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T$$

$$\nu_{k,t-1} - \nu_{k,t} \leq 1 - \nu_{k,\tau}, \quad \tau = t + 1, \dots, \min\{t + \tau - 1, T\}, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T$$

$$P\left[\bigcap_{t=1}^T \left[\sum_{k=1}^K P_{k,t} \geq l_t\right]\right] \geq 1 - \alpha$$

$$\nu_{k,t} \in \{0, 1\}, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T$$

$$P_{k,t} \geq 0, \quad k = 1, 2, \dots, K, \quad t = 1, 2, \dots, T$$

3.4 The Effect of Considering the Correlated Load Structure on the UC

Solution

It was mentioned that the load can be approximated by a multivariate normal distribution with a specific correlation structure. In solving problem **UC-S** this correlation structure can be considered in the solution methodology developed. If the stochastic model **UC-S** is solved assuming a correlation coefficient of zero then the objective function value of the solutions obtained are guaranteed to be greater than or equal to the objective function value of the solutions obtained using the positive correlation structure. Thus if in the real problem the load data is correlated then the solution obtained by considering this fact will be at least as good as that found by assuming zero correlation for the load data. This result requires the following well-known inequality whose proof can be found in the original paper by Slepian ^[61] and in the book by Tong ^[62].

Theorem 1 (*Slepian's inequality*). *Let X be a T -dimensional multi-normal random vector with mean μ and covariance matrix $\Sigma = (\sigma_{ij})$, and let Y be a T -dimensional multi-normal random vector with mean μ and covariance matrix $\Gamma = (\gamma_{ij})$. If $\sigma_{ii} = \gamma_{ii}$ for $i = 1, \dots, T$, and $\sigma_{ij} \geq \gamma_{ij}$ for all $i \neq j$, then*

$$P\left(\bigcap_{t=1}^T \{X_t > a_t\}\right) \geq P\left(\bigcap_{t=1}^T \{Y_t > a_t\}\right)$$

and

$$P\left(\bigcap_{t=1}^T \{X_t < a_t\}\right) \geq P\left(\bigcap_{t=1}^T \{Y_t < a_t\}\right)$$

Proposition 2 Consider two solutions obtained to the problem **UC-S**, where the first one considers a positive correlation coefficient while solving the problem whereas the second one assumes the correlation to equal zero, then:

$$\sum_{k=1}^K \sum_{t=1}^T \{CF_{k,t}(P_{k,t}^c, \nu_{k,t}^c) + S_{kt}(\nu_{k,t}^c)\} \leq \sum_{k=1}^K \sum_{t=1}^T \{CF_{k,t}(P_{k,t}^{nc}, \nu_{k,t}^{nc}) + S_{kt}(\nu_{k,t}^{nc})\} \quad (3-23)$$

where $\nu_{k,t}^c$ and $P_{k,t}^c$ represent the decision variables obtained considering positive correlation and $\nu_{k,t}^{nc}$ and $P_{k,t}^{nc}$ are the decision variables obtained if the problem is solved assuming zero correlation.

Proof: This can be proved using Theorem 1. Assume X and Y correspond to the load vectors of the problem with positive correlation and zero correlation, respectively. When the optimization problem is solved for zero correlation one obtains a UC schedule where a_t correspond to the total power available at time period t . By the second expression in Theorem 1, the probability of satisfying constraint (3-5) is higher when $\sigma_{ij} \geq \gamma_{ij}$ for all $i \neq j$ as shown below for this schedule.

$$P\left(\bigcap_{t=1}^T \{X_t < a_t\}\right) \geq P\left(\bigcap_{t=1}^T \{Y_t < a_t\}\right) \cong (1 - \alpha)$$

Since we are solving for $1 - \alpha$ in both cases it follows that the optimum schedule for the positive correlation case will be different where the total power available at time period t (a_t^c) satisfy the following inequality:

$$a_t^c \leq a_t. \quad (3-24)$$

In other words reducing a_t will reduce the satisfaction probability of the case where there is positive correlation. Therefore the number of generators that should be committed for time period t (or the total power output) when there is correlation should be either less than or equal to the number of generators that should be committed (or the total power output) when there is zero correlation. Since the objective function is an increasing function of the number of units and their total power output the proposition follows. \square

This proposition justifies considering correlation when solving problem **UC-S** since the objective function value is guaranteed to be either the same or better due to the drop in power allocated for all periods. This is an additional reason why the stochastic model presented in the previous section should be considered in solving the UC problem.

4.0 A METHODOLOGY FOR SOLVING THE CHANCE CONSTRAINED PROGRAM

This Chapter provides a description of a methodology for solving the chance constrained program introduced in Chapter 3. First the convexity of the search space is established which ensures that the proposed methodology can solve the problem to optimality. The rest of this chapter presents details of the LR based methodology that is used for solving the stochastic UC problem. Recall that LR is one of the most widely used methods for solving integer programs with linking constraints. Relaxing of the coupling constraints of the UC problem is commonly done in order to solve large problems. The idea behind using LR is to solve the dual problem obtained by relaxing the coupling constraints in order to obtain a solution for the primal problem. However, the solution to the dual obtained by using the subgradient algorithm will not always be feasible for the primal problem. In this case, a heuristic is needed to derive a primal optimal solution from the dual solution. As shown in Figure 4.1 the solution methodology takes into consideration the stochasticity of load and its correlation structure while solving the UC problem for a prescribed probability level.

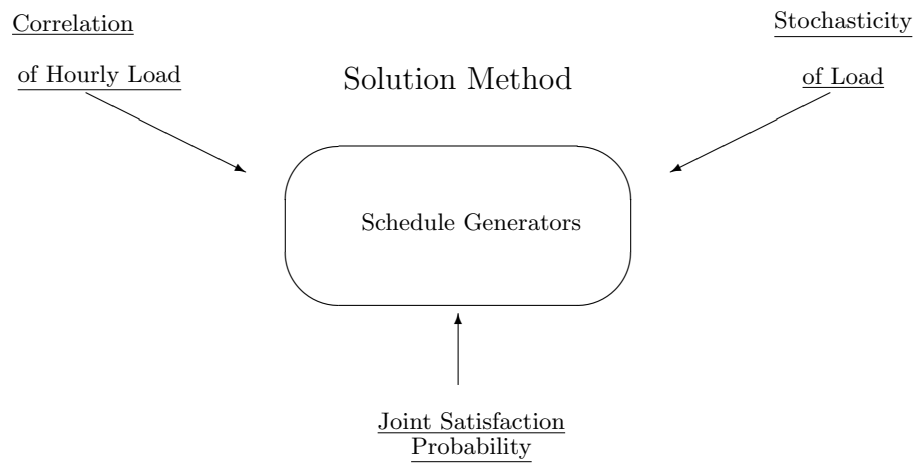


Figure 4.1 Overview of the Solution Method

4.1 Preliminaries

Usually there is no guarantee that the resulting polyhedra for a stochastic program is convex even when the individual constraints are linear. Consider the definitions below for the UC problem.

- $P \in \mathfrak{R}^{K \times T}$ is a decision variable matrix.
- $l \in \Omega \subset \mathfrak{R}^T$ is a random vector.
- Assume that $(\Omega, \mathcal{F}, \mathcal{P})$ is a known probability space, where a family of events are given.
- $(1 - \alpha)$ is a prescribed level of probability.
- J_{tk} is the coefficient matrix with t rows and k columns for P which is equivalent to the unit matrix for the UC problem.

Definition 1 *Probability measure \mathcal{P} on \mathfrak{R}^r is called a logarithmic concave probability measure, if*

$$\mathcal{P}(\lambda A + (1 - \lambda)B) \geq \{\mathcal{P}(A)\}^\lambda \{\mathcal{P}(B)\}^{1-\lambda}$$

holds $\forall A, B \subseteq \mathfrak{R}^r, \forall \lambda : 0 \leq \lambda \leq 1$, where $\lambda A + (1 - \lambda)B = \{\lambda x + (1 - \lambda)y : x \in A, y \in B\}$.

Taking advantage of this definition the following theorem can be stated which allows the application of traditional optimization methods. This theorem shows that the resulting problem has a convex constraint set. ^[52]

Theorem 2 *If l has a logarithmic concave probability measure on \mathfrak{R}^r and has a distribution function F , then the set of feasible points of $F(J_{tk}P) \geq (1 - \alpha)$ is convex.*

The proof for this theorem can be found in the work by Prekopa. Since the normal distribution has a logarithmic concave probability measure and the objective function is convex, the resulting constraint set is jointly convex which means that the algorithm that will be presented can solve the problem to optimality.

4.2 Overview of the Methodology

An overview of the algorithm used in solving the stochastic UC problem can be seen in Figure 4.2. The algorithm needs a z -value so the right hand side of constraint 3-22 can be determined. Next the λ multipliers are initialized so the dual function can be calculated. The dual problem is stated in expression 4-1.

$$q^*(\lambda) = \max_{\lambda \geq 0} q(\lambda) \quad (4-1)$$

where the dual function is

$$q(\lambda) = \min_{P, \nu} L(P, \nu, \lambda). \quad (4-2)$$

The Lagrangian function is the following:

$$L(P, \nu, \lambda) = \sum_{k=1}^K \sum_{t=1}^T \{CF_k(P_{k,t}, \nu_{k,t}) + S(\nu_{k,t})\} + \sum_{t=1}^T \lambda^t (\mu_t + z\sigma_t - \sum_{k=1}^K P_{k,t}). \quad (4-3)$$

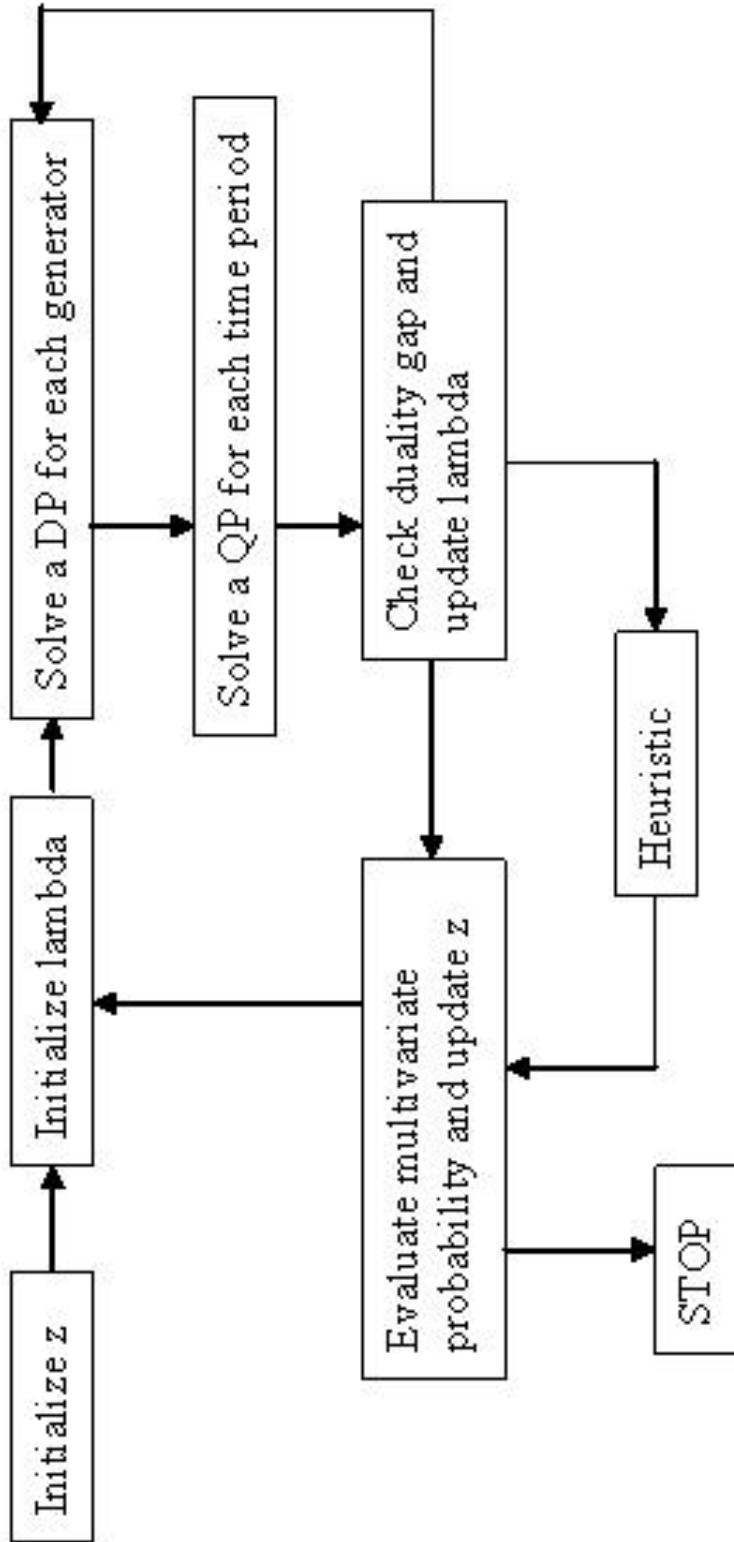


Figure 4.2 Block Diagram of the Proposed Method

When the linking constraint is relaxed the dual problem can be solved separately for different units. Note that for a given λ the term including μ_t and σ_t can be dropped while solving the DP to commit units since λ is a constant and the objective is minimization. Thus the resulting objective is given in (4-4):

$$L(P, \nu, \lambda) = \sum_{k=1}^K \left(\sum_{t=1}^T \{CF_k(P_{k,t}, \nu_{k,t}) + S(\nu_{k,t})\} - \lambda^t P_{k,t} \right). \quad (4-4)$$

The unit sub problem solved for each unit k using DP is presented in problem **U-SP**. Once all unit sub problems are solved then $q^*(\lambda)$ can be calculated for the current iteration.

U-SP

$$\text{Min} \sum_{t=1}^T \{CF_k(P_{k,t}, \nu_{k,t}) + S(\nu_{k,t})\} - \lambda^t P_{k,t}$$

subject to

$$\nu_{k,t} P_{k,min} \leq P_{k,t} \leq \nu_{k,t} P_{k,max} \quad t = 1, 2, \dots, T$$

$$\nu_{k,t} - \nu_{k,t-1} \leq \nu_{k,\kappa}, \quad \kappa = t + 1, \dots, \min\{t + \kappa_k - 1, T\}, \quad t = 1, 2, \dots, T$$

$$\nu_{k,t-1} - \nu_{k,t} \leq 1 - \nu_{k,\tau}, \quad \tau = t + 1, \dots, \min\{t + \tau_k - 1, T\}, \quad t = 1, 2, \dots, T$$

$$\nu_{k,t} \in \{0, 1\}, \quad t = 1, 2, \dots, T$$

$$P_{k,t} \geq 0, \quad t = 1, 2, \dots, T$$

Then in the next step the algorithm solves the period subproblems (**P-SP**) (quadratic programs) using CPLEX 7.0 for each time unit t . This subproblem is presented below. Note that here the only decision variables are $P_{k,t}$'s and $\nu_{k,t}$'s are already determined solving **U-SP**.

SP. P-SP

$$\text{Min} \sum_{k=1}^K CF_k(P_{k,t}, \nu_{k,t})$$

subject to

$$\nu_{k,t}P_{k,min} \leq P_{k,t} \leq \nu_{k,t}P_{k,max} \quad t = 1, 2, \dots, T$$

$$\sum_{k=1}^K P_{k,t} \geq \mu_t + z\sigma_t$$

$$P_{k,t} \geq 0, \quad k = 1, 2, \dots, K$$

Theoretically, in the optimal solution there should be no constraint violation and therefore the Lagrangian function value should equal the primal objective function value. In the algorithm the primal feasible result, J^* , is defined as below:

$$J^* = \min L(P, \nu, \lambda^*) \tag{4-5}$$

Using the $P_{k,t}$'s obtained solving **P-SP** and $\nu_{k,t}$'s obtained solving **U-SP** the value of J^* is calculated. In the next step a subgradient method updates λ and the duality gap is

calculated to check convergence. If the convergence is not achieved the problem is solved again with the new set of λ 's, otherwise a follow-up heuristic achieves feasibility if necessary. In the next step the multivariate probability of (3-10) is calculated and if this value is not close to $p_{target} = (1 - \alpha)$ then an update methodology modifies the value of z that is used to calculate the right hand side value of constraint (3-22) and the algorithm solves the whole problem using this new z . If the convergence to the prescribed probability level is achieved the algorithm terminates with a solution that satisfies load with the given probability value. All of these steps are explained in more detail in the next section.

