THREE ESSAYS ON INFORMATION TRANSMISSION IN ASYMMETRIC INFORMATION GAMES

by

Yeol Yong Sung

BA in Economics, Sungkyunkwan University, 1999

MA in Economics, Sungkyunkwan University, 2004

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This dissertation was presented

by

Yeol Yong Sung

It was defended on

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and approved by

Andreas Blume, Professor, Department of Economics

James Cassing, Professor, Department of Economics

John Duffy, Professor, Department of Economics

Esther Gal-Or, Professor, Katz School of Business

Dissertation Director: Andreas Blume, Professor, Department of Economics

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Yeol Yong Sung, PhD

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This dissertation consists of three chapters where we study information transmission in various environments.

The first chapter analyzes the effect of the presence of an uninformed sender on the information transmission between an informed sender and the receiver. The sender is uninformed with a positive probability and it is not verifiable whether she is informed or not. In almost all equilibria, the uninformed sender pools with a subset of types of the informed sender. We show that there exists an equilibrium in which the informed sender's cheap talk message conveys more precise information and the informed sender is better off by the presence of the uninformed sender.

In the second chapter, a buyer is uncertain of information on product qualities. We introduce a variable that generates social value of information, which is buyer's action such as the usage and maintenance of a product after purchase. If the buyer is concerned about his action, the seller has more incentive to reveal product information. Furthermore, more information is revealed as the variance of the quality is larger or as the average quality is lower. In this model, the certification cost is increasing in the sense that a better certificate is more costly. Then, there are multiple equilibria and the least level of revelation is *ex ante* Pareto optimal.

In the third chapter, we study firms' voluntary disclosure in an oligopoly market for differentiated products in which firms are allowed to advertise a rival's product as well as their own product. We show that full information is revealed by a high quality firm's comparative advertisement, where the advertisement on the rival's product is negative. Moreover, full revelation is the unique equilibrium outcome. The results imply that by allowing for negative advertisement on rivals' products, a society can increase consumers' welfare without mandatory disclosure laws.

Keywords: cheap talk, uninformed sender, certification, advertisement, disclosure.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

PREFACE

I am deeply indebted to Andreas Blume and Oliver Board for their help, advice, and encouragement. I am also grateful to my committee members, James Cassing, John Duffy, and Esther Gal-Or for their guidance and support. Lastly, I would like to thank Sourav Bhattacharya, Wonjea Huh, Wooyoung Lim, and Jipeng Zhang for their useful comments.

1.0 INFORMATION TRANSMISSION WHEN UNINFORMED SENDERS ARE PRESENT

1.1 INTRODUCTION

Most literature regarding communication games has studied an environment where the receiver is sure of how informed the sender is. Crawford and Sobel [9] (CS) present a canonical model, in which the sender is fully informed for certain and her preference is biased away from the receiver's. However, if informed experts and uninformed charlatans coexist and they cannot verify or strategically do not reveal their informedness, then the receiver (or decision maker) is uncertain of who is informed. For example, consider a customer in a retail shop, who may want to ask an assistant for information about a product that he considers purchasing. Because assistants work for the retailer and are not directly concerned with the product, they may not be informed of the product. Thus, the customer bears uncertainty about the assistant's informedness as well as the product information. The customer must take this into account when deciding to purchase the product. Other examples of uncertain expertise can be found in lobbyist-legislator and medical doctor-patient relationships. Lobbyists may have to give advice without full information on the state of the world, and medical doctors may not catch up with up-to-date knowledge and technology. We model this kind of receiver uncertainty by introducing an uninformed sender.

This chapter analyzes the effect of the presence of an uninformed sender on the information transmission between an informed sender and the receiver. The model is based on CS, but the sender is uninformed with a positive probability and, consequently, the receiver is uncertain of the quality of information that he receives. Formally, the sender is an informed expert with probability $p \in (0, 1)$ and uninformed with $1-p$. Whether the sender is informed or not is not verifiable to the receiver.

If the receiver could distinguish the expert from the uninformed sender, he could get a higher expected payoff by consulting the informed expert rather than the uninformed sender. As a result, the uninformed sender would play no parts and the outcome would be just like the CS one. However, in the case in which the receiver is uncertain of the sender's informedness, the model has different outcomes. In almost all equilibria, the uninformed sender pretends to be informed, that is, she behaves as a charlatan. Therefore, the informed sender is willing to distinguish herself from the uninformed sender and for some $p \in (0, 1)$ there exists an equilibrium in which the informed sender's cheap talk message conveys more precise information. Because of this informational effect, the informed sender is better off than she would be in the best CS equilibrium. This model is a one-shot game and the sender does not care about future reputation. If the informed sender had a reputation for being well informed, she would not have many incentives to transmit precise information (See Morris [35] and Ottaviani and Sørensen [39, 40]).

We say that an equilibrium is *influential* if at least two actions are realized with a positive probability as the outcome and that equilibrium η is *more influential* than equilibrium η' if more actions are realized in η than η'. An equilibrium is often said to be *informative* if the receiver's updated belief after receiving a message is distinct from his prior belief. An influential equilibrium is informative but the converse is not always true. The informativeness can be measured by the receiver's expected utility in cheap talk models with the uniform-quadratic specification because its absolute value equals the residual variance of the receiver's expectation on the sender's type. If information that the receiver receives is distorted by the uninformed sender, a more influential equilibrium can be less informative than a CS equilbrium. Thus, we borrow the influentialness concept in arguing improvement over the CS model but will use the both concepts if there is no confusion.[1]

In this model with the uniform-quadratic specification, no information is conveyed when the preference bias between the players is greater than or equal to 1/4 as in the CS model. However, for example, a 3-step equilibrium exists even when the bias is greater than or equal

[1] See Austen-Smith [1, p.958] and Austen-Smith and Banks [2, p.5] for the concept of 'influential' versus 'informative'.

to $1/12$ if p is not very high. For those values of b, there does not exist such an equilibrium in the CS model. Generally speaking, for some $p \in (0,1)$, a more influential equilibrium exists than the best (most influential) CS equilibrium when the uninformed sender pools with middle interval types of the informed sender. In this equilibrium, the informed sender's expected utility is greater than that in the best CS equilibrium. However, the receiver's expected utility is less if the uninformed sender distorts the receiver's inference by a large amount although it is more influential.

Austen-Smith [1] analyzes the same situation with a verifiability assumption. In his model, the informed sender can verify that she is informed so that the uninformed sender cannot pretend to be informed. The sender's information acquisition is costly and the probability of being informed is determined by the price of information. In that case, it is reasonable that the informed sender's claim that she is informed is verifiable. With this assumption, there exists an influential equilibrium even for the bias between $1/4$ and $1/2$. However, without verifiability, since the uninformed sender is able to mimic any types of the informed sender, her message must be optimal over the messages that the informed sender sends, which restricts the possible values of the bias that support influential equilibria.

Fischer and Stocken [11] and Ivanov [21] argue that more informative (or possibly more influential) equilibrium can be attained by imposing restriction on the sender's information and reducing the information quality. On the other hand, our model studies the effect of uncertainty about the sender's information quality on the influentialness of equilibria. Obviously, equilibria become less informative from the receiver's standpoint because the information is distorted by the uninformed sender pretending to be informed. However, the informed sender's message tends to be more informative and as a result more influential equilibrium can be attained. Kurino and Lai [26] work on a similar model to this chapter but the sender's information structure is totally imperfect.

There is some literature on communication in uncertain environment. Blume *et al.* [5] introduce noise in communication process and Goltsman *et al.* [14] identifies that an optimal communication mechanism can be attained by the introduction of noise. Blume and Board [4] show that the optimal outcome can be attained as well if the sender has private information about message availability from a limited message space. In their another paper, [3], the sender takes vague messages strategically in communication to mitigate conflict, which leads to welfare enhancement. Krishna and Morgan [25] allow for mutual communication between the players, the outcome of which is uncertain, before the receiver takes an action. Morgan and Stocken [34], Li [28], and Li and Madarász [29] introduce receiver uncertainty about the preference bias. All of them show that informativeness can improve under the uncertain environment, assuming that the receiver knows how informed the sender is.

The rest of the chapter is as follows: Section 1.2 presents the formal model and Section 1.3 characterizes the properties of equilibria. Section 1.4 analyzes the model in the uniform-quadratic case in comparison with CS and Section 1.5 shows the equilibrium welfare results with a particular example. Lastly, Section 1.6 summarizes the main results and some proofs are in the appendix.

1.2 MODEL

This is an extension of the sender-receiver communication game. The sender learns her type, $t \in T = [0,1]$, with probability $p \in (0,1)$, where t is distributed according to a common prior density f with $f(t) > 0$ for all $t \in [0,1]$. Whether she knows her type or not, the sender sends a message, m, to the receiver, who takes an action, $a \in \mathbb{R}$, after observing a message. Messages are irrelevant to the payoffs and taken from an arbitrary set M which is large enough to convey whatever needs to be conveyed. The sender's and receiver's utilities are given by $u^S(a,t)$ and $u^R(a,t)$ respectively. The utility functions are twice continuously differentiable in both arguments and we assume that $u^i_{11} < 0$ and $u^i_{12} > 0$ for $i = S, R$ with subscripts describing partial derivatives. Define $y^S(t)$, $y^R(t)$ for given t and $y^R(\underline{t}, \bar{t})$ for $\underline{t} < \bar{t}$ as

$$y^i(t) := \operatorname*{argmax}_{a \in \mathbb{R}} u^i(a,t), \quad i = S, R,$$

$$y^R(\underline{t}, \bar{t}) := \operatorname*{argmax}_{a \in \mathbb{R}} \int_{\underline{t}}^{\bar{t}} u^R(a,t) f(t)\, \mathrm{d}t.$$

Assuming that for every $t \in [0,1]$, $u^i_1(a,t) = 0$ for some $a \in \mathbb{R}$, then $u^i_{11} < 0$ and $u^i_{12} > 0$ imply that $y^i(t)$ and $y^R(\underline{t}, \bar{t})$ are well-defined and strictly increasing in all arguments. Another crucial assumption on the preferences is that $y^S(t) \neq y^R(t)$ for any $t \in [0,1]$, implying that there is a conflict of interests between the sender and the receiver. By the continuity of $y^S(\cdot)$ and $y^R(\cdot)$, we assume without loss of generality that $y^R(t) < y^S(t)$ for all $t \in [0,1]$.