4.3 Solving the Optimization Problem

In this section the steps of the optimization part of the algorithm can be found followed by a detailed description of each step. Here superscript i represents the i^{th} iteration. Steps of the chance constrained optimization (CCO) algorithm is presented below. A detailed description of the algorithm follows.

Algorithm CCO

Step 1 Choose an initial z -value.

Step 2 Choose a starting set of λ .

Step 3 For each unit k solve a dynamic program with $4T$ states and T stages; obtain $q^*(\lambda^i)$.

Step 4 Solve the economic dispatch problem for each hour using the scheduled units and obtain J^* .

Step 5 Check the relative duality gap $\frac{J^* - q^*(\lambda^i)}{q^*(\lambda^i)}$.

Step 6 Update λ using $\lambda^{i+1} = \lambda^i + s^i g^i$, where:

$$s^i = \frac{\alpha^i (J^* - q^*(\lambda^i))}{\|g^i\|^2}, \quad (4-6)$$

$$\alpha^i = \frac{1 + m}{i + m} \quad (4-7)$$

and g^i is the subgradient. If the gap is not small enough then go back to step 3.

Otherwise continue.

Step 7 If the final solution set is feasible go to step 8. Otherwise use the heuristic algorithm to derive a feasible solution.

Step 8 Evaluate the multivariate probability, if convergence to the prescribed probability level (p_{target}) is not achieved then update z and go back to step 2, otherwise STOP.

The algorithm starts by choosing a high value for the initial z -value, which makes the corresponding solution satisfy the load with a probability level higher than $p_{target} = (1 - \alpha)$. This z -value can be selected differently depending on the dimension of the problem. For example, $z = 5.0$ is chosen for 96 dimensions. At step 2 all λ multipliers are set to 0.0. Then at step 3 the dual problem is solved using DP and $q^*(\lambda)$ is obtained. At this step the scheduling problem for each generator is solved separately to decide which generators should be turned on. The forward recursion DP has $4T$ states and T stages. Here state corresponds to how long a generator has been up if it is a positive number or down if it is a negative number. On the other hand stage corresponds to the time period. The initial state of a generator is assumed to be between $-T$ and T . Since the time horizon for the problem is T , the final state has to be between $-2T$ and $2T$. Figure 4.3 demonstrates what

the state space would look like if the time horizon is $T=2$. Note that in the diagram there is no state corresponding to 0. This is due to the following reason. Assume a generator has been up for 4 hours at time period t then the corresponding state is 4. If the generator is turned off at that state then at time period $t + 1$ it has been down for 1 hour and therefore the corresponding state is -1, not 0. The same applies for transition to state 1 from negative states.

For all problems the initial states of the generators are given. The DP takes these as inputs and at each stage calculates for all different possible actions. If the unit is currently down and has been down for less than τ_k time units then the only possible action is to preserve the unit's off status. If it has been down for more than τ_k time units then there are two possible actions. The cost for preserving unit's off status and turning it on are both calculated. When the cost for transition to an upstate is calculated the start up cost is also taken into consideration. The question that arises when a unit is being turned on is the level it should operate under. This power value is found in the following manner. The function to be minimized is $q(\lambda)$. When the first derivative is taken with respect to P the following expression is obtained:

$$\frac{d}{dP_{k,t}}[CF_k(P_{k,t}, \nu_{k,t}) - \lambda^t P_{k,t}]. \quad (4-8)$$

Assuming a quadratic fuel cost function (4-9), the optimal $P_{k,t}$ is shown in (4-10).

$$CF_k(P_{k,t}) = a_0 + a_1 P_{k,t} + a_2 P_{k,t}^2 \quad (4-9)$$

$$P_{k,t} = \frac{\lambda^t - a_1}{2a_2}. \quad (4-10)$$

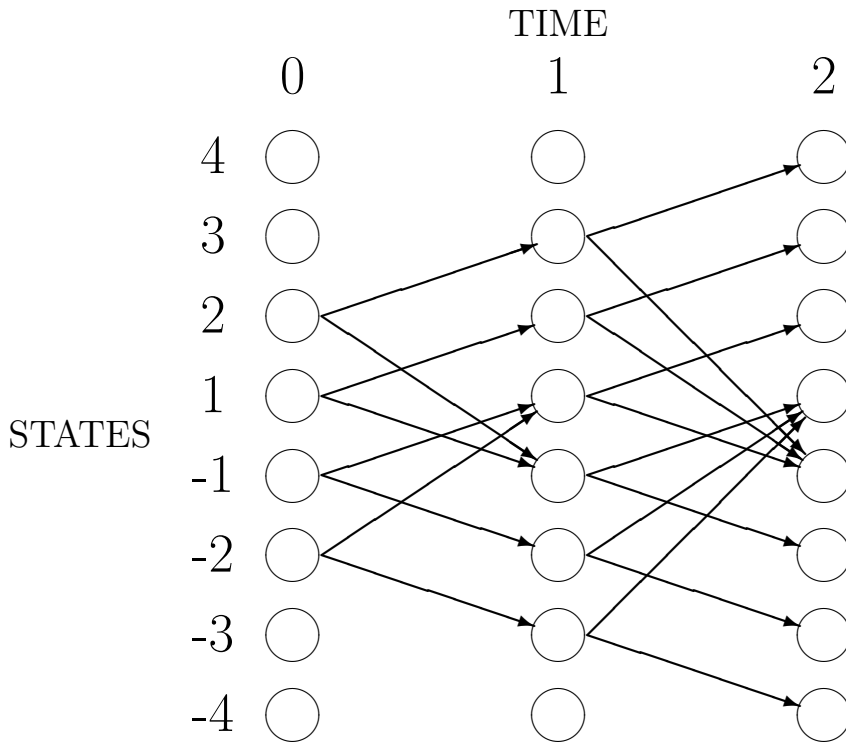


Figure 4.3 DP State Space for a Single Unit and $T=2$

Once the optimal value of $P_{k,t}$ is determined one needs to check if this value is within the capacity limit of the corresponding unit k . If $P_{k,t} < P_{k,min}$ then the unit's operating level is set to $P_{k,min}$. Similarly if $P_{k,t} > P_{k,max}$ the unit's operating level is set to $P_{k,max}$. Otherwise the unit's operating level is set to $P_{k,t}$. Once all of the nodes' costs are calculated the algorithm back-traces to find the optimal schedule. In the initial iteration of the algorithm since the Lagrange multipliers are 0 none of the units will be turned on. However once the Lagrange multipliers are updated at step 6 using the subgradient this will result with a change in the schedule at the second iteration in step 3.

At step 4 the period subproblems are solved using QP since the fuel cost is in quadratic form. At this step an economic dispatch problem is solved for each time period separately.

Solving the economic dispatch the algorithm obtains the operation levels of all the scheduled generators at step 3. J^* is calculated using these operation levels for corresponding units at this step. At step 5 the duality gap is checked and if it is less than ϵ then the algorithm proceeds with step 7, otherwise the λ multipliers are updated using a subgradient method which determines the improving direction at step 6. The ϵ used for this problem is 0.05%. Before proceeding to evaluate the multivariate probability one needs to check if the final UC schedule is feasible since LR techniques typically provide infeasible solutions where some time periods do not have enough generators turned on. If the result is feasible the algorithm continues to step 8; otherwise, a heuristic is used to derive a feasible solution and the algorithm proceeds to step 8 after this. The heuristic applied here is simply to turn on the cheapest generator available for the time periods that have a shortage of power. The heuristic makes sure that the duality gap is still less than ϵ , after modifying the schedule.

4.4 Evaluating Multivariate Normal Probabilities

At step 8 of the CCO algorithm one needs to calculate the multivariate normal probability. This is needed to ensure that the probabilistic constraint 3-10 is satisfied with the prescribed joint probability over the entire time horizon and load is met with a high probability. This can become time consuming especially when the dimension is as high as 96 and this needs to be done at each iteration. It is shown in Proposition 1 that the stochastic load constraint used in this model provides conservative solutions; therefore, one needs to check the original probabilistic constraint and ensure that the calculated probability level is in the ϵ neighborhood of p_{target} . To do this one needs to evaluate with what joint probability the load is satisfied. To evaluate the multivariate probability a subregion adaptive algorithm is

employed which is a multivariate integration technique due to Genz. ^[11] If the calculated probability level is in the ϵ neighborhood of p_{target} the algorithm terminates since the goal of finding a schedule that satisfies load with a probability of p_{target} is accomplished, otherwise the z -value is updated and the previous steps are repeated to obtain another schedule.

To update the z -value the following methodology is used which are outlined in algorithm z -update:

The goal is to find a z -value that provides a schedule where the load can be satisfied with a probability of p_{target} . Given a multivariate z -value that is used to calculate the right hand side of (3-22), algorithm CCO will calculate the corresponding probability value. Then the corresponding univariate z -value will be calculated. Next using interpolation a new z -value will be calculated and the algorithm will iterate until convergence to p_{target} is achieved.

The algorithm begins with finding the corresponding univariate z -value (z_{target}) for the p_{target} value at step 1. Since it is not possible to write a closed form expression for this, the following approximation to the inverse of the cumulative univariate normal distribution function is used. This approximation appears in Abramowitz and Stegun. ^[63]

The goal is to find a z such that $F(z) = p$ for a given p . According to this approximation ^[63] z can be approximated by the following equation:

$$z = t(p) - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}, \quad (4-11)$$

where $t(p)$ is:

$$t(p) = \sqrt{\ln \frac{1}{p^2}}. \quad (4-12)$$

The values of the constants are shown in Table 4.1:

Table 4.1 Values of Constants Used in Approximating the Inverse of the Cumulative Normal Distribution Function

c_0	2.515517	d_0	1.432788
c_1	0.802853	d_1	0.189269
c_2	0.010328	d_2	0.001308

Since the corresponding multivariate z -value for p_{target} has to be greater than this z -value, z_{lower} is set to be equal to z_{target} . At step 2, a high z -value such as 5.0 for 96 dimensions is selected and then at step 3 the CCO algorithm is run to obtain a schedule and a corresponding p_{upper} (the joint satisfaction probability of the load constraint when the z -value is 5.0). A corresponding univariate z -value is next found for p_{upper} which is referred to as z_2 . The CCO algorithm is run a second time at step 4 to find p_{lower} for z_{lower} . As before, a corresponding univariate z -value is found for p_{lower} and z_1 is set to this value. Now all the values are established to find the approximate z -value to evaluate next to find a new probability value that is closer to p_{target} . These values are all listed in Table 4.2.

Table 4.2 Parameters of Algorithm z -Update

multivariate z -value	multivariate probability	univariate z -value
z_{upper}	p_{upper}	z_2
z_{new}	p_{target}	z_{target}
z_{lower}	p_{lower}	z_1

$$z_{new} = z_{lower} + \left(\frac{z_{target} - z_1}{z_2 - z_1} \right) (z_{upper} - z_{lower}) \quad (4-13)$$

Using the linear interpolation given in (4-13), at step 5 the value of z_{new} is set and the algorithm iterates until the calculated probability level is in the ϵ neighborhood of p_{target} which is checked at step 7. During this process every time a new schedule is found at step 6 with a corresponding p the algorithm checks at steps 8 and 9 to see if its corresponding univariate z -value is lower than z_2 and higher than z_{target} , or higher than z_1 and lower than z_{target} . In both of these cases the algorithm replaces the corresponding z -value. Algorithm z -update contains a step by step description of the method used in updating z . In this algorithm the ϵ used changes with p_{target} and these values are listed in Table 4.3.

Table 4.3 ϵ Values Used in Checking Convergence of Algorithm z -update

p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
ϵ	0.005	0.005	0.005	0.005	0.0005	0.00005

Algorithm z-update

Step 1 Find the univariate z -value for $p_{target} = (1 - \alpha)$. Set this to z_{target} and z_{lower} .

Step 2 Initialize z_{upper} to a large value.

Step 3 Run the CCO algorithm to obtain a schedule and a corresponding probability value(p_{upper}) and its corresponding univariate z -value = z_2 .

Step 4 Run the CCO algorithm using z_{lower} to obtain a schedule and a corresponding probability value(p_{lower}) and its corresponding univariate z -value = z_1 .

Step 5 Find z_{new} using: $z_{new} = z_{lower} + \left(\frac{z_{target}-z_1}{z_2-z_1}\right)(z_{upper} - z_{lower})$.

Step 6 Run the CCO algorithm using z_{new} to obtain a schedule and a corresponding probability value, p , and its corresponding univariate z -value, z_{temp} .

Step 7 If $|p - p_{target}| \leq \epsilon$ then terminate. Otherwise continue.

Step 8 If $z_{temp} \leq z_{target}$ then set $z_1 = z_{temp}$, $z_{lower} = z_{new}$ goto Step 10, otherwise goto step 9.

Step 9 Else set $z_2 = z_{temp}$, $z_{lower} = z_{new}$.

Step 10 Goto step 5.

5.0 TEST PROBLEMS FOR THE UC PROBLEM

This chapter will describe the data sets used in this dissertation. The data for six different systems are listed in sections 5.1 through 5.6. The appendix to this dissertation provides the optimal operating schedules, and the next chapter contains the objective function values obtained for these problems. The correlation matrix used is shown below. This matrix is used as an input to the multivariate evaluation algorithm. This specific form can be used due to the AR(1) structure of the residual demand. Here the value of ρ is set to be 0.8. In addition to the original problem data, specific values for standard deviations for each time period are assumed and shown below for the respective systems.

$$R = \begin{pmatrix} 1 & \rho & \rho^2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & \dots & \rho^{T-2} \\ \rho^2 & \rho & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ \cdot & & & & \cdot \\ \rho^{T-1} & \rho^{T-2} & \dots & \dots & \dots & 1 \end{pmatrix}$$

All the problems have a quadratic fuel cost function in the form shown below:

$$a_0 + a_1 P_{k,t} + a_2 P_{k,t}^2 \tag{5-1}$$

All the systems summarized in this chapter originally contain approximated load values. The standard deviation values are added to capture the stochasticity. While doing this it

is ensured that $\mu_t + z\sigma_t$ is not more than 80% of the total generating capacity suggested by Sandell et al. [64]

For Systems B through F the following notation will be used: t_{up} represents the minimum number of hours required for a generator to stay up once it is turned on, similarly, t_{down} represents the corresponding minimum time it should stay down once it is turned off. If a generator is turned on when it is in a hot state the corresponding start up cost is represented by S_h , otherwise the assigned cost is S_c . A generator is considered to be in a hot state during the next $t_{coldstart}$ hours after the time it is turned off. The initial state shows how long a generator has been up if it is positive and how long it has been down if it is negative.

5.1 System A: A 3-Unit System for a Time Horizon of Four Hours without Uptime and Downtime Constraints

This is the simplest problem considered. This problem appears in Wood and Wollenberg. [65] The fuel costs are in quadratic form and there is no cost associated with starting up a generator. There are also no uptime and downtime constraints. The system characteristics are summarized in Table 5.1. Table 5.2 contains the load profile. The system contains three generators and the time horizon is four hours. Since in the original problem the system capacity is not large enough to satisfy the load with high probability, the capacities of the individual units are modified.

Table 5.1 Characteristics of the Generators of System A

	Unit 1	Unit 2	Unit 3
$P_{max}(MW)$	1200	150	240
$P_{min}(MW)$	100	100	50
a_0	500	300	100
a_1	10	8	6
a_2	0.002	0.0025	0.005

Table 5.2 Load Profile of System A

Hour	1	2	3	4
Mean Load (MWH)	170	520	1100	330
Standard Deviation (MWH)	13.6	41.6	88	26.4

5.2 System B: A 4-Unit System for a Time Horizon of Eight Hours

This system is the smallest pool of generators considered in this dissertation that has up-time and downtime constraints and start up costs. It appears in Wood and Wollenberg. ^[65] The system characteristics and load profile are shown in Tables 5.3 and 5.4 respectively. This system has four thermal units and the scheduling time horizon is eight hours.