The informed sender's pure strategy is a function of types, $m^I : T \to M$, and the uninformed sender's pure strategy is just to choose a message, $m^U \in M$. Without loss of generality, we assume that the informed sender only uses pure strategies.[2] The uninformed sender is allowed to mix messages so that her strategy is a probability distribution, $q(\cdot)$, over the message space. The receiver will only use a pure strategy in equilibrium because the utility function is strictly concave in a for every $t \in T$. The pure strategy is represented by a real-valued function of messages, $a : M \to \mathbb{R}$. We say that an action $\bar{a}$ is *induced* if $a(m^I(t)) = \bar{a}$ for some $t \in T$ or $a(m^U) = \bar{a}$ for $m^U \in M$ such that $q(m^U) > 0$.

The receiver is uncertain of both the sender's informedness and the informed sender's type, but the probability p and the distribution of t are common knowledge. Therefore, although the receiver observes neither the true value of the type nor even whether the sender is informed or not, he can infer them from messages that he receives. On observing message $m \in M$, the receiver's posterior belief about the sender's type is denoted by $\mu(\cdot|m)$. Then, a perfect Bayesian equilibrium of the model is a profile of strategies and the receiver's belief, $(m^I(\cdot), q(\cdot), a(\cdot), \mu(\cdot|\cdot))$, which satisfies:

(E1) $\forall\, t \in [0,1],\; m^I(t) \in \operatorname*{argmax}_{m \in M} u^S(a(m), t),$

(E2) $\forall\, m^U \in \operatorname{supp}(q),\; m^U \in \operatorname*{argmax}_{m \in M} \int_0^1 u^S(a(m), t) f(t)\, \mathrm{d}t,$

(E3) $\forall\, m \in M,\; a(m) \in \operatorname*{argmax}_{a \in \mathbb{R}} \int_0^1 u^R(a, t) \mu(t|m)\, \mathrm{d}t,$

(E4) $\mu(\cdot|m)$ is derived by Bayes' rule if $m = m^I(t)$ for some $t \in [0,1]$ or $q(m) > 0$.

[2] If the uninformed sender is separating from the informed sender with a nondegenerate probability $q \in (0,1)$, then the types pooling with the uninformed sender with a positive probability must induce $a = y^R(0,1)$ in equilibrium whatever message they send. This is outcome equivalent to an equilibrium in which those types pool with the uninformed sender on one message with probability 1. See Lemma 1 and Example 1. In other cases, we can apply the same reasoning as in Theorem 1 and footnote 4 in CS. Thus, any equilibrium is outcome equivalent to an equilibrium in which every informed type sends one message with probability 1.

1.3 EQUILIBRIUM

In this section, we characterize the equilibrium. First, we begin with a lemma showing a generic property of equilibria. For the lemma, we assume that the monotonicity condition, (M), in CS is satisfied if the uninformed sender is not present.

Lemma 1. *Suppose that* $m^I(\cdot)$, $q(\cdot)$, *and* $a(\cdot)$ *constitute an equilibrium and* $q(m^U) > 0$ *for some* $m^U \in M$. *Then,* $m^U \neq m^I(t)$ *for any* $t \in [0,1]$ *only if* $a(m^I(t)) = y^R(0,1)$ *for some* $t \in [0,1]$. *Moreover, this equilibrium is outcome equivalent to an equilibrium in which* $m^U = m^I(t)$ *for that* t.

Proof. Suppose that $q(m^U) > 0$ and $m^U \neq m^I(t)$ for any $t \in [0,1]$. Then, $a(m^U) = y^R(0,1)$. If $a(m^I(t)) = y^R(0,1)$ for some $t \in [0,1]$, then the type t and the uninformed sender freely pool together without the outcome changed. This proves the second statement.

Suppose that $a(m^I(t)) \neq y^R(0,1)$ for any $t \in [0,1]$. This implies that $q(m^I(t)) = 0$ for all $t \in [0,1]$ and that at least two actions are induced by the informed sender. Since the sender reveals her informedness according to the strategy, the type space is partitioned in the same manner in CS. Using the CS Corollary 1 under the monotonicity condition, we want to show that

$$\exists\, t \in [0,1] \text{ such that } y^S(t) = y^R(0,1),$$

because then the type t absolutely prefers m^U to $m^I(t)$, which leads to a contradiction. Suppose that $y^S(t) \neq y^R(0,1)$ for any $t \in [0,1]$. Since $y^S(\cdot)$ is continuous, we have two cases: either

$$\begin{aligned} &\text{(i)} \quad y^S(t) > y^R(0,1) \text{ for all } t \in [0,1]; \quad \text{or} \\ &\text{(ii)} \quad y^S(t) < y^R(0,1) \text{ for all } t \in [0,1]. \end{aligned}$$