Table 5.3 Characteristics of the Generators of Systems B and C

	Unit 1	Unit 2	Unit 3	Unit 4
$P_{max}(MW)$	300	250	80	60
$P_{min}(MW)$	75	60	25	20
a_0	684.74	585.62	213.00	252.00
a_1	16.83	16.95	20.74	23.60
a_2	0.0021	0.0042	0.0018	0.0034
$t_{up}(h)$	5	5	4	1
$t_{down}(h)$	4	3	2	1
$S_h(\text{\$})(\text{hot start})$	500	170	150	0.00
$S_c(\text{\$})(\text{cold start})$	1100	400	350	0.02
$t_{coldstart}(h)$	5	5	4	0
Initial State	8	8	-5	-6

Table 5.4 Load Profile of System B

Hour	1	2	3	4	5	6	7	8
Mean Load (MWH)	450	530	600	540	400	280	290	500
Standard Deviation (MWH)	11.25	13.25	15	13.5	10	7	7.25	12.5

5.3 System C: A 4-Unit System for a Time Horizon of One Day

This system has the same generators as System B except the time horizon is 1 day. To obtain the load profile for this system the load profile of System B is repeated 6 times. Below is the load profile used for System C.

Table 5.5 Load Profile of System C

Hour	1	2	3	4	5	6	7	8
Mean Load (MWH)	450	530	600	540	400	280	290	500
Standard Deviation (MWH)	11.25	13.25	15	13.5	10	7	7.25	12.5
Hour	9	10	11	12	13	14	15	16
Mean Load (MWH)	450	530	600	540	400	280	290	500
Standard Deviation (MWH)	11.25	13.25	15	13.5	10	7	7.25	12.5
Hour	17	18	19	20	21	22	23	24
Mean Load (MWH)	450	530	600	540	400	280	290	500
Standard Deviation (MWH)	11.25	13.25	15	13.5	10	7	7.25	12.5

5.4 System D: A 10-Unit System for a Time Horizon of One Day

System D appears in Kazarlis et al. ^[66] It also has a quadratic cost function. The data is listed in Tables 5.6 and 5.7. The system consists of ten thermal units. The time horizon is one day.

Table 5.6 Characteristics of the Generators of System D, E, and F

Unit	P_{max} (MW)	P_{min} (MW)	a_0	a_1	a_2	t_{up} hour	t_{down} hour	S_h (\$)	S_c (\$)	$t_{coldstart}$ hour	Initial State
1	455	150	1000	16.19	0.00048	8	8	4500	9000	5	8
2	455	150	970	17.26	0.00031	8	8	5000	10000	5	8
3	130	20	700	16.60	0.00200	5	5	550	1100	4	-5
4	130	20	680	16.50	0.00211	5	5	560	1120	4	-5
5	162	25	450	19.70	0.00398	6	6	900	1800	4	-6
6	80	20	370	22.26	0.00712	3	3	170	340	2	-3
7	85	25	480	27.74	0.00079	3	3	260	520	2	-3
8	55	10	660	25.92	0.00413	1	1	30	60	0	-1
9	55	10	665	27.27	0.00222	1	1	30	60	0	-1
10	55	10	670	27.79	0.00173	1	1	30	60	0	-1

Table 5.7 Load Profile of System D

Hour	1	2	3	4	5	6	7	8
Mean Load (MWH)	700	750	850	950	1000	1100	1150	1200
Standard Deviation (MWH)	8.75	9.375	10.75	11.875	12.5	13.75	14.375	15
Hour	9	10	11	12	13	14	15	16
Mean Load (MWH)	1300	1400	1450	1500	1400	1300	1200	1050
Standard Deviation (MWH)	16.25	17.5	18.125	18.75	17.5	16.25	15	13.125
Hour	17	18	19	20	21	22	23	24
Mean Load (MWH)	1000	1100	1200	1400	1300	1100	900	800
Standard Deviation (MWH)	12.5	13.75	15	17.5	16.25	13.75	11.25	10

5.5 System E: A 10-Unit System for a Time Horizon of Four Days

The load in system D is repeated four times to obtain the data set for System E. The generators and their characteristics are the same as System D. The time horizon is four days.

5.6 System F: A 100-Unit System for a Time Horizon of Four Days

The generators in system D are duplicated 10 times to obtain the generators in System F which has 100 thermal units. Their characteristics stay the same. The load profile is modified to match this change in total capacity and is shown below. Note that only the load profile for the first day is shown since the load profile for the remaining days is a repetition of the first day. The scheduling time horizon is four days.

Table 5.8 Partial Load Profile of System F

Hour	1	2	3	4	5	6	7	8
Load (MWH)	6204.8	6648	7534.4	8420.8	8864	9750.4	10193.6	10636.8
SD (MWH)	186.144	199.44	226.032	252.624	265.92	292.512	305.808	319.104
Hour	9	10	11	12	13	14	15	16
Load (MWH)	11523.2	12409.6	12852.8	13296	12409.6	11523.2	10636.8	9307.2
SD (MWH)	345.696	372.288	385.584	398.88	372.288	345.696	319.104	279.216
Hour	17	18	19	20	21	22	23	24
Load (MWH)	8864	9750.4	10636.8	12409.6	11523.2	9750.4	7977.6	7091.2
SD (MWH)	265.92	292.512	319.104	372.288	345.696	292.512	239.328	212.736

6.0 SOLUTION OF THE STOCHASTIC UC MODEL

In this chapter the results obtained for the problems listed in the previous chapter are presented. The largest problem solved is System F. This problem has 19,200 decision variables (9,600 binary variables) and 78,816 constraints. All of the schedules obtained are listed in the Appendix to this dissertation.

Two versions of each problem one with positive correlation and another with zero correlation are solved for 6 different probability levels ranging from 0.80 to 0.9999 by modifying constraint (3-21). First the results assuming zero correlation are listed. They are followed by the results obtained when a positive correlation matrix is included in the calculations. The comparisons and an analysis follows.

6.1 Results for the Test Problems

Systems A, B, and C have minor differences in their schedules for the different probability levels. Considering correlation also does not affect the schedules significantly. This is due to the small number of generators present in the power pool. Systems D, E, and F show significant differences in schedules for different probability levels and presence of correlation affects the schedules significantly. The schedule obtained for System D is shown in Table 6.1. In the table a row corresponds to a generator and a column corresponds to a time period, where a value of 1 indicates that the corresponding unit is up at that time period.

This schedule is guaranteed to satisfy the electricity load profile shown in Figure 6.1 with 99.99% probability. Table 6.2 summarizes the objective function values obtained for different problems. Here for each system the first row lists the specified probability level p_{target} . All values given are in dollars and represent the total cost of operation. A comparison is made for all problems between correlated and uncorrelated versions. Figures 6.2 and 6.3 show one such comparison for System F for lower and higher probability levels, respectively. There are two important observations one can derive from these results. First, the schedules obtained using the correlation information have either lower objective function values or have the same value as the solutions obtained without considering correlation. This is expected due to Proposition 2. This observation is important because it shows that one should not simply ignore the correlation of load between time periods. Second, the rate of increase in total cost gets higher as the specified probability level approaches 1.0. This second observation proves that one cannot arbitrarily choose a high value SR and operate under that since the total operating cost will undergo a steep increase. This shows the importance of determining the precise z -value for a system.

Demand Profile for System D

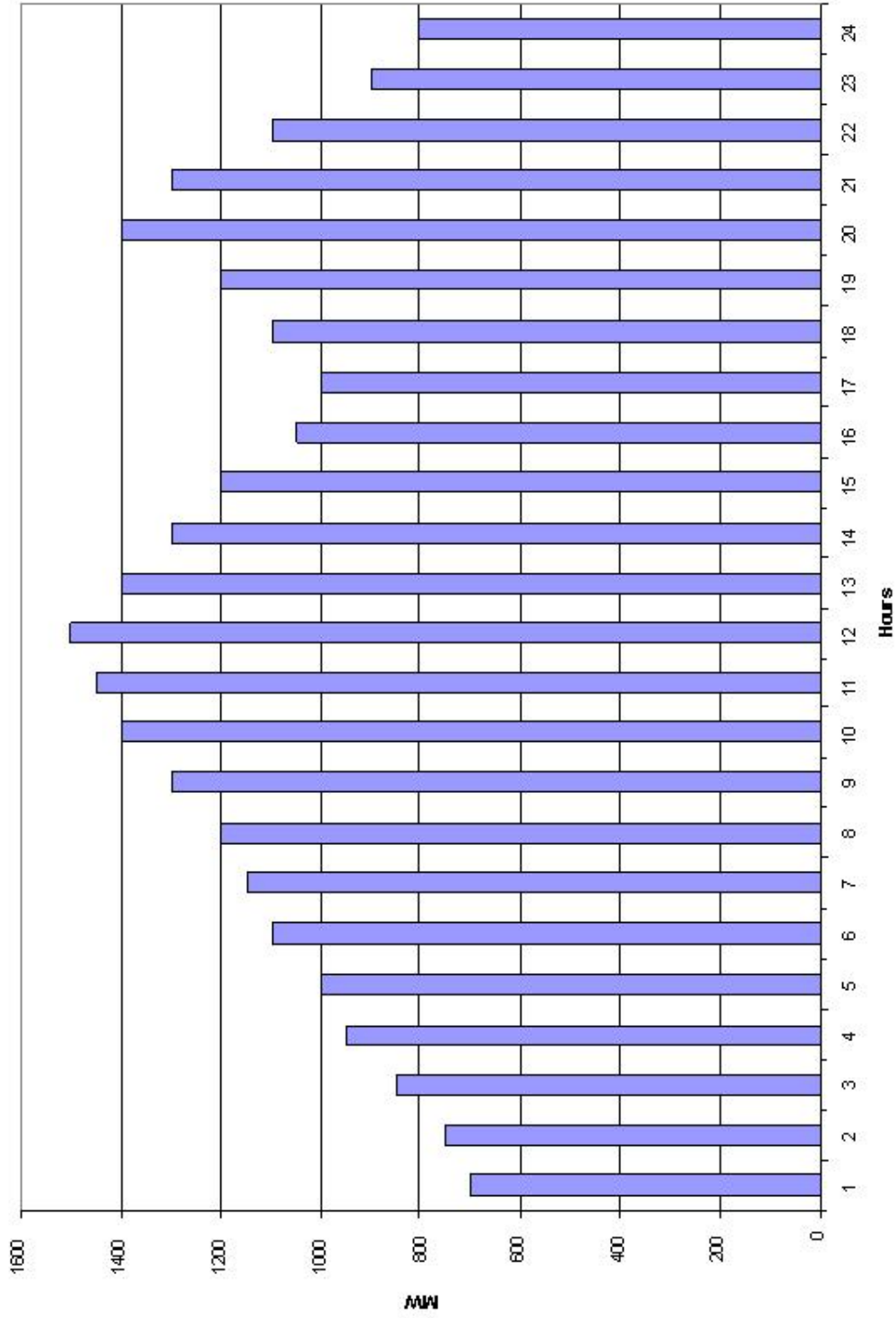


Figure 6.1 Demand Profile for System D

Table 6.2 Optimum Objective Function Values (in \$) for All Systems for p_{target} Values of 0.80-0.9999

System A	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	23837.3	24590.0	25242.8	27045.9	28470.7	29693.5
	positive correlation	23505.2	24322.5	25019.2	26972	28429.3	29344.8
System B	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	76545.6	77553.7	77946.1	79067.5	80413.3	81506.1
	positive correlation	76545.6	77553.7	77946.1	79067.5	80413.3	81506.1
System C	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	232091.0	236456.0	237426.3	240301.6	243131.6	247974.6
	positive correlation	231335.7	235404.2	235183.8	239499.2	242819.7	247842
System D	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	559833.7	563016.1	563308.4	569408.5	572812	575190.7
	positive correlation	557585.4	560566.1	560692.1	569063.6	570096.5	572801.7
System E	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	2264783.5	2267920.8	2269417.6	2302925.1	2304112.6	2338114.7
	positive correlation	2249392.5	2252754.0	2263945.5	2287348.6	2304112.6	2320934.7
System F	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	20641925.4	20850148.5	21013662.8	21405827.9	21688391.5	22143540.3
	positive correlation	20529473.2	20739383.8	20988190.3	21400685.6	21672225.6	21995314.5

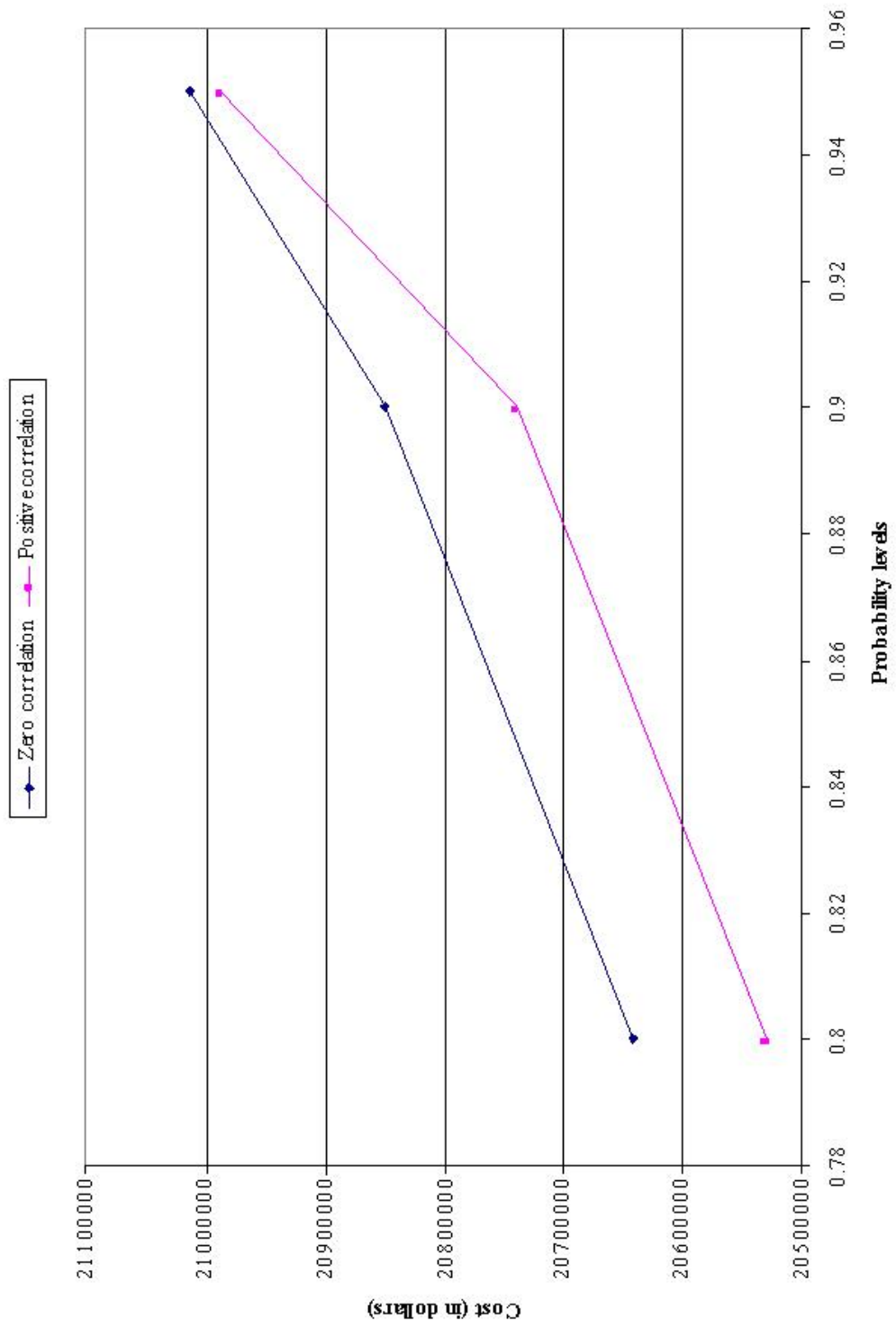


Figure 6.2 Comparison of Optimum Objective Function Values at Probability Levels Less Than 0.95 for System F



Figure 6.3 Comparison of Optimum Objective Function Values at Probability Levels Greater Than 0.95 for System F

To interpret the magnitude of savings consider system F, which is capable of meeting load in the state of Pennsylvania. Observing the objective function values obtained for the probability level of 0.9999 for this system one can conclude that only considering the correlation improves the objective function value by \$148,225.8 for the time horizon of four days. This corresponds to a savings of \$13,525,604.25 in one year only for the state of Pennsylvania, which is significant. The cost savings coupled with the benefits of having a reliable schedule that operates with a given probability level p_{target} demonstrates the value of using the methodology proposed in this dissertation. Final z -values obtained are also summarized in Table 6.3.

Table 6.3 Optimum z -values for All Systems for p_{target} Values of 0.80-0.9999

System A	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	1.28	1.69	2.05	2.57	3.29	3.89
	positive correlation	1.12	1.56	1.93	2.54	3.27	3.78
System B	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	0.84	1.28	1.65	2.33	3.10	3.78
	positive correlation	0.84	1.28	1.65	2.33	3.10	3.78
System C	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	1.22	2.00	2.25	2.84	3.42	4.06
	positive correlation	1.13	1.77	2.00	2.67	3.35	4.00
System D	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	1.16	1.48	1.72	2.23	2.64	3.10
	positive correlation	1.09	1.39	1.52	2.10	2.59	2.76
System E	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	1.88	1.97	2.27	2.70	3.09	3.79
	positive correlation	1.68	1.89	1.99	2.60	3.09	3.72
System F	p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	zero correlation	2.24	2.45	2.71	3.24	3.84	4.30
	positive correlation	2.12	2.44	2.69	3.24	3.76	4.27

Another point of interest is the amount of SR dedicated for varying levels of satisfaction probabilities. Figure 6.4 demonstrates one such comparison for system F considering correlation. Here it is observed that the average SR level per hour changes with increasing levels of probability. For instance, an increase of 4% in the probability level results in a 7%

increase in the SR level. This shows the importance of choosing the correct level of probability that the utility should operate under and the effect of choosing an arbitrary level of operation. Another important observation that can be made is that the level of SR depends on system characteristics, load profile, and the standard deviation of the load. Simply using the same SR level of 5% or 10% of peak load will not be appropriate. Figure 6.5 shows how the average level of SR level per hour changes in percentage of peak load for different probability levels for systems D, E, and F. It is observed that the SR levels for systems D and E are close to each other since they possess the same power pool. System F needs to carry a higher level of SR since the standard deviation is higher for system F compared to D and E. Even a 1% change in SR can change the probability level as much as 5%. And this means that it is important for utilities to establish the level of satisfaction probability and then solve the UC problem to obtain a schedule.

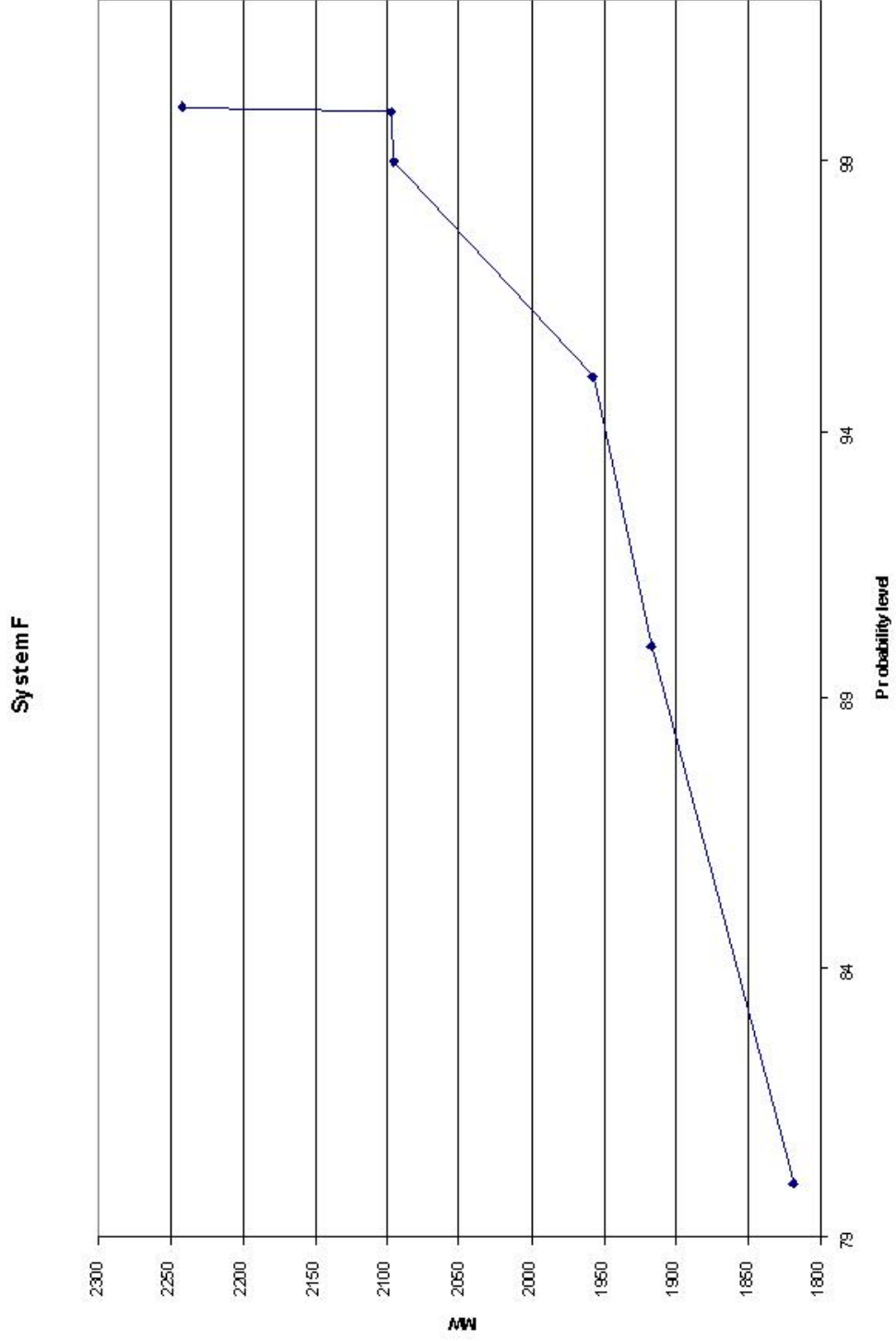


Figure 6.4 Average Spinning Reserve Levels in MW for Different Probability Levels for System F Considering Positive Correlation

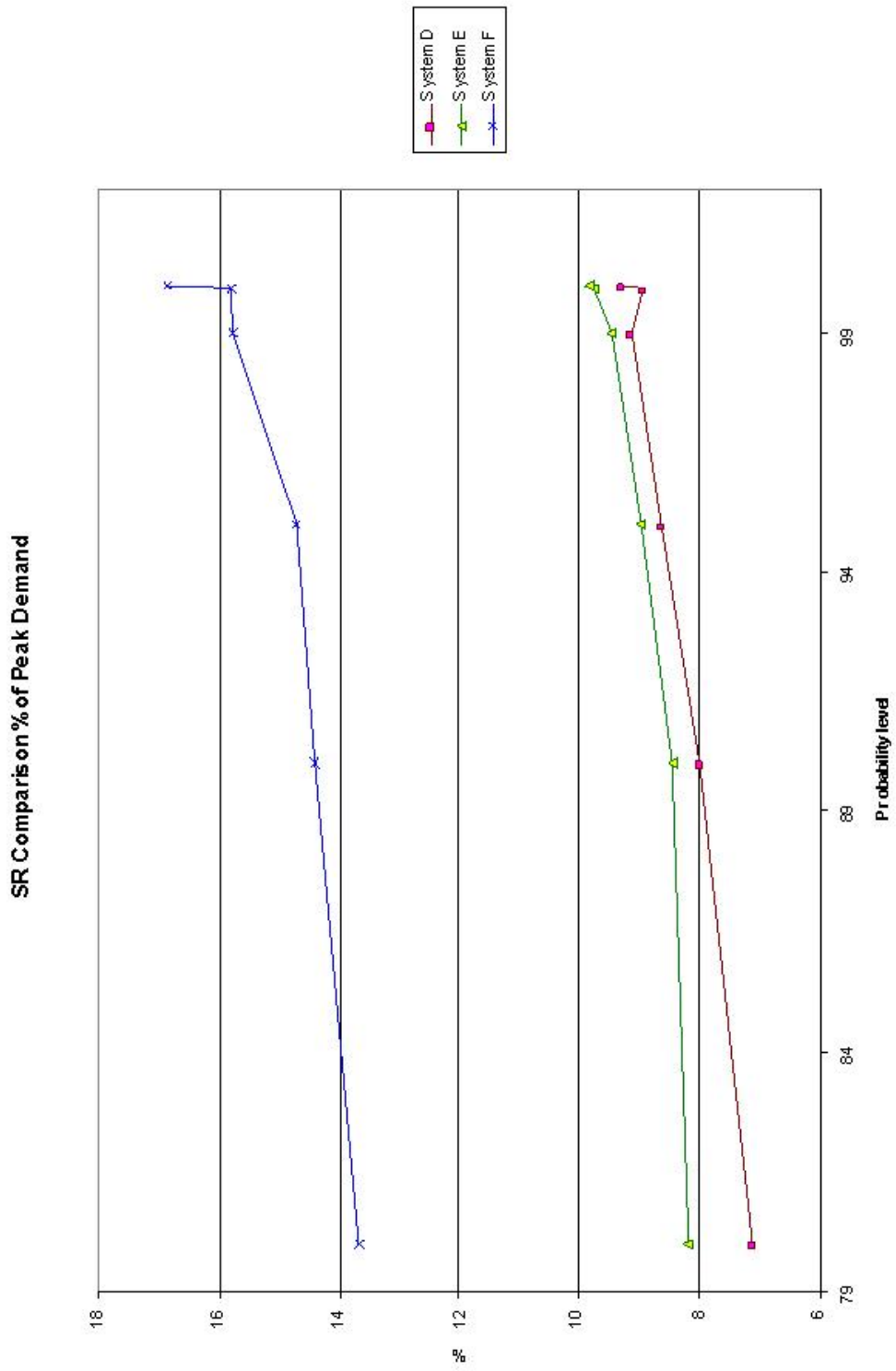


Figure 6.5 Average Spinning Reserve Comparison in Percentage of Peak Demand for Systems D, E and F

6.2 Algorithm Performance

Table 6.4 shows the average CPU time it took to solve the problems in this dissertation for different systems using a PC with a 550 MHz Pentium III processor. For comparison, a UC problem with a linear cost function, for a system with 17 generators and a time horizon of 1 week takes about 25 minutes to solve using CPLEX 7.0. ^[5] Using the method developed in this dissertation the largest problem solved takes about 40 minutes per iteration and it has 100 generators and the time horizon is 4 days.

The number of main iterations it took for the algorithm z -update to converge to the prescribed probability level is dependent on a few factors. The first factor is the initial value chosen for z . A second factor is the approximation method chosen in updating the z -values. The last factor is the size of the problem that is being considered. As seen in Table 6.4, in conformity with expectations, the number of iterations in the algorithm z -update increases with the problem size. The CPU time also shows the same tendency.

Table 6.4 Performance Statistics of the Algorithm for Different Systems

	System A	System B	System C	System D	System E	System F
CPU time (in mins)	0.28	0.34	4.8	6.3	32.5	188.8
# of main iterations	2.92	2.00	4.75	5.25	6.08	4.75

7.0 A SIMULATION ANALYSIS OF THE UC SCHEDULES

The correctness of the optimization algorithm has been verified by comparing the operating schedules with known solutions for the deterministic versions. To verify the effectiveness of the optimum UC schedules provided by the chance constrained optimization algorithm Monte Carlo simulations are used in which the simulation generates the realized load according to the assumed multivariate normal distribution with a specific correlation structure. Averaging over an appropriate number of simulation runs an estimate of the satisfaction probability corresponding to this schedule and the assumed multivariate distribution for the load is found. Corresponding to a given value of the preassigned probability level, p_{target} , and the system parameters the simulation provides an estimate of the expected satisfaction probability.

7.1 Verification Using Monte Carlo Simulation

Schedules obtained using the algorithm explained in this dissertation are simulated to obtain an estimate of the violation probability, $1 - P_s$. To do this the number of occurrences in 100,000 realizations are counted where the scheduled load is not sufficient to meet the realized load in at least one time period in a given realization.

Table 7.1 demonstrates how reliable the schedules obtained by the algorithm in this dissertation are. The multivariate probability evaluation algorithm by Genz is suitable

for the purposes of this algorithm since the principle emphasis of this dissertation is on tail probabilities and the algorithm by Genz is also designed towards evaluating these tail probabilities. It is expected that Genz’s algorithm’s error will increase when the probability requirement is low as 0.8. Therefore it is not unrealistic to see high variations between the probability level obtained by Genz’s algorithm and the simulated results in this probability range. However this difference is negligible when the focus of evaluation moves to the extreme tail probabilities. Table 7.1 shows this variation for System D.

Table 7.1 P_s Values for Varying p_{target} Values (80-99.99) for System D Considering Zero and Positive Correlation

values in %	80	90	95	99	99.9	99.99
positive correlation	79.45	90.33	95.01	98.73	99.93	100.00
zero correlation	83.86	90.13	94.65	99.43	99.93	99.995

Table 7.2 shows the values of average absolute difference between P_s and p_{target} over different problems. This deviation increases for the smaller problems since it is harder to obtain a schedule that satisfies the load with a precise probability. The average values in Table 7.2 are also high for larger systems but note that it includes probability levels 0.8 to 0.95 and this deviation is not significant when only tail probabilities are considered.

Table 7.2 Average $|P_s - p_{target}|$ for All Systems for Varying p_{target} Values

values in %	System A	System B	System C	System D	System E	System F
average deviation	5.50	6.02	5.26	0.64	1.10	1.30

7.2 Cost Analysis of Simulated Schedules

In addition to verification of probability levels, a simulation cost analysis is also completed for System F. First a deterministic UC problem is solved for varying levels of SR ranging from 0.1 to 4σ with 0.1σ increments. Then using the schedules obtained from these 40 runs, system F is simulated. Whenever a schedule is not capable of satisfying demand in a time period, a penalty cost is calculated using a quadratic cost function of five times the most expensive unit in the system. For each time period, in each simulation run, an appropriate economic dispatch problem is solved. Since the cost function is quadratic, a quadratic program is solved for each time period using CPLEX 7.0 to determine the operating levels of the scheduled generators. Averaging over 200 simulation runs, an estimate of the cost for that schedule is obtained. The number of simulation runs is determined by first analyzing the change in simulation cost in a sample problem by varying the simulation runs between 100 and 1500. Table 7.3 lists a portion of these results for the corresponding probability levels. It is observed that by increasing the level of satisfaction probability from 81.2% to 99.8%, the expected operating cost increased by 1.64%. This cost increase results with a satisfaction probability increase of 18.6%. A utility needs to decide if this probability increase is worth the corresponding increase in cost and proceed accordingly. For instance, if certain customers agree to having power outages in return for paying a lower rate for the utility bill, a low probability solution may become feasible. This is a trade off the utility might consider.

Table 7.3 Estimates of the Operating Cost Obtained by Simulation for Different Schedules of System F

<i>p_{target}</i>	0.812	0.916	0.956	0.99	0.998	1.0
Cost (in million dollars)	1.967	1.974	1.978	1.982	1.999	2.001

8.0 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

Reliable power production is critical to the profitability of electricity utilities. Generators that produce electricity need to be scheduled efficiently to meet customer demand. This dissertation introduces a solution method for obtaining the operation schedule of generators while determining the estimated amount of surplus power each generator should produce. This is done taking into consideration the stochasticity of load and its correlation structure. In this problem it is assumed that the generation system is an isolated one such that it does not have access to an electricity market. The method proposed in this dissertation allows the utility to specify the probability level of load the system should operate under.