If the latter case, (ii), holds, then $y^S(1) < y^R(0,1)$ for $t = 1$, but $y^R(0,1) < y^R(1)$ and this contradicts $y^R(1) < y^S(1)$. Thus, $y^S(t) > y^R(0,1)$ for all $t \in [0,1]$. By the strictly increasing $y^S(\cdot)$, it is equivalent to that $y^S(0) > y^R(0,1)$. Since $u^S(y^S(0),0) > u^S(y^R(0,1),0)$ by the definition of $y^S(\cdot)$, $y^S(0) > y^R(0,1) > y^R(0)$ and $u^S_{11} < 0$ imply

$$u^S(y^R(0,1),0) > u^S(y^R(0),0).$$

Then, under condition (M) and by the CS Corollary 1, the equilibrium must be uninfluential. This is a contradiction. □

This is true because there exists $t \in T$ that strictly prefers to pool with the uninformed sender as long as $a = y^R(0,1)$ is not induced by the informed sender. To understand the lemma, take an example of uniform f and quadratic loss utility functions, $u^S(a,t) = -(a-(t+b))^2$ and $u^R(a,t) = -(a-t)^2$, where $b > 0$. If the uninformed sender separates, then her message induces the receiver to take $a = 1/2$, and $t = 1/2 - b$, if $b \leq 1/2$, certainly prefers this action.[3] In equilibrium, the uninformed sender separates only if $a = 1/2$ is induced by the informed sender. Example 1 shows such an equilibrium.

Example 1. Suppose that f is uniform and the utility functions are of the quadratic loss forms as specified in the above. Then, for $b = 1/56$, the following strategies constitute an equilibrium in which the uninformed sender separates.

$$m^I(t) = \begin{cases} m_1, & \text{if } t \in [0, 2/14), \\ m_2, & \text{if } t \in [2/14, 5/14), \\ m_3, & \text{if } t \in [5/14, 9/14), \\ m_4, & \text{if } t \in [9/14, 1]. \end{cases}$$

$$m^U = m'$$

$$a(m) = \begin{cases} 2/28, & \text{if } m = m_1, \\ 7/28, & \text{if } m = m_2, \\ 14/28\,(=1/2), & \text{if } m = m_3 \text{ or } m', \\ 23/28, & \text{if } m = m_4, \\ a(m_1), & \text{otherwise.} \end{cases}$$

In Example 1, both the informed sender of type $t \in [5/14, 9/14)$ and the uninformed sender induce the same action $a = 1/2$. Thus, even if any of them randomly chooses between m_3 and m', it does not affect the equilibrium partition of the type space and the players'

[3] For $b > 1/2$, it is obvious that communication plays no role and every informed type induces $a = 1/2$ in equilibrium.

payoffs. In fact, if $a = 1/2$ is induced by some informed types, they can freely pool together with the uninformed sender on the same message without the outcome changed.

By Lemma 1, any equilibrium in which the uninformed sender separates from the informed sender is outcome equivalent to an equilibrium in which the uninformed sender pools with a subset of types of the informed sender. Moreover, the nonseparating uninformed sender is a generic property of equilibria. Thus, we assume without loss of generality that the uninformed sender pretends to be informed unless she is strictly better off by revealing herself.

The following lemma shows another property of equilibria: the actions induced in an equilibrium are nondecreasing in types. It guarantees that the informed sender partitions the type space into subintervals and reveals in which interval her type lies.

Lemma 2. *Suppose that $m^I(\cdot)$ and $a(\cdot)$ constitute an equilibrium. Then, $a(m^I(t))$ is nondecreasing in t.*

Proof. Let $t_1 < t_2$ and suppose that $a(m^I(t_1)) = a_1$ and $a(m^I(t_2)) = a_2$. The informed sender's incentive compatibility, (E1), requires that

$$u^S(a_1, t_1) \geq u^S(a_2, t_1), \text{ and}$$
$$u^S(a_2, t_2) \geq u^S(a_1, t_2).$$

Adding the two inequalities and rearranging the terms, we have

$$u^S(a_2, t_2) - u^S(a_2, t_1) \geq u^S(a_1, t_2) - u^S(a_1, t_1). \tag{1.1}$$

Given t_1 and t_2, differentiating $u^S(a, t_2) - u^S(a, t_1)$ with respect to a,

$$\frac{\partial[u^S(a, t_2) - u^S(a, t_1)]}{\partial a} = u_1^S(a, t_2) - u_1^S(a, t_1) > 0$$

because $t_1 < t_2$ and $u_{12}^S > 0$. Thus, $u^S(a, t_2) - u^S(a, t_1)$ is increasing in a and (1.1) implies that $a_1 \leq a_2$. □

Lemma 2 guarantees that the set of types that induce the same action in an equilibrium is convex, so that all equilibria are partitional. If an action is induced by at most one type, then the convexity is trivial. Let $\underline{t} < \bar{t}$ and suppose that both $\underline{t}$ and $\bar{t}$ induce an action $\tilde{a}$. Then, for any $t \in (\underline{t}, \bar{t})$, we must have $a(m^I(\underline{t})) \leq a(m^I(t)) \leq a(m^I(\bar{t}))$ and since $a(m^I(\underline{t})) = a(m^I(\bar{t})) = \tilde{a}$, t also induces $\tilde{a}$. Thus, by this convexity and the monotone $a(m^I(\cdot))$ together with the assumption that $y^R(t) < y^S(t)$ for all $t \in T$, the informed sender's type space is partitioned into a finite number N of subintervals with boundary types $\langle t_0 = 0, t_1, \ldots, t_N = 1\rangle$ and N actions are induced as in the CS model. In fact, Lemma 2 holds regardless of the presence of the uninformed sender.