8.1 Concluding Remarks

This dissertation has made substantial contributions in the following areas:

- This dissertation combines probabilistic analysis with optimization to solve a difficult but important problem.
- The proposed method represents the first effort to apply chance constrained programming to the UC problem.

- The contributions in multivariate analysis, and large-scale stochastic optimization should also be applicable in other problem contexts. In this dissertation a stochastic optimization methodology is developed that is applicable to a large-scale system.
- This dissertation also represents an important synergy between the tools of probabilistic analysis and optimization used within industrial engineering with the analysis tools and power planning methods of electrical engineering.
- The proposed approach uses the correlation matrix for the load between time periods when calculating the value function at each iteration in the proposed algorithm. Considering these correlations improves the solution to the problem. This is shown both in implementation and theory.
- Instead of using a set level of SR, the proposed model determines the level probabilistically which is superior to using an arbitrary approximation. Quantifying costs of increased satisfaction probability level can help the utility decide at which level to operate.

In this dissertation a computer program is developed using the C Language that solves the optimization problem which includes a forward recursion dynamic program. A program developed by Alan Genz in FORTRAN is used to evaluate high dimensional multivariate probabilities. To solve the quadratic programs CPLEX 7.0 is employed. Finally, a computer program is developed using the C language to simulate the load and verify the results obtained by the optimization methodology.

The following conclusions are reached from the results obtained in this dissertation:

- Load correlation is a significant factor for profitability and cannot be ignored while determining a schedule for the pool of generators a utility possesses. Observing the objective function values obtained for the probability level of 0.9999 for this system one can conclude that only considering the correlation improves the optimum value of the objective function by \$148,225.8 for the time horizon of four days. This corresponds to a saving of \$13,525,604.25 in one year, which is substantial. This conclusion is supported by a theoretical proof.
- It is very risky to choose an arbitrary level of SR when solving the UC problem. The SR is dependent on many factors such as the characteristics of the generators possessed, load profile parameters including the mean and standard deviation for different time periods and the correlation in load between the time periods for the time horizon considered. Figure 6.4 shows how average SR level per hour changes with increasing levels of satisfaction probability. An increase of 4% in probability level results in a 7% increase in SR level.
- Chance constrained programming is an appropriate way of modeling the UC problem when the system is isolated or operating as a stand alone system.

8.2 Limitations

This solution method is not suitable for all kinds of systems. This section summarizes the limitations of the method developed in this dissertation including the restrictive assumptions needed.

- The algorithm is designed for an isolated power system in which satisfying the load with a high probability has the utmost priority. If the goal is to only minimize the expected operating cost or if there is an accessible power market, then a different optimization method is needed such as multi-stage stochastic programming.
- If the power system includes wind power generators then this method can only be used under the assumption that residual demand ($d_t - w_t$) follows an AR(1) structure or has a symmetric positive definite covariance matrix. Otherwise the methods used in the evaluation of the multivariate probabilities are not applicable.
- If the underlying distribution for the residual load is non-normal then this method can not be used and a different approach should be considered.
- If the estimates for the temperature is not reliable then a different approach should be considered that includes this uncertainty into this method.
- The method developed in this dissertation ignores the stochasticity introduced by the failure of generating units since the time horizon is short and units are not expected to fail during this short period. If the stochasticity introduced by the failure of generating units is significant then the model and the solution method requires modification.

8.3 Future Research Directions

In this section specific research directions following from this dissertation are discussed:

1. Enhancement of the Optimization Algorithm

To improve the efficiency of the solution algorithm one can consider two research efforts. First, the algorithm presented in this dissertation uses a straightforward LR implementation. One can investigate methods of improving the convergence rate for this part of the algorithm by exploring alternative subgradient methods that have proven to be effective for the UC problem. One can also consider the use of ALR.

2. Period Specific z -values

In the development of the algorithm it is assumed that the z -values are the same in each time period. This need not necessarily be so and therefore one can permit the z values to be different for different time periods. Considering a vector of z -values will complicate the structure of the problem. It will be necessary to explore the characteristics of the resulting objective function to determine if it is jointly convex. If the function is jointly convex then it may be possible to utilize gradient based methods to solve the problem. Otherwise heuristic methods can be employed to solve the problem. In either case, determining the optimal values for this substantially larger search space will require an alternative solution strategy. It will be useful to determine whether this approach results in a still lower value for the system operating costs.

3. Period Specific Probability Requirements for Meeting Load

It is conceivable that from the point of view of a user some time periods are more important than others. The formulation can be modified to insure that the load for

certain periods is met with a higher level of probability and that the load for other periods is met with lower probability. For example, if the UC problem is solved repeatedly on a rolling horizon basis one could require the first few time periods to have a high probability of meeting load and have a lower probability of meeting load for the later periods. Additionally, the model can be updated as time moves forward and more information, such as load data, becomes available. One can modify the chance constrained optimization problem to reflect this situation. Note that this will give rise to more than one probabilistic constraint.

4. Accounting for Other Sources of Uncertainty

One can enhance the chance constrained optimization formulation so as to include uncertainty arising from sources other than the load and wind power generation. One can investigate to what extent thermal generator failures can be accommodated within this formulation. Frequently the operating states of a thermal generator are modeled as a two-state continuous Markov chain for which the auto-correlation function is well known. However, it is not clear whether the total hourly power generated from an ensemble of plants whose operation can be modeled by such stochastic processes can be accurately approximated by a suitable multivariate normal distribution. If it can be done in this manner then the amount of available power (accounting for forced outages) can be accommodated within this dissertation's framework. Under deregulation, a producer may choose to purchase power from the market instead of generating the power itself. One can also investigate how the additional source of uncertainty associated with market prices may alter the formulation of the optimization problem as well as the subsequent Monte Carlo production simulation.

5. Stochastic Economic Dispatch

After the UC schedule is determined, the second step in planning is to determine the amount of power generated by the committed generators in real time. This is the task of the system operator. Given the previous time period's load and trend one can estimate the next time period's load distribution and variance. Then a similar approach employed in handling stochasticity in the UC problem can be used to solve the economic dispatch problem. To be more specific, one can employ a single chance constraint for demand and convert it to a deterministic form. Adding this to the constraints of the economic dispatch problem one can utilize the same methods used for solving the economic dispatch problem to obtain a load level to be carried in the next time period instead of using a percentage of peak load as SR.

APPENDIX

APPENDIX

A1 - Schedules for System A

Table A.1 Optimal Schedule for System A for Zero Correlation

p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
1	0 0 1	0 0 1	0 0 1	0 0 1	0 0 1	0 0 1
2	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
3	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
4	0 1 1	0 1 1	0 1 1	1 1 1	1 1 1	1 1 1

Table A.2 Optimal Schedule for System A for Positive Correlation

p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
1	0 0 1	0 0 1	0 0 1	0 0 1	0 0 1	0 0 1
2	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
3	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
4	0 1 1	0 1 1	0 1 1	1 1 1	1 1 1	1 1 1

A2 - Schedules for System B

Table A.3 Optimal Schedule for System B for Zero Correlation

p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
1	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
2	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
3	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
4	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
5	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
6	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
7	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
8	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0

Table A.4 Optimal Schedule for System B for Positive Correlation

p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
1	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
2	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
3	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
4	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
5	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
6	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
7	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
8	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 1 0

A3 - Schedules for System C

Table A.5 Optimal Schedule for System C for Zero Correlation

p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
1	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 1 0
2	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
3	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
4	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
5	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 0 0
6	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
7	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
8	1 1 1 0	1 1 1 0	1 1 1 0	1 1 0 0	1 1 0 0	1 1 1 0
9	1 1 1 0	1 1 1 0	1 1 1 0	1 1 0 0	1 1 0 0	1 1 1 0
10	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 1
11	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
12	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
13	1 1 1 0	1 1 0 0	1 1 0 0	1 1 1 0	1 1 1 0	1 1 0 0
14	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
15	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
16	1 1 0 0	1 1 1 0	1 1 1 0	1 1 0 0	1 1 0 0	1 1 1 0
17	1 1 0 0	1 1 1 0	1 1 1 0	1 1 0 0	1 1 0 0	1 1 1 0
18	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
19	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
20	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
21	1 1 1 0	1 1 0 0	1 1 0 0	1 1 1 0	1 1 1 0	1 1 0 0
22	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
23	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
24	1 1 0 0	1 1 1 0	1 1 1 0	1 1 0 0	1 1 0 0	1 1 1 0

Table A.6 Optimal Schedule for System C for Positive Correlation

p_{target}	0.8	0.9	0.95	0.99	0.999	0.9999
	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
1	1 1 0 0	1 1 1 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 1 0
2	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
3	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
4	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
5	1 1 1 0	1 1 0 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 0 0
6	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
7	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
8	1 1 1 0	1 1 1 0	1 1 0 0	1 1 1 0	1 1 0 0	1 1 1 0
9	1 1 1 0	1 1 1 0	1 1 0 0	1 1 1 0	1 1 0 0	1 1 1 0
10	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 1
11	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 1	1 1 1 1	1 1 1 1
12	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
13	1 1 0 0	1 1 0 0	1 1 1 0	1 1 0 0	1 1 1 0	1 1 0 0
14	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
15	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
16	1 1 1 0	1 1 1 0	1 1 0 0	1 1 1 0	1 1 0 0	1 1 1 0
17	1 1 1 0	1 1 1 0	1 1 0 0	1 1 1 0	1 1 0 0	1 1 1 0
18	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
19	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
20	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0	1 1 1 0
21	1 1 0 0	1 1 0 0	1 1 1 0	1 1 0 0	1 1 1 0	1 1 1 0
22	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
23	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 0 0
24	1 1 0 0	1 1 1 0	1 1 0 0	1 1 0 0	1 1 0 0	1 1 1 0

A4 - Schedules for System D

Table A.7 Optimal Schedule for System D for Zero Correlation - Lower Probabilities (0.80-0.95)

p_{target}	0.8	0.9	0.95
	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 1 1 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 1 1 1	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
4	1 1 0 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
5	1 1 0 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 1 0 0	1 1 1 1 0 0 0 1 0 0
8	1 1 1 1 1 0 0 0 0 0	1 1 1 1 0 0 0 1 0 0	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
10	1 1 1 1 1 1 0 1 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 0 1 0 0
11	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0
12	1 1 1 1 1 1 1 1 1 0	1 1 1 1 1 1 1 1 1 0	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 0 1 0 0	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 0 0 1 1 0	1 1 1 1 1 1 0 1 1 0	1 1 1 1 1 1 0 1 1 0
21	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0

Table A.8 Optimal Schedule for System D for Zero Correlation- Higher Probabilities (0.99-0.9999)

p_{target}	0.99	0.999	0.9999
	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 1 1 1	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
4	1 1 0 1 0 0 0 0 0 0	1 1 0 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
5	1 1 0 1 0 0 0 0 0 0	1 1 0 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 0 0 1 0 0	1 1 1 1 1 1 0 0 0 0
10	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 1 0 0	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 0 1 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 0 0 1 0 0	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 1 1 1	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 1 1 0
17	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 1 0 1 1 0	1 1 1 1 1 1 0 1 1 0	1 1 1 1 1 1 0 1 0 0
21	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	1 1 0 1 0 0 0 0 0 0	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0

Table A.9 Optimal Schedule for System D for Positive Correlation- Lower Probabilities (0.80-0.95)

p_{target}	0.8	0.9	0.95
	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
4	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 1 1 0	1 1 0 1 0 0 0 0 0 0
5	1 1 0 1 0 0 0 0 0 0	1 1 0 0 0 0 0 1 1 0	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	1 1 0 1 0 0 0 1 1 0	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 1 0 0	1 1 1 1 0 0 0 1 0 0
8	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
10	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 0 1 0 0	1 1 1 1 1 1 0 1 0 0
11	1 1 1 1 1 1 1 1 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0
12	1 1 1 1 1 1 1 1 1 0	1 1 1 1 1 1 1 1 1 0	1 1 1 1 1 1 1 1 0 0
13	1 1 1 1 1 1 0 1 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 0 0 1 1 0	1 1 1 1 1 1 0 1 1 0	1 1 1 1 1 1 0 1 0 0
21	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 1 0 0	1 1 1 1 0 0 0 0 0 0
24	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0

Table A.10 Optimal Schedule for System D for Positive Correlation- Higher Probabilities (0.99-0.9999)

p_{target}	0.99	0.999	0.9999
	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10	1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 1 1 1	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0
4	1 1 1 1 0 0 0 0 0 0	1 1 0 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
5	1 1 1 1 0 0 0 0 0 0	1 1 0 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 0 0 0 1 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 0 0 0 1 0 0
8	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 1 0 0	1 1 1 1 1 0 0 1 0 0	1 1 1 1 1 0 0 1 0 0
10	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 1 0 0	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 0	1 1 1 1 1 1 1 1 1 0	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 0 0 1 0 0
15	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 0 0 0 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 0 0 1 1 0	1 1 1 1 1 1 0 1 1 0	1 1 1 1 1 1 0 1 1 0
21	1 1 1 1 1 0 0 1 0 0	1 1 1 1 1 1 0 0 0 0	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 1 0 0	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0

A5 - Schedules for System E

Table A.11 Optimal Schedule for System E for Zero Correlation- $p_{target} = 0.80$

	1	2	3	4	5	6	7	8	9	10		1	2	3	4	5	6	7	8	9	10	
1	1	1	0	0	0	0	0	0	0	0	49	1	1	0	0	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0	0	0	50	1	1	0	0	0	0	0	0	0	0	0
3	1	1	0	0	0	0	0	0	0	0	51	1	1	0	0	0	0	0	0	0	0	0
4	1	1	0	0	0	0	0	1	1	0	52	1	1	0	0	0	0	0	1	1	0	0
5	1	1	0	0	0	0	0	1	1	1	53	1	1	1	1	0	0	0	0	0	0	0
6	1	1	1	1	0	0	0	0	0	0	54	1	1	1	1	0	0	0	0	0	0	0
7	1	1	1	1	0	0	0	1	0	0	55	1	1	1	1	0	0	0	1	0	0	0
8	1	1	1	1	1	0	0	0	0	0	56	1	1	1	1	1	0	0	0	0	0	0
9	1	1	1	1	1	0	0	0	0	0	57	1	1	1	1	1	0	0	0	0	0	0
10	1	1	1	1	1	1	0	0	0	0	58	1	1	1	1	1	1	0	0	0	0	0
11	1	1	1	1	1	1	0	0	0	0	59	1	1	1	1	1	1	1	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	60	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	0	0	0	0	61	1	1	1	1	1	1	1	0	0	0	0
14	1	1	1	1	1	1	0	0	0	0	62	1	1	1	1	1	0	0	0	0	0	0
15	1	1	1	1	1	0	0	0	0	0	63	1	1	1	1	1	0	0	0	0	0	0
16	1	1	1	1	1	0	0	0	0	0	64	1	1	1	1	1	0	0	0	0	0	0
17	1	1	1	1	1	0	0	0	0	0	65	1	1	1	1	1	0	0	0	0	0	0
18	1	1	1	1	1	0	0	0	0	0	66	1	1	1	1	1	0	0	0	0	0	0
19	1	1	1	1	1	1	0	0	0	0	67	1	1	1	1	1	0	0	0	0	0	0
20	1	1	1	1	1	0	1	1	1	1	68	1	1	1	1	0	0	1	1	1	1	1
21	1	1	1	1	1	1	0	0	0	0	69	1	1	1	1	1	0	0	0	0	0	0
22	1	1	1	1	0	0	0	0	0	0	70	1	1	1	0	0	0	0	0	0	0	0
23	1	1	0	0	0	0	1	0	0	0	71	1	1	0	1	0	0	0	0	0	0	0
24	1	1	0	0	0	0	0	0	0	0	72	1	1	0	0	0	0	0	0	0	0	0
25	1	1	0	0	0	0	0	0	0	0	73	1	1	0	0	0	0	0	0	0	0	0
26	1	1	0	0	0	0	0	0	0	0	74	1	1	0	0	0	0	0	0	0	0	0
27	1	1	0	0	0	0	0	0	0	0	75	1	1	0	0	0	0	0	0	0	0	0
28	1	1	1	1	0	0	0	0	0	0	76	1	1	0	0	0	0	0	1	1	0	0
29	1	1	1	1	0	0	0	0	0	0	77	1	1	0	0	0	0	0	1	1	1	1
30	1	1	1	1	0	0	0	0	0	0	78	1	1	1	1	0	0	0	0	0	0	0
31	1	1	1	1	0	0	0	1	0	0	79	1	1	1	1	0	0	0	1	0	0	0
32	1	1	1	1	1	0	0	0	0	0	80	1	1	1	1	1	0	0	0	0	0	0
33	1	1	1	1	1	0	0	0	0	0	81	1	1	1	1	1	0	0	0	0	0	0
34	1	1	1	1	1	1	0	0	0	0	82	1	1	1	1	1	1	0	0	0	0	0
35	1	1	1	1	1	1	1	0	0	0	83	1	1	1	1	1	1	1	0	0	0	0
36	1	1	1	1	1	1	1	1	0	0	84	1	1	1	1	1	1	1	1	1	1	1
37	1	1	1	1	1	1	1	0	0	0	85	1	1	1	1	1	1	1	0	0	0	0
38	1	1	1	1	1	0	0	0	0	0	86	1	1	1	1	1	1	0	0	0	0	0
39	1	1	1	1	1	0	0	0	0	0	87	1	1	1	1	1	0	0	0	0	0	0
40	1	1	1	1	1	0	0	0	0	0	88	1	1	1	1	1	0	0	0	0	0	0
41	1	1	1	1	1	0	0	0	0	0	89	1	1	1	1	1	0	0	0	0	0	0
42	1	1	1	1	1	0	0	0	0	0	90	1	1	1	1	1	0	0	0	0	0	0
43	1	1	1	1	1	1	0	0	0	0	91	1	1	1	1	1	0	0	0	0	0	0
44	1	1	1	1	1	0	1	1	1	1	92	1	1	1	1	0	0	1	1	0	0	0
45	1	1	1	1	1	1	0	0	0	0	93	1	1	1	1	1	0	0	0	0	0	0
46	1	1	1	1	0	0	0	0	0	0	94	1	1	1	0	0	0	0	0	0	0	0
47	1	1	0	0	0	0	0	1	0	0	95	1	1	0	1	0	0	0	0	0	0	0
48	1	1	0	0	0	0	0	0	0	0	96	1	1	0	0	0	0	0	0	0	0	0

Table A.12 Optimal Schedule for System E for Zero Correlation- $p_{target} = 0.90$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 1 0 0	51	1 1 0 0 0 0 0 1 0 0
4	1 1 0 0 0 0 0 1 1 0	52	1 1 0 1 0 0 0 0 0 0
5	1 1 0 0 0 0 0 1 1 1	53	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 1 0 0 0 0 0	55	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 1 0 0	57	1 1 1 1 1 0 0 1 0 0
10	1 1 1 1 1 1 0 1 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 0 0 0	59	1 1 1 1 1 1 1 0 0 0
12	1 1 1 1 1 1 1 1 0 0	60	1 1 1 1 1 1 1 1 0 0
13	1 1 1 1 1 1 1 1 0 0	61	1 1 1 1 1 1 1 0 1 0
14	1 1 1 1 1 1 0 0 1 0	62	1 1 1 1 1 1 1 0 0 0
15	1 1 1 1 1 1 0 0 0 0	63	1 1 1 1 1 1 0 0 0 0
16	1 1 1 1 1 1 0 0 0 0	64	1 1 1 1 1 1 0 0 1 1
17	1 1 1 1 1 1 0 0 0 0	65	1 1 1 1 1 1 0 0 0 0
18	1 1 1 1 1 1 0 0 0 0	66	1 1 1 1 1 1 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 1 1 1 1 1 0 0 0
20	1 1 1 1 1 1 0 0 1 1	68	1 1 1 1 1 1 1 0 0 1
21	1 1 1 1 1 1 0 0 1 0	69	1 1 1 1 1 1 1 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	71	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 1 0 0	75	1 1 0 0 0 0 0 1 0 0
28	1 1 1 1 0 0 0 0 0 0	76	1 1 1 1 0 0 0 0 0 0
29	1 1 1 1 0 0 0 0 0 0	77	1 1 1 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 1 0 0 0 0 0	79	1 1 1 1 1 0 0 0 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 0 0 1 0 0	81	1 1 1 1 1 0 0 1 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 0 0 0	83	1 1 1 1 1 1 1 0 0 0
36	1 1 1 1 1 1 1 1 1 1	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 1 0	85	1 1 1 1 1 1 1 0 1 0
38	1 1 1 1 1 1 0 0 1 0	86	1 1 1 1 1 1 1 0 0 0
39	1 1 1 1 1 1 0 0 0 0	87	1 1 1 1 1 1 0 0 0 0
40	1 1 1 1 1 1 0 0 0 0	88	1 1 1 1 1 1 0 0 1 1
41	1 1 1 1 1 1 0 0 0 0	89	1 1 1 1 1 1 0 0 0 0
42	1 1 1 1 1 1 0 0 0 0	90	1 1 1 1 1 1 0 0 0 0
43	1 1 1 1 1 1 1 0 0 0	91	1 1 1 1 1 1 1 0 0 0
44	1 1 1 1 1 1 1 0 1 0	92	1 1 1 1 1 1 1 0 1 0
45	1 1 1 1 1 1 1 0 0 0	93	1 1 1 1 1 1 1 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 0 0 0 0 0 1 0 0	95	1 1 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.13 Optimal Schedule for System E for Zero Correlation- $p_{target} = 0.95$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	51	1 1 0 0 0 0 0 0 0 0
4	1 1 1 1 0 0 0 0 0 0	52	1 1 0 0 0 0 0 1 1 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 0 0 0 0 0 1 1 1
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 1 0 0 0 0 0	55	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 1 0 0	57	1 1 1 1 1 0 0 1 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 0 0 0	59	1 1 1 1 1 1 1 0 0 0
12	1 1 1 1 1 1 1 1 0 0	60	1 1 1 1 1 1 1 1 0 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 1 0 1 1 1	68	1 1 1 1 1 1 0 0 1 1
21	1 1 1 1 1 1 0 0 0 0	69	1 1 1 1 0 1 0 1 1 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	71	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 1 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 0 0 0	75	1 1 0 0 0 0 0 0 0 0
28	1 1 0 0 0 0 0 1 1 0	76	1 1 0 1 0 0 0 0 0 0
29	1 1 1 1 0 0 0 0 0 0	77	1 1 0 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 1 0 0 0 0 0	79	1 1 1 1 1 0 0 0 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 0 0 1 0 0	81	1 1 1 1 1 0 0 1 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 0 0 0	83	1 1 1 1 1 1 1 0 0 0
36	1 1 1 1 1 1 1 1 1 1	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 1 0 1 1 1	92	1 1 1 1 1 1 0 1 0 0
45	1 1 1 1 0 1 0 1 1 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 0 0 0 0 0 1 0 0	95	1 1 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.14 Optimal Schedule for System E for Zero Correlation- $p_{target} = 0.99$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 1 0 0	51	1 1 0 0 0 0 0 1 0 0
4	1 1 0 1 0 0 0 0 0 0	52	1 1 0 1 0 0 0 0 0 0
5	1 1 0 1 0 0 0 0 0 0	53	1 1 0 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 0 0 0 1 0 0	55	1 1 1 1 0 0 0 1 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 1 0 0	57	1 1 1 1 1 0 0 1 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 1 0 0	59	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 0	60	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 1 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 1 0 1 1 0	68	1 1 1 1 1 1 0 1 1 0
21	1 1 1 1 1 1 0 0 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	71	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 1 0 0	75	1 1 0 0 0 0 0 1 0 0
28	1 1 1 1 0 0 0 0 0 0	76	1 1 1 1 0 0 0 0 0 0
29	1 1 1 1 0 0 0 0 0 0	77	1 1 1 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 0 0 0 1 0 0	79	1 1 1 1 0 0 0 1 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 1 0 0 0 0	81	1 1 1 1 1 1 0 0 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 1 0 0	83	1 1 1 1 1 1 1 1 0 0
36	1 1 1 1 1 1 1 1 1 0	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 1 0 1 0 0	92	1 1 1 1 1 1 0 1 0 0
45	1 1 1 1 1 1 0 0 0 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 0 1 0 0 0 1 0 0
47	1 1 0 0 0 0 0 1 0 0	95	1 1 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.15 Optimal Schedule for System E for Zero Correlation- $p_{target} = 0.999$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	51	1 1 0 0 0 0 0 0 0 0
4	1 1 1 1 0 0 0 0 0 0	52	1 1 0 0 0 0 0 1 1 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 0 0 0 0 0 1 1 1
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 1 0 0 0 0 0	55	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 1 0 0 0 0	57	1 1 1 1 1 1 0 0 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 1 0 0	59	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 1	60	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 1 0 1 0 0	68	1 1 1 1 1 1 0 1 1 1
21	1 1 1 1 1 1 0 0 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 1 1 0 0 0 0 0 0	71	1 1 0 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 0 0 0	75	1 1 0 0 0 0 0 0 0 0
28	1 1 0 0 0 0 0 1 1 0	76	1 1 1 1 0 0 0 0 0 0
29	1 1 0 0 0 0 0 1 1 1	77	1 1 1 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 1 0 0 0 0 0	79	1 1 1 1 1 0 0 0 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 1 0 0 0 0	81	1 1 1 1 1 1 0 0 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 1 0 0	83	1 1 1 1 1 1 1 1 0 0
36	1 1 1 1 1 1 1 1 1 1	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 1 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 1 0 1 0 0	92	1 1 1 1 1 1 0 1 1 0
45	1 1 1 1 1 1 0 0 0 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 1 1 0 0 0 0 0 0	95	1 1 0 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.16 Optimal Schedule for System E for Zero Correlation- $p_{target} = 0.9999$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	51	1 1 0 0 0 0 0 0 0 0
4	1 1 1 1 0 0 0 0 0 0	52	1 1 1 1 1 0 0 0 0 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 1 1 1 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 1 0 0 0 0 0
7	1 1 1 1 1 0 0 0 0 0	55	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 1 0 0 0 0	57	1 1 1 1 1 0 0 1 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 1 0 0	59	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 1	60	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 0 0 1 0 0	67	1 1 1 1 1 0 0 0 0 0
20	1 1 1 1 1 0 0 1 1 1	68	1 1 1 1 1 0 0 1 1 1
21	1 1 1 1 1 0 0 1 0 0	69	1 1 1 1 1 0 0 1 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	71	1 1 1 0 0 0 0 0 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 0 0 0	75	1 1 0 0 0 0 0 0 0 0
28	1 1 1 1 1 0 0 0 0 0	76	1 1 0 1 0 0 0 0 0 0
29	1 1 1 1 1 0 0 0 0 0	77	1 1 0 1 0 0 0 1 0 0
30	1 1 1 1 1 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 1 0 0 0 0 0	79	1 1 1 1 1 0 0 0 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 0 0 1 0 0	81	1 1 1 1 1 0 0 1 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 1 0 0	83	1 1 1 1 1 1 1 1 0 0
36	1 1 1 1 1 1 1 1 1 0	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 1 0 1 1 0	92	1 1 1 1 1 1 0 1 1 0
45	1 1 1 1 1 1 0 0 0 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 0 0 0 0 0 1 0 0	95	1 1 0 1 0 0 0 0 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.17 Optimal Schedule for System E for Positive Correlation- $p_{target} = 0.80$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	51	1 1 0 0 0 0 0 0 0 0
4	1 1 0 1 0 0 0 0 0 0	52	1 1 0 1 0 0 0 0 0 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 0 0 0 1 0 0	55	1 1 1 1 0 0 0 1 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 0 0 0	57	1 1 1 1 1 0 0 0 0 0
10	1 1 1 1 1 1 0 1 0 0	58	1 1 1 1 1 1 0 1 0 0
11	1 1 1 1 1 1 1 0 0 0	59	1 1 1 1 1 1 1 0 0 0
12	1 1 1 1 1 1 1 1 0 0	60	1 1 1 1 1 1 1 1 0 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 0 0 0 0 0	62	1 1 1 1 1 0 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 1 0 0 1 0	68	1 1 1 1 1 1 0 1 1 1
21	1 1 1 1 1 1 0 0 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	71	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 0 0 0	75	1 1 0 0 0 0 0 0 0 0
28	1 1 0 0 0 0 0 1 1 0	76	1 1 1 1 0 0 0 0 0 0
29	1 1 1 1 0 0 0 0 0 0	77	1 1 1 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 0 0 0 1 0 0	79	1 1 1 1 0 0 0 1 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 0 0 0 0 0	81	1 1 1 1 1 0 0 0 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 0 1 0 0
35	1 1 1 1 1 1 1 0 0 0	83	1 1 1 1 1 1 1 0 0 0
36	1 1 1 1 1 1 1 1 1 0	84	1 1 1 1 1 1 1 0 1 1
37	1 1 1 1 1 1 0 1 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 0 0 0 0 0	86	1 1 1 1 1 0 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 1 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 0 0 0 0 0
44	1 1 1 1 1 1 0 1 0 0	92	1 1 1 1 1 0 0 1 1 0
45	1 1 1 1 1 1 0 0 0 0	93	1 1 1 1 1 0 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 0 0 0 0 0 1 0 0	95	1 1 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.18 Optimal Schedule for System E for Positive Correlation- $p_{target} = 0.90$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 1 1 1	49	1 1 0 0 0 0 0 1 1 1
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	51	1 1 0 0 0 0 0 0 0 0
4	1 1 1 1 0 0 0 0 0 0	52	1 1 1 1 0 0 0 0 0 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 0 0 0 1 0 0	55	1 1 1 1 0 0 0 1 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 0 0 0	57	1 1 1 1 1 0 0 0 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 0 0 0	59	1 1 1 1 1 1 1 0 0 0
12	1 1 1 1 1 1 1 1 1 1	60	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 0 1 0 0	61	1 1 1 1 1 1 0 1 0 0
14	1 1 1 1 1 0 0 0 0 0	62	1 1 1 1 1 0 0 0 0 0
15	1 1 0 1 1 0 0 1 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 0 1 1 0 0 1 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 0 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 0 0 0
18	1 1 0 1 1 0 0 1 1 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 0 1 1 0 0 1 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 0 0 1 1 0	68	1 1 1 1 1 1 0 0 1 1
21	1 1 1 1 1 0 0 0 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 1 0 0 0 0 0 0 0	71	1 1 0 0 0 0 0 1 0 0
24	1 1 1 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 1 1 1	73	1 1 0 0 0 0 0 1 1 1
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 0 0 0	75	1 1 0 0 0 0 0 0 0 0
28	1 1 0 1 0 0 0 0 0 0	76	1 1 0 1 0 0 0 0 0 0
29	1 1 0 1 0 0 0 0 0 0	77	1 1 0 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 0 1 0 0 0 1 1 0
31	1 1 1 1 0 0 0 1 0 0	79	1 1 1 1 0 0 0 1 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 0 0 0 0 0	81	1 1 1 1 1 0 0 0 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 0 1 0 0
35	1 1 1 1 1 1 1 0 0 0	83	1 1 1 1 1 1 1 0 0 0
36	1 1 1 1 1 1 1 1 1 1	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 0 1 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 0 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 0 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 0 0 1 1 1	92	1 1 1 1 1 1 0 1 0 0
45	1 1 1 1 1 0 0 0 0 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 0 1 0 0 0 1 1 0
47	1 1 0 0 0 0 0 1 0 0	95	1 1 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.19 Optimal Schedule for System E for Positive Correlation- $p_{target} = 0.95$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	51	1 1 0 0 0 0 0 0 0 0
4	1 1 0 1 0 0 0 0 0 0	52	1 1 0 1 0 0 0 0 0 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 1 1 0 0 0 0 0 0
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 1 0 0 0 0 0	55	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 1 0 0 0 0	57	1 1 1 1 1 0 0 1 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 0 0 0	59	1 1 1 1 1 1 1 0 0 0
12	1 1 1 1 1 1 1 1 1 0	60	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 1 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 1 1 1	65	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 0 0 1 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 0 0 1 1 0	68	1 1 1 1 1 1 0 0 1 1
21	1 1 1 1 1 0 0 1 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	71	1 1 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 0 0 0	75	1 1 0 0 0 0 0 0 0 0
28	1 1 0 1 0 0 0 0 0 0	76	1 1 1 1 0 0 0 0 0 0
29	1 1 0 1 0 0 0 0 0 0	77	1 1 1 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 1 0 0 0 0 0	79	1 1 1 1 1 0 0 0 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 1 0 0 0 0	81	1 1 1 1 1 0 0 1 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 0 0 0	83	1 1 1 1 1 1 1 0 0 0
36	1 1 1 1 1 1 1 1 1 0	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 0 0 0 0 0
44	1 1 1 1 1 1 0 0 1 1	92	1 1 1 1 1 0 0 1 1 0
45	1 1 1 1 0 1 0 1 1 0	93	1 1 1 1 1 0 0 1 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 0 0 0 0 0 1 0 0	95	1 1 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.20 Optimal Schedule for System E for Positive Correlation- $p_{target} = 0.99$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 1 1 0	51	1 1 0 0 0 0 0 1 1 0
4	1 1 0 1 0 0 0 0 0 0	52	1 1 0 0 0 0 0 1 1 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 0 0 0 0 0 1 1 1
6	1 1 1 1 0 0 0 0 0 0	54	1 1 0 1 0 0 0 1 1 0
7	1 1 1 1 0 0 0 1 0 0	55	1 1 1 1 0 0 0 1 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 0 0 1 0 0	57	1 1 1 1 1 1 0 0 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 1 0 0	59	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 0	60	1 1 1 1 1 1 1 1 0 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 0 1 1 0 0 1 1 1
16	1 1 1 1 1 0 0 0 0 0	64	1 1 0 1 1 0 0 1 0 0
17	1 1 1 1 1 0 0 0 0 0	65	1 1 0 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 0 1 1 0 0 1 1 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 0 1 1 1 0 1 1 0
20	1 1 1 1 1 1 0 1 1 0	68	1 1 1 1 1 1 0 1 1 1
21	1 1 1 1 1 1 0 0 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 0 0 0 0 0 1 0 0	71	1 1 1 0 0 0 0 0 0 0
24	1 1 0 0 0 0 0 1 1 1	72	1 1 1 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 1 1 0	75	1 1 0 0 0 0 0 1 1 0
28	1 1 1 1 0 0 0 0 0 0	76	1 1 0 1 0 0 0 0 0 0
29	1 1 1 1 0 0 0 0 0 0	77	1 1 0 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 0 0 0 1 0 0	79	1 1 1 1 0 0 0 1 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 1 0 0 0 0	81	1 1 1 1 1 0 0 1 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 1 0 0	83	1 1 1 1 1 1 1 1 0 0
36	1 1 1 1 1 1 1 1 0 0	84	1 1 1 1 1 1 1 1 1 1
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 1 0 1 0 0	92	1 1 1 1 1 1 0 1 0 0
45	1 1 1 1 1 1 0 0 0 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 1 0 0 0 0 0
47	1 1 0 1 0 0 0 0 0 0	95	1 1 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.21 Optimal Schedule for System E for Positive Correlation- $p_{target} = 0.999$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 0 0 0	51	1 1 0 0 0 0 0 0 0 0
4	1 1 1 1 0 0 0 0 0 0	52	1 1 0 0 0 0 0 1 1 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 0 0 0 0 0 1 1 1
6	1 1 1 1 0 0 0 0 0 0	54	1 1 1 1 0 0 0 0 0 0
7	1 1 1 1 1 0 0 0 0 0	55	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 1 0 0 0 0	57	1 1 1 1 1 1 0 0 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 1 0 0	59	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 1	60	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 1 1 1 1 0 0 0 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 1 1 1 0 0 0 0 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 1 1 1 0 0 0 0 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 1 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 0 0 0
18	1 1 1 1 1 0 0 0 0 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 1 0 1 0 0	68	1 1 1 1 1 1 0 1 1 1
21	1 1 1 1 1 1 0 0 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 1 1 0 0 0 0 0 0	71	1 1 0 0 0 0 0 0 1 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 0 0 0	75	1 1 0 0 0 0 0 0 0 0
28	1 1 0 0 0 0 0 1 1 0	76	1 1 1 1 0 0 0 0 0 0
29	1 1 0 0 0 0 0 1 1 1	77	1 1 1 1 0 0 0 0 0 0
30	1 1 1 1 0 0 0 0 0 0	78	1 1 1 1 0 0 0 0 0 0
31	1 1 1 1 1 0 0 0 0 0	79	1 1 1 1 1 0 0 0 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 1 0 0 0 0	81	1 1 1 1 1 1 0 0 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 1 0 0	83	1 1 1 1 1 1 1 1 0 0
36	1 1 1 1 1 1 1 1 1 1	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 1 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 1 0 1 0 0	92	1 1 1 1 1 1 0 1 1 0
45	1 1 1 1 1 1 0 0 0 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 1 1 0 0 0 0 0 0	95	1 1 0 0 0 0 0 0 1 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.22 Optimal Schedule for System E for Positive Correlation- $p_{target} = 0.9999$