By Lemma 1 and Lemma 2, we are now ready to characterize the equilibrium when the sender is uninformed with a positive probability. Given $p \in (0, 1)$, there exists an integer N such that the type space is partitioned into N subintervals, the informed types in the same interval induce the same action, and the uninformed sender pools with a subset of informed types. Essentially, any equilibrium belongs to one in this class. Obviously, the uninformative equilibrium ($N = 1$) always exists in which $m^I(t) = m^\circ$ for all $t \in T$, $q(\cdot)$ is arbitrary, and $a(m) = y^R(0, 1)$ with $\mu(\cdot|m) = f(\cdot)$ for all $m \in M$. In the rest of this section, we characterize the general form of equilibrium.

First, consider the sender's strategy. Suppose that the informed sender of type t sends $m^I(t) = m_i$ if $t \in [t_{i-1}, t_i)$, $i = 1, 2, \ldots, N$, where $t_0 = 0$ and $t_N = 1$, and the uninformed sender's strategy $q(\cdot)$ is such that if $q(m^U) > 0$ then $m^U = m_k$ for some $k \in \{1, 2, \ldots, N\}$. The uninformed sender will mix over at most two messages because the utility is strictly concave in a. Moreover, the randomization of the two messages constitutes an equilibrium only if they induce adjacent two actions.

Given $m^I(\cdot)$ and $q(\cdot)$, the receiver forms his belief on t after observing m_i as

$$\mu(t|m_i) = \begin{cases} \phi(m_i) \dfrac{f(t)}{\int_{t_{i-1}}^{t_i} f(\tau)\, \mathrm{d}\tau} + [1 - \phi(m_i)] f(t) & \text{if } t \in [t_{i-1}, t_i), \\ \\ \phi(m_i) \cdot 0 + [1 - \phi(m_i)] f(t) & \text{if } t \notin [t_{i-1}, t_i), \end{cases}$$

where $\phi(m_i)$ is the receiver's posterior belief that the sender is informed on observing the message m_i. $\phi(m_i)$ is derived by Bayes' rule as

$$\phi(m_i) = \frac{p\int_{t_{i-1}}^{t_i} f(\tau)\,\mathrm{d}\tau}{p\int_{t_{i-1}}^{t_i} f(\tau)\,\mathrm{d}\tau + (1-p)q(m_i)}.$$

In words, if the receiver observes m_i, he believes that t is distributed on $[t_{i-1}, t_i)$ according to the density $f(t)/\int_{t_{i-1}}^{t_i} f(\tau)\,\mathrm{d}\tau$ with probability $\phi(m_i)$ and on $[0,1]$ according to the prior density $f(t)$ with $1-\phi(m_i)$. Thus, given the beliefs, for all $i = 1, 2, \ldots, N$, the receiver's optimal action $a(m_i)$ when observing m_i solves

$$\max_{a\in\mathbb{R}} \left\{ \phi(m_i) \int_{t_{i-1}}^{t_i} u^R(a,t) \frac{f(t)}{\int_{t_{i-1}}^{t_i} f(\tau)\,\mathrm{d}\tau}\,\mathrm{d}t + [1-\phi(m_i)] \int_0^1 u^R(a,t) f(t)\,\mathrm{d}t \right\}$$

and therefore,

$$a(m_i) = \phi(m_i)\, y^R(t_{i-1}, t_i) + [1-\phi(m_i)]\, y^R(0,1).$$

Let $a(m_i) \equiv a_i$. For any unsent message $m \in M \setminus \{m_1, m_2, \ldots, m_N\}$, specify $a(m)$ as any $a \in \{a_1, a_2, \ldots, a_N\}$ with the receiver believing that t is in one of the intervals, $[t_{i-1}, t_i)$, with probability $\int_{t_{i-1}}^{t_i} \mu(t|m_i)\,\mathrm{d}t$.

For $m^I(\cdot)$ and $q(\cdot)$ to be the best responses to $a(\cdot)$, (E1) and (E2) require that

(i) (Arbitrage condition) For the informed sender,

$$u^S(a_i, t_i) = u^S(a_{i+1}, t_i) \quad \text{for all } i = 1, 2, \ldots, N-1.$$

(ii) For the uninformed sender, if $q(m_k) > 0$ for some $k \in \{1, 2, \ldots, N\}$,

$$\int_0^1 u^S(a_k, t) f(t)\,\mathrm{d}t \geq \int_0^1 u^S(a_i, t) f(t)\,\mathrm{d}t \quad \text{for all } i = 1, 2, \ldots, N.$$

Thus, the profile of the specified strategies and beliefs, $(m^I(\cdot), q(\cdot), a(\cdot), \mu(\cdot|\cdot))$, is an equilibrium if the two conditions, (i) and (ii), are satisfied.