	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
1	1 1 0 0 0 0 0 0 0 0	49	1 1 0 0 0 0 0 0 0 0
2	1 1 0 0 0 0 0 0 0 0	50	1 1 0 0 0 0 0 0 0 0
3	1 1 0 0 0 0 0 1 0 0	51	1 1 0 0 0 0 0 1 0 0
4	1 1 1 1 0 0 0 0 0 0	52	1 1 0 0 0 0 0 1 1 0
5	1 1 1 1 0 0 0 0 0 0	53	1 1 0 1 0 0 0 1 0 0
6	1 1 1 1 0 0 0 0 0 0	54	1 1 0 1 0 0 0 1 1 1
7	1 1 1 1 1 0 0 0 0 0	55	1 1 1 1 1 0 0 0 0 0
8	1 1 1 1 1 0 0 0 0 0	56	1 1 1 1 1 0 0 0 0 0
9	1 1 1 1 1 1 0 0 0 0	57	1 1 1 1 1 0 0 1 0 0
10	1 1 1 1 1 1 1 0 0 0	58	1 1 1 1 1 1 1 0 0 0
11	1 1 1 1 1 1 1 1 0 0	59	1 1 1 1 1 1 1 1 0 0
12	1 1 1 1 1 1 1 1 1 0	60	1 1 1 1 1 1 1 1 1 0
13	1 1 1 1 1 1 1 0 0 0	61	1 1 1 1 1 1 1 0 0 0
14	1 1 0 1 1 1 0 1 1 0	62	1 1 1 1 1 1 0 0 0 0
15	1 1 0 1 1 0 0 1 1 0	63	1 1 1 1 1 0 0 0 0 0
16	1 1 0 1 1 0 0 1 1 0	64	1 1 1 1 1 0 0 0 0 0
17	1 1 0 1 1 0 0 0 0 0	65	1 1 1 1 1 0 0 0 0 0
18	1 1 0 1 1 0 0 1 1 0	66	1 1 1 1 1 0 0 0 0 0
19	1 1 1 1 1 1 0 0 0 0	67	1 1 1 1 1 1 0 0 0 0
20	1 1 1 1 1 1 0 1 1 0	68	1 1 1 1 1 1 0 1 1 0
21	1 1 1 1 1 1 0 0 0 0	69	1 1 1 1 1 1 0 0 0 0
22	1 1 1 1 0 0 0 0 0 0	70	1 1 1 1 0 0 0 0 0 0
23	1 1 1 1 0 0 0 0 0 0	71	1 1 1 1 0 0 0 0 0 0
24	1 1 0 0 0 0 0 0 0 0	72	1 1 0 0 0 0 0 0 0 0
25	1 1 0 0 0 0 0 0 0 0	73	1 1 0 0 0 0 0 0 0 0
26	1 1 0 0 0 0 0 0 0 0	74	1 1 0 0 0 0 0 0 0 0
27	1 1 0 0 0 0 0 1 0 0	75	1 1 0 0 0 0 0 1 0 0
28	1 1 0 0 0 0 0 1 1 0	76	1 1 0 0 0 0 0 1 1 0
29	1 1 0 1 0 0 0 1 0 0	77	1 1 0 1 0 0 0 1 0 0
30	1 1 0 1 0 0 0 1 1 1	78	1 1 0 1 0 0 0 1 1 1
31	1 1 1 1 1 0 0 0 0 0	79	1 1 1 1 1 0 0 0 0 0
32	1 1 1 1 1 0 0 0 0 0	80	1 1 1 1 1 0 0 0 0 0
33	1 1 1 1 1 1 0 0 0 0	81	1 1 1 1 1 1 0 0 0 0
34	1 1 1 1 1 1 1 0 0 0	82	1 1 1 1 1 1 1 0 0 0
35	1 1 1 1 1 1 1 1 0 0	83	1 1 1 1 1 1 1 1 0 0
36	1 1 1 1 1 1 1 1 0 1	84	1 1 1 1 1 1 1 1 1 0
37	1 1 1 1 1 1 1 0 0 0	85	1 1 1 1 1 1 1 0 0 0
38	1 1 1 1 1 1 0 0 0 0	86	1 1 1 1 1 1 0 0 0 0
39	1 1 1 1 1 0 0 0 0 0	87	1 1 1 1 1 0 0 0 0 0
40	1 1 1 1 1 0 0 0 0 0	88	1 1 1 1 1 0 0 0 0 0
41	1 1 1 1 1 0 0 0 0 0	89	1 1 1 1 1 0 0 0 0 0
42	1 1 1 1 1 0 0 0 0 0	90	1 1 1 1 1 0 0 0 0 0
43	1 1 1 1 1 1 0 0 0 0	91	1 1 1 1 1 1 0 0 0 0
44	1 1 1 1 1 1 0 1 1 1	92	1 1 1 1 1 1 0 1 1 0
45	1 1 1 1 1 1 0 0 0 0	93	1 1 1 1 1 1 0 0 0 0
46	1 1 1 1 0 0 0 0 0 0	94	1 1 1 1 0 0 0 0 0 0
47	1 1 1 1 0 0 0 0 0 0	95	1 1 0 1 0 0 0 0 0 0
48	1 1 0 0 0 0 0 0 0 0	96	1 1 0 0 0 0 0 0 0 0

Table A.24 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.80$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	0100100100100100100100100100100
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0100100100100100100100100100100
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	00000000000000000000000000000000
26	00000000000000000000000000000000
27	11111100000000000000000000000000
28	00000000000000000000000000000000
29	11000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	11110000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0100100100100100100100100100100
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0100100100100100100100100100100
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.26 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.80$ for Generators 76-100

	76-100
49	00000000000000000000000000000000
50	00000000000000000000000000000000
51	11111100000000000000000000000000
52	00000000000000000000000000000000
53	11000000000000000000000000000000
54	00000000000000000000000000000000
55	00000000000000000000000000000000
56	00000000000000000000000000000000
57	00000000000000000000000000000000
58	00000000000000000000000000000000
59	00000000000000000000000000000000
60	0100100100100100100100100100100
61	00000000000000000000000000000000
62	00000000000000000000000000000000
63	00000000000000000000000000000000
64	00000000000000000000000000000000
65	00000000000000000000000000000000
66	00000000000000000000000000000000
67	00000000000000000000000000000000
68	0100100100100100100100100100100
69	00000000000000000000000000000000
70	00000000000000000000000000000000
71	00000000000000000000000000000000
72	00000000000000000000000000000000
73	00000000000000000000000000000000
74	00000000000000000000000000000000
75	11111100000000000000000000000000
76	00000000000000000000000000000000
77	11000000000000000000000000000000
78	00000000000000000000000000000000
79	00000000000000000000000000000000
80	00000000000000000000000000000000
81	11110000000000000000000000000000
82	00000000000000000000000000000000
83	00000000000000000000000000000000
84	0100100100100100100100100100100
85	00000000000000000000000000000000
86	00000000000000000000000000000000
87	00000000000000000000000000000000
88	00000000000000000000000000000000
89	00000000000000000000000000000000
90	00000000000000000000000000000000
91	00000000000000000000000000000000
92	0100100100100100100100100100100
93	00000000000000000000000000000000
94	00000000000000000000000000000000
95	00000000000000000000000000000000
96	00000000000000000000000000000000

Table A.28 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.90$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	0100100100100100100100100100100
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	00000000000000000000000000000000
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	11111100000000000000000000000000
26	11111111111000000000000000000000
27	00000000000000000000000000000000
28	00000000000000000000000000000000
29	00000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0100100100100100100100100100100
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	00000000000000000000000000000000
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.30 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.90$ for Generators 76-100

	76-100
49	11111111000000000000000000000000
50	11111111111111110000000000000000
51	00000000000000000000000000000000
52	00000000000000000000000000000000
53	00000000000000000000000000000000
54	00000000000000000000000000000000
55	00000000000000000000000000000000
56	00000000000000000000000000000000
57	00000000000000000000000000000000
58	00000000000000000000000000000000
59	00000000000000000000000000000000
60	0110110110110110110110110110110
61	00000000000000000000000000000000
62	00000000000000000000000000000000
63	00000000000000000000000000000000
64	00000000000000000000000000000000
65	00000000000000000000000000000000
66	00000000000000000000000000000000
67	00000000000000000000000000000000
68	0100100100100100100100100100100
69	00000000000000000000000000000000
70	00000000000000000000000000000000
71	00000000000000000000000000000000
72	00000000000000000000000000000000
73	11111100000000000000000000000000
74	11111111111100000000000000000000
75	00000000000000000000000000000000
76	00000000000000000000000000000000
77	00000000000000000000000000000000
78	00000000000000000000000000000000
79	00000000000000000000000000000000
80	00000000000000000000000000000000
81	00000000000000000000000000000000
82	00000000000000000000000000000000
83	00000000000000000000000000000000
84	0100100100100100100100100100100
85	00000000000000000000000000000000
86	00000000000000000000000000000000
87	00000000000000000000000000000000
88	00000000000000000000000000000000
89	11111100000000000000000000000000
90	00000000000000000000000000000000
91	00000000000000000000000000000000
92	00000000000000000000000000000000
93	11111100000000000000000000000000
94	00000000000000000000000000000000
95	00000000000000000000000000000000
96	00000000000000000000000000000000

Table A.32 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.95$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	11111111111111111111111111111111
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0110110110110110110110110110110
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	11100000000000000000000000000000
26	00000000000000000000000000000000
27	00000000000000000000000000000000
28	00000000000000000000000000000000
29	00000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0110110110110110110110110110110
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	11111110000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0100100100100100100100100100100
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.36 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.99$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	0100100100100100100100100100100
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	11000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0100100100100100100100100100100
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	00000000000000000000000000000000
26	00000000000000000000000000000000
27	00000000000000000000000000000000
28	00000000000000000000000000000000
29	00000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	0100100100100100100100100100100
36	0100100100100100100100100100100
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	11111111111111111111111111111111
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0100100100100100100100100100100
45	00000000000000000000000000000000
46	11000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.38 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.99$ for Generators 76-100