If $u^S(a,t)$ satisfies certainty equivalence, we can say more about the uninformed sender's incentive. Certainty equivalence of an objective function is a convenient property to analyze the optimization problem in uncertain environment. For an uncertain parameter t, we say that $u^S(a,t)$ satisfies *certainty equivalence* if

$$\operatorname*{argmax}_a \int_0^1 u^S(a,t) f(t)\,\mathrm{d}t \equiv \operatorname*{argmax}_a u^S(a, E[t]).$$

The property says that the optimization problem under uncertainty is equivalent to the problem with the certain parameter $t = E[t]$. If $u^S(a,t)$ has the certainty equivalence property, then the uninformed sender has the same incentives as the type $t = E[t]$ of the informed sender because her expected type is $E[t]$. Then, it is straightforward that the uninformed sender pools with $t = E[t]$ in equilibrium. Therefore, the uninformed sender's incentive compatibility condition, (ii), amounts to Proposition 1 with the help of (E1).

Proposition 1. *Assume that $u^S(a,t)$ has the certainty equivalence property. Then, if $q(\cdot)$ constitutes an equilibrium and $q(m_k) > 0$ for some $k \in \{1, 2, \ldots, N\}$, then $t_{k-1} \leq E[t] \leq t_k$. Moreover, if $q(\cdot)$ is nondegenerate so that $q(m_k) = q_k \in (0,1)$ and $q(m_{k+1}) = 1 - q_k$ for some $k \in \{1, 2, \ldots, N-1\}$, then $E[t] = t_k$.*

Proof. If $q(\cdot)$ constitutes an equilibrium, then by (E2), $q(m_k) > 0$ implies that

$$a_k \in \operatorname*{argmax}_{a \in \{a_1, a_2, \ldots, a_N\}} \int_0^1 u^S(a,t) f(t)\,\mathrm{d}t.$$

Then, by the certainty equivalence property of $u^S(a,t)$,

$$a_k \in \operatorname*{argmax}_{a \in \{a_1, a_2, \ldots, a_N\}} u^S(a, E[t]).$$

This implies that by (E1), $E[t] \in [t_{k-1}, t_k]$. If $q(m_k) = q_k$ and $q(m_{k+1}) = 1 - q_k$ for some $q_k \in (0,1)$, then by the same argument,

$$\{a_k, a_{k+1}\} = \operatorname*{argmax}_{a \in \{a_1, a_2, \ldots, a_N\}} \int_0^1 u^S(a,t) f(t)\,\mathrm{d}t$$

and equivalently,

$$\{a_k, a_{k+1}\} = \underset{a\in\{a_1,a_2,\ldots,a_N\}}{\operatorname{argmax}} u^S(a, E[t]).$$

This implies that the type $t = E[t]$ is indifferent between a_k and a_{k+1} and we must have $E[t] = t_k$. □

The certainty equivalence property is satisfied in examples of the quadratic utility functions prevailing in literature. With $u^S(a,t) = -(a-(t+b))^2$, it is true that $E[u^S(a,t)] = u^S(a, E[t]) - \text{Var}(t)$ and the certainty equivalence property is satisfied for any f. More generally, with $u^S(a,t) = -|a-(t+b)|^n$, the certainty equivalence property holds for any positive integer n if f is symmetric. Under the certainty equivalence property, the uninformed sender always pools with the informed sender of type $t = E[t]$. Therefore, when the uninformed sender uses a pure strategy $m^U = m_k$, it must be that $t_{k-1} \leq E[t] \leq t_k$ in equilibrium, which describes the uninformed sender's incentive compatibility. As for mixed strategy equilibrium, the uninformed sender randomizes adjacent two messages, say m_k and m_{k+1}, only if the type $t = E[t]$ is indifferent over the actions induced by those messages. This implies that $t = E[t]$ is the boundary type t_k between the subintervals sending those messages.

1.4 THE UNIFORM-QUADRATIC CASE

In this section, we analyze the model with the uniform-quadratic example. We assume that $f(t) = 1$ for all $t \in [0,1]$ and the utility functions are given by

$$u^S(a,t) = -(a-(t+b))^2,$$
$$u^R(a,t) = -(a-t)^2,$$

where $b \in (0,\infty)$. The interest conflict between the sender and the receiver is measured by a parameter b because $y^S(t) - y^R(t) = b$ for all $t \in [0,1]$. The quadratic loss form of the utility functions satisfy all the assumptions on the preferences, condition (M) without the uninformed sender, and the certainty equivalence property.

To recap, for given $p \in (0,1)$ and $b \in (0,\infty)$, there exists an N-step equilibrium such that

(i) $m^I(t) = m_i$ if $t \in [t_{i-1}, t_i)$, $i = 1, 2, \ldots, N$, where $t_0 = 0$ and $t_N = 1$,

(ii) $q(\cdot)$ is such that if $q(m^U) > 0$ then $m^U = m_k$ for some $k \in \{1, 2, \ldots, N\}$, and

(iii) $a(m_i) \equiv a_i = \phi(m_i)\dfrac{t_{i-1} + t_i}{2} + [1 - \phi(m_i)]\dfrac{1}{2}$, $i = 1, 2, \ldots, N$, where

$$\phi(m_i) = \frac{p(t_i - t_{i-1})}{p(t_i - t_{i-1}) + (1-p)q(m_i)}.$$

In equilibrium, the uninformed sender mixes over at most adjacent two messages, say m_k and m_{k+1}, and for expositional convenience, we denote her strategy by $m^U = q_k m_k + (1-q_k)m_{k+1}$ hereafter, describing that $q(m_k) = q_k$ and $q(m_{k+1}) = 1 - q_k$. For the specified strategies to constitute an equilibrium, the incentive compatibilities require that

(iv) $-(a_i - (t_i + b))^2 = -(a_{i+1} - (t_i + b))^2$, $i = 1, 2, \ldots, N-1$, and

(v) if $m^U = m_k$, then $t_{k-1} \leq 1/2 \leq t_k$; and if $m^U = q_k m_k + (1-q_k)m_{k+1}$ for $q_k \in (0,1)$, then $t_k = 1/2$.

First, we check the condition for the existence of an influential and so informative equilibrium.

Proposition 2. *When $b \geq 1/4$, all equilibria are uninformative for any $p \in (0,1)$.*

Proof. Construct the sender's strategies as $m^I(t) = m_1$ if $t \in [0, t_1)$, $m^I(t) = m_2$ if $t \in [t_1, 1]$, and $m^U = q_1 m_1 + (1-q_1)m_2$, where $q_1 \in [0,1]$. Then, the receiver's optimal actions are

$$a_1 = \frac{pt_1}{pt_1 + (1-p)q_1} \cdot \frac{t_1}{2} + \frac{(1-p)q_1}{pt_1 + (1-p)q_1} \cdot \frac{1}{2},$$

$$a_2 = \frac{p(1-t_1)}{p(1-t_1) + (1-p)(1-q_1)} \cdot \frac{t_1 + 1}{2} + \frac{(1-p)(1-q_1)}{p(1-t_1) + (1-p)(1-q_1)} \cdot \frac{1}{2}.$$

From the arbitrage condition for t_1, $-(a_1 - (t_1 + b))^2 = -(a_2 - (t_1 + b))^2$, we have

$$t_1 = \frac{a_1 + a_2}{2} - b.$$

Note from the above expression for a_1 and a_2 that $0 \leq a_1 \leq 1/2$ and $1/2 \leq a_2 \leq 1$ for any $p \in (0,1)$ and $q_1 \in [0,1]$. This implies that

$$\frac{1}{4} - b \leq t_1 \leq \frac{3}{4} - b.$$

For a 2-step equilibrium to exist, t_1 must lie in the interior of the type space and we need $1/4 - b > 0$ and $3/4 - b < 1$. The latter is always true for $b > 0$, and thus, if the former does not hold, then a 2-step equilibrium does not exit at all. Therefore, if $b \geq 1/4$, only equilibrium is uninformative. □

Proposition 2 is evident when we construct 2-step equilibria and identify the values of b supporting the equilibria. There are three different 2-step equilibria depending on b. First, when $b \leq \frac{1-p}{4(2-p)}$ for given $p \in (0,1)$, there exists a 2-step equilibrium in which the uninformed sender pretends that she is informed of a low type with a positive probability:

$$q_1 = 1 \quad \text{and} \quad t_1 = \frac{1}{4p}\left(-3 + 4p - 4bp + \sqrt{9 - 8p - 8bp + 16b^2p^2}\right) \ ; \quad \text{or}$$

$$q_1 = \frac{-p + 8b - 8bp + \sqrt{p^2 + 64b^2}}{16b(1-p)} \quad \text{and} \quad t_1 = \frac{1}{2}.$$

Since $\frac{1-p}{4(2-p)}$ cannot exceed $1/8$, if $b \geq 1/8$, there does not exist a 2-step equilibrium with $q_1 > 0$ for any $p \in (0,1)$.

If the informed sender can verify that she is informed as in Austen-Smith [1], we can show that for any $p \in (0,1)$, a 2-step equilibrium exists for $b < 1/2$ as long as low types pool with the uninformed sender. This is because the uninformed sender has no choice over messages and we only need the condition that t_1 should lie in $(0,1)$. However, without verifiability, the uninformed sender is free to mimic any types so that her choice of message must be incentive compatible over messages that the informed sender sends. This additional constraint requires that $t_1 \geq 1/2$ and shrinks the range of b that supports the equilibria.