	76-100
49	00000000000000000000000000000000
50	00000000000000000000000000000000
51	00000000000000000000000000000000
52	00000000000000000000000000000000
53	00000000000000000000000000000000
54	00000000000000000000000000000000
55	00000000000000000000000000000000
56	00000000000000000000000000000000
57	00000000000000000000000000000000
58	00000000000000000000000000000000
59	0100100100100100100100100100100
60	11111111111111111111111111111111
61	00000000000000000000000000000000
62	00000000000000000000000000000000
63	00000000000000000000000000000000
64	00000000000000000000000000000000
65	00000000000000000000000000000000
66	00000000000000000000000000000000
67	00000000000000000000000000000000
68	0100100100100100100100100100100
69	00000000000000000000000000000000
70	11000000000000000000000000000000
71	00000000000000000000000000000000
72	00000000000000000000000000000000
73	00000000000000000000000000000000
74	00000000000000000000000000000000
75	00000000000000000000000000000000
76	00000000000000000000000000000000
77	00000000000000000000000000000000
78	11000000000000000000000000000000
79	11111111110000000000000000000000
80	00000000000000000000000000000000
81	00000000000000000000000000000000
82	00000000000000000000000000000000
83	00000000000000000000000000000000
84	0010010010010010010010010010010
85	00000000000000000000000000000000
86	00000000000000000000000000000000
87	00000000000000000000000000000000
88	00000000000000000000000000000000
89	11111111111111111111111111000000
90	00000000000000000000000000000000
91	00000000000000000000000000000000
92	0100100100100100100100100100100
93	00000000000000000000000000000000
94	00000000000000000000000000000000
95	00000000000000000000000000000000
96	00000000000000000000000000000000

Table A.40 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.999$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	11000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	1100100100100100100100100100100
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	11111111111111111111111111110000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0100100100100100100100100100100
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	00000000000000000000000000000000
26	00000000000000000000000000000000
27	00000000000000000000000000000000
28	10000000000000000000000000000000
29	00000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0110110110110110110110110110110
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	11111111111111000000000000000000
44	0100100100100100100100100100100
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.42 Optimal Schedule for System F for Zero Correlation - $p_{target} = 0.999$ for Generators 76-100

	76-100
49	0000000000000000000000000000000000
50	0000000000000000000000000000000000
51	0000000000000000000000000000000000
52	0000000000000000000000000000000000
53	0000000000000000000000000000000000
54	0000000000000000000000000000000000
55	0000000000000000000000000000000000
56	0000000000000000000000000000000000
57	0000000000000000000000000000000000
58	0000000000000000000000000000000000
59	0100100100100100100100100100100100
60	0110110110110110110110110110110
61	0000000000000000000000000000000000
62	0000000000000000000000000000000000
63	0000000000000000000000000000000000
64	0000000000000000000000000000000000
65	0000000000000000000000000000000000
66	0000000000000000000000000000000000
67	0000000000000000000000000000000000
68	0100100100100100100100100100100
69	0000000000000000000000000000000000
70	0000000000000000000000000000000000
71	0000000000000000000000000000000000
72	0000000000000000000000000000000000
73	0000000000000000000000000000000000
74	0000000000000000000000000000000000
75	0000000000000000000000000000000000
76	0000000000000000000000000000000000
77	0000000000000000000000000000000000
78	0000000000000000000000000000000000
79	0000000000000000000000000000000000
80	0000000000000000000000000000000000
81	0000000000000000000000000000000000
82	0000000000000000000000000000000000
83	0100100100100100100100100100100
84	0110110110110110110110110110110
85	0000000000000000000000000000000000
86	0000000000000000000000000000000000
87	0000000000000000000000000000000000
88	0000000000000000000000000000000000
89	0000000000000000000000000000000000
90	0000000000000000000000000000000000
91	0000000000000000000000000000000000
92	0100100100100100100100100100100
93	0000000000000000000000000000000000
94	0000000000000000000000000000000000
95	0000000000000000000000000000000000
96	0000000000000000000000000000000000

Table A.44 Optimal Schedule for System F for Zero Correlation - $P_{target} = 0.9999$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	0100100100100100100100100100100
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	0100100100100100100100100100100
12	0110110110110110110110110110110
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	11111111111111111111111111111111
19	00000000000000000000000000000000
20	0100100100100100100100100100100
21	00000000000000000000000000000000
22	11111000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	11111100000000000000000000000000
26	11111111111111111111111111110000
27	00000000000000000000000000000000
28	00000000000000000000000000000000
29	00000000000000000000000000000000
30	11111100000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	0100100100100100100100100100100
36	0100100100100100100100100100100
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0110110110110110110110110110110
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	11111000000000000000000000000000

Table A.46 Optimal Schedule for System F for Zero Correlation - $P_{target} = 0.9999$ for Generators 76-100

	76-100
49	111111100000000000000000000000
50	111111111111111111111111111000
51	000000000000000000000000000000
52	000000000000000000000000000000
53	000000000000000000000000000000
54	000000000000000000000000000000
55	000000000000000000000000000000
56	000000000000000000000000000000
57	000000000000000000000000000000
58	000000000000000000000000000000
59	000000000000000000000000000000
60	1101101101101101101101101101101
61	000000000000000000000000000000
62	000000000000000000000000000000
63	000000000000000000000000000000
64	000000000000000000000000000000
65	000000000000000000000000000000
66	000000000000000000000000000000
67	000000000000000000000000000000
68	0100100100100100100100100100100
69	000000000000000000000000000000
70	000000000000000000000000000000
71	000000000000000000000000000000
72	1111111111111111111111111111111
73	111111100000000000000000000000
74	111111111111111111111111111000
75	000000000000000000000000000000
76	000000000000000000000000000000
77	000000000000000000000000000000
78	111111000000000000000000000000
79	000000000000000000000000000000
80	000000000000000000000000000000
81	000000000000000000000000000000
82	000000000000000000000000000000
83	0100100100100100100100100100100
84	0110110110110110110110110110110
85	000000000000000000000000000000
86	000000000000000000000000000000
87	000000000000000000000000000000
88	000000000000000000000000000000
89	000000000000000000000000000000
90	000000000000000000000000000000
91	000000000000000000000000000000
92	0100100100100100100100100100100
93	000000000000000000000000000000
94	000000000000000000000000000000
95	000000000000000000000000000000
96	000000000000000000000000000000

Table A.48 Optimal Schedule for System F for Positive Correlation - $p_{target} = 0.80$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	00000000000000000000000000000000
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0110110110110110110110110110110
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	111111111111111111111111111111100
25	00000000000000000000000000000000
26	00000000000000000000000000000000
27	00000000000000000000000000000000
28	00000000000000000000000000000000
29	00000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0100100100100100100100100100100
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	00000000000000000000000000000000
45	11111000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.52 Optimal Schedule for System F for Positive Correlation - $p_{target} = 0.90$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	0110110110110110110110110110110
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	00000000000000000000000000000000
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	11111100000000000000000000000000
25	00000000000000000000000000000000
26	00000000000000000000000000000000
27	00000000000000000000000000000000
28	00000000000000000000000000000000
29	11100000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0100100100100100100100100100100
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	00000000000000000000000000000000
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.54 Optimal Schedule for System F for Positive Correlation for Zero Correlation - $p_{target} = 0.90$ for Generators 76-100

	76-100
49	00000000000000000000000000000000
50	00000000000000000000000000000000
51	00000000000000000000000000000000
52	00000000000000000000000000000000
53	11100000000000000000000000000000
54	00000000000000000000000000000000
55	00000000000000000000000000000000
56	00000000000000000000000000000000
57	00000000000000000000000000000000
58	00000000000000000000000000000000
59	00000000000000000000000000000000
60	0100100100100100100100100100100
61	00000000000000000000000000000000
62	00000000000000000000000000000000
63	00000000000000000000000000000000
64	00000000000000000000000000000000
65	00000000000000000000000000000000
66	00000000000000000000000000000000
67	00000000000000000000000000000000
68	0100100100100100100100100100100
69	00000000000000000000000000000000
70	00000000000000000000000000000000
71	00000000000000000000000000000000
72	00000000000000000000000000000000
73	00000000000000000000000000000000
74	00000000000000000000000000000000
75	00000000000000000000000000000000
76	00000000000000000000000000000000
77	11100000000000000000000000000000
78	00000000000000000000000000000000
79	00000000000000000000000000000000
80	00000000000000000000000000000000
81	00000000000000000000000000000000
82	00000000000000000000000000000000
83	00000000000000000000000000000000
84	0100100100100100100100100100100
85	00000000000000000000000000000000
86	00000000000000000000000000000000
87	00000000000000000000000000000000
88	00000000000000000000000000000000
89	00000000000000000000000000000000
90	00000000000000000000000000000000
91	00000000000000000000000000000000
92	0100100100100100100100100100100
93	00000000000000000000000000000000
94	00000000000000000000000000000000
95	00000000000000000000000000000000
96	00000000000000000000000000000000

Table A.56 Optimal Schedule for System F for Positive Correlation - $p_{target} = 0.95$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	0100100100100100100100100100100
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	0010010010010010010010010010010
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	11111110000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0100100100100100100100100100100
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	00000000000000000000000000000000
26	11111111111000000000000000000000
27	00000000000000000000000000000000
28	00000000000000000000000000000000
29	00000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0010010010010010010010010010010
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0110110110110110110110110110110
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.60 Optimal Schedule for System F for Positive Correlation - $p_{target} = 0.99$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	0100100100100100100100100100100
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	0010010010010010010010010010010
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0100100100100100100100100100100
21	00000000000000000000000000000000
22	10000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	00000000000000000000000000000000
26	11111111111111111111111111111111
27	11111111111111111111111111111111
28	00000000000000000000000000000000
29	00000000000000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0010010010010010010010010010010
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	11111111111111111111111111111111
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0100100100100100100100100100100
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.62 Optimal Schedule for System F for Positive Correlation for Zero Correlation - $p_{target} = 0.99$ for Generators 76-100

	76-100
49	00000000000000000000000000000000
50	11111111111111111111111111111111
51	11111111111111111111111111111111
52	00000000000000000000000000000000
53	00000000000000000000000000000000
54	00000000000000000000000000000000
55	00000000000000000000000000000000
56	00000000000000000000000000000000
57	00000000000000000000000000000000
58	00000000000000000000000000000000
59	00000000000000000000000000000000
60	01001001001001001001001001001001
61	00000000000000000000000000000000
62	00000000000000000000000000000000
63	00000000000000000000000000000000
64	00000000000000000000000000000000
65	11111111111111111111111111111111
66	00000000000000000000000000000000
67	00000000000000000000000000000000
68	11001001001001001001001001001001
69	11111111111111111111111111111111
70	00000000000000000000000000000000
71	00000000000000000000000000000000
72	00000000000000000000000000000000
73	00000000000000000000000000000000
74	11111111111111111111111111111111
75	11111111111111111111111111111111
76	00000000000000000000000000000000
77	00000000000000000000000000000000
78	00000000000000000000000000000000
79	00000000000000000000000000000000
80	00000000000000000000000000000000
81	00000000000000000000000000000000
82	00000000000000000000000000000000
83	00000000000000000000000000000000
84	00100100100100100100100100100100
85	00000000000000000000000000000000
86	00000000000000000000000000000000
87	00000000000000000000000000000000
88	00000000000000000000000000000000
89	00000000000000000000000000000000
90	00000000000000000000000000000000
91	00000000000000000000000000000000
92	01001001001001001001001001001001
93	00000000000000000000000000000000
94	10000000000000000000000000000000
95	00000000000000000000000000000000
96	00000000000000000000000000000000

Table A.64 Optimal Schedule for System F for Positive Correlation - $p_{target} = 0.999$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	00000000000000000000000000000000
12	0110110110110110110110110110
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0100100100100100100100100100
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	111111111110000000000000000000
25	00000000000000000000000000000000
26	00000000000000000000000000000000
27	00000000000000000000000000000000
28	10000000000000000000000000000000
29	111111111000000000000000000000
30	00000000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0010010010010010010010010010
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0100100100100100100100100100
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

Table A.66 Optimal Schedule for System F for Positive Correlation for Zero Correlation - $p_{target} = 0.999$ for Generators 76-100

	76-100
49	00000000000000000000000000000000
50	00000000000000000000000000000000
51	00000000000000000000000000000000
52	10000000000000000000000000000000
53	00000000000000000000000000000000
54	00000000000000000000000000000000
55	00000000000000000000000000000000
56	00000000000000000000000000000000
57	00000000000000000000000000000000
58	00000000000000000000000000000000
59	0100100100100100100100100100100
60	0110110110110110110110110110110
61	00000000000000000000000000000000
62	00000000000000000000000000000000
63	00000000000000000000000000000000
64	00000000000000000000000000000000
65	00000000000000000000000000000000
66	00000000000000000000000000000000
67	00000000000000000000000000000000
68	0100100100100100100100100100100
69	00000000000000000000000000000000
70	00000000000000000000000000000000
71	00000000000000000000000000000000
72	00000000000000000000000000000000
73	00000000000000000000000000000000
74	00000000000000000000000000000000
75	00000000000000000000000000000000
76	00000000000000000000000000000000
77	00000000000000000000000000000000
78	00000000000000000000000000000000
79	00000000000000000000000000000000
80	00000000000000000000000000000000
81	00000000000000000000000000000000
82	00000000000000000000000000000000
83	0100100100100100100100100100100
84	0110110110110110110110110110110
85	00000000000000000000000000000000
86	00000000000000000000000000000000
87	00000000000000000000000000000000
88	00000000000000000000000000000000
89	00000000000000000000000000000000
90	00000000000000000000000000000000
91	00000000000000000000000000000000
92	0100100100100100100100100100100
93	00000000000000000000000000000000
94	00000000000000000000000000000000
95	00000000000000000000000000000000
96	00000000000000000000000000000000

Table A.68 Optimal Schedule for System F for Positive Correlation - $p_{target} = 0.9999$ for Generators 76-100

	76-100
1	00000000000000000000000000000000
2	00000000000000000000000000000000
3	00000000000000000000000000000000
4	00000000000000000000000000000000
5	00000000000000000000000000000000
6	00000000000000000000000000000000
7	00000000000000000000000000000000
8	00000000000000000000000000000000
9	00000000000000000000000000000000
10	00000000000000000000000000000000
11	11000000000000000000000000000000
12	111100100100100100100100100100
13	00000000000000000000000000000000
14	00000000000000000000000000000000
15	00000000000000000000000000000000
16	00000000000000000000000000000000
17	00000000000000000000000000000000
18	00000000000000000000000000000000
19	00000000000000000000000000000000
20	0100100100100100100100100100100
21	00000000000000000000000000000000
22	00000000000000000000000000000000
23	00000000000000000000000000000000
24	00000000000000000000000000000000
25	11111111110000000000000000000000
26	00000000000000000000000000000000
27	00000000000000000000000000000000
28	11000000000000000000000000000000
29	11111111110000000000000000000000
30	11111000000000000000000000000000
31	00000000000000000000000000000000
32	00000000000000000000000000000000
33	00000000000000000000000000000000
34	00000000000000000000000000000000
35	00000000000000000000000000000000
36	0110110110110110110110110110
37	00000000000000000000000000000000
38	00000000000000000000000000000000
39	00000000000000000000000000000000
40	00000000000000000000000000000000
41	00000000000000000000000000000000
42	00000000000000000000000000000000
43	00000000000000000000000000000000
44	0100100100100100100100100100100
45	00000000000000000000000000000000
46	00000000000000000000000000000000
47	00000000000000000000000000000000
48	00000000000000000000000000000000

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GLOSSARY

Auto Regressive Moving Average Model: This is a parametric method that finds a statistical relationship in sets of data to utilize in forecasting future values of a variable.

Chance Constrained Programming: Mathematical programming employing chance constraints, the violation of which are dependent on a defined probability level.

Dynamic Programming: An optimization technique that iteratively solves shorter time horizon problems iteratively to obtain a solution for the original problem.

Economic Dispatch Problem: Power planning problem for the short term (seconds to minutes) where operators determine the operating levels of the scheduled power generators.

Lagrangian Relaxation: An optimization technique that relaxes a complicating set of constraints and solves the relaxed problem to find an optimum solution to the original problem.

Recourse Based Modeling: A stochastic model that assumes a corrective set of decisions can be made once the outcomes of random variables are observed.

Spinning Reserve: The additional electricity available (synchronized) to serve load immediately is defined as the spinning reserve.

Stochastic Programming: A programming technique where some or all the parameters are modeled considering their underlying probability distribution.

Unit Commitment Problem: Determining the mix of generators and their estimated output level to meet the expected demand of electricity over a given time horizon (a day or a week), while satisfying constraints such as ramp rate limits, uptime and downtime constraints, reserve and energy requirements.