We now construct the other kind of 2-step equilibrium, in which the uninformed sender pretends to be a high type with probability 1. Such an equilibrium exists for any $p \in (0,1)$ if $b < 1/4$, where

$$q_1 = 0 \quad \text{and} \quad t_1 = \frac{1}{4p}\left(3 - 4bp - \sqrt{9 - 8p + 8bp + 16b^2p^2}\right).$$

From the construction of all 2-step equilibria, we can see that if $b \geq 1/4$, cheap talk messages cannot be informative for any $p \in (0,1)$ no matter what types the uninformed sender pretends to be. For those values of b, as Crawford and Sobel [9] show, the only equilibrium is totally uninformative without the uninformed sender as well. Thus, with the uniform-quadratic specification, if $b \geq 1/4$, no information is transmitted whether the sender is informed for certain or not.[4]

However, for some $p \in (0,1)$, there exists $b < 1/4$ that supports more influential equilibrium than the most influential equilibrium without the uninformed sender. An equilibrium can be more influential when the uninformed sender pools with middle interval types. First, we consider the two extreme cases in which the uninformed sender pools with either the highest interval types or the lowest interval types. These equilibria can never improve in influentialness.

Proposition 3. *For $N \geq 2$, there exists an N-step equilibrium in which $m^U = m_1$ if*

$$b \leq \frac{2 - pN}{4(2-p)N(N-1)}.$$

Proof. See the appendix. □

Proposition 4. *For $N \geq 3$, there exists an N-step equilibrium in which $m^U = m_N$ if*

$$\frac{pN - 2}{4(2-p)N(N-1)} \leq b < \frac{N - 1 - \sqrt{(1-p)N(N-2)+1}}{2pN(N-1)(N-2)}.$$

Proof. See the appendix. □

We can easily check that for any $p \in (0,1)$, the supremum of b that supports N-step equilibrium in either case is less than that in the CS N-step equilibrium. Let $\bar{b}_k^N(p)$ be the supremum of b that supports N-step equilibrium with $m^U = m_k$ for given $p \in (0,1)$, and $\bar{b}_{CS}^N$ be the supremum of b that supports CS N-step equilibrium. Notice that $\bar{b}_1^N(p)$ in Proposition 3 is decreasing in p, and as p goes to zero, it goes up to $1/[4N(N-1)]$, which is a half of $\bar{b}_{CS}^N = 1/[2N(N-1)]$. Also, notice that in Proposition 4, $\bar{b}_N^N(p)$ is increasing in p, and that as p goes to 1, the equilibrium partition and $\bar{b}_N^N(p)$ converge to those in the CS model. Thus, if the uninformed sender mimics the lowest types or the highest types, then for any $p \in (0,1)$, given $b > 0$, the equilibria cannot be more influential than the best CS equilibrium. A 2-step equilibrium belongs to either of the extreme cases (although Proposition 4 holds for $N \geq 3$) and $\bar{b}_k^2(p)$ cannot exceed 1/4 for any $k = 1, 2$ and $p \in (0,1)$.

[4]When $b = 1/4$, there exists an equilibrium in which only the type $t = 0$ sends m_1 and all other types send m_2. This event occurs with probability zero and $t = 0$ is indifferent between revealing and disguising herself. Therefore, this equilibrium yields the essentially equivalent outcome to the uninformative equilibrium.

Moreover, the largest N is bounded for any positive value of b, implying that the informativeness of cheap talk messages is limited even if the interest conflict is very small.

Corollary 1. *Given $p \in (0,1)$, N is bounded for any $b \in (0,\infty)$ if $m^U = m_1$ or $m^U = m_N$.*

Proof. Recall that $b > 0$. For given $p \in (0,1)$, such b that supports the equilibria exists when $N < 2/p$ in Proposition 3 and when $N < 4/p$ in Proposition 4. □

The largest N cannot exceed $2/p$ when $m^U = m_1$ and $4/p$ when $m^U = m_N$ even as b goes to zero. Since the largest N is nonincreasing in p, a low probability $(1-p)$ of the existence of the uninformed sender disturbs information flow significantly. For example, consider p close to one. When $m^U = m_1$, if $p \in [2/3, 1)$, $N < 2/p$ implies that the most influential equilibrium is not more than 2-step for any $b \in (0,\infty)$. As for the case that $m^U = m_N$, the most influential equilibrium is not more than 4-step for p close to one. We constructed the 2-step equilibrium in the above for $p \in (0,1)$ when $b < 1/4$. For $N \geq 3$, $N < 4/p$ implies that $N < 5$ if $p \in [0.8, 1)$.

Recall that the incentive compatibilities require that the uninformed sender pool with the type $t = E[t]$. The equilibria with the uninformed sender mimicking the lowest types or the highest types cannot be more influential at all than the best CS equilibrium. This is because in the two extreme cases, the left-end interval or the right-end interval must contain $E[t] = 1/2$ so that the complement segment of the type space is relatively small to be divided into subintervals. On the other hand, it is conceivable that given b, the type space can be partitioned into more number of elements when the uninformed sender pools with middle interval types. Only in the case, the largest N goes to infinity as b goes to zero. Moreover, there exists $b > 0$ for which more influential equilibrium exists than the best CS equilibrium for some $p \in (0,1)$.

Proposition 5. *Suppose that $N \geq 3$. For some $p \in (0,1)$, there exists k such that $1 < k < N$ and $\bar{b}_k^N(p) > \bar{b}_{CS}^N$.*

Proof. See the appendix. □

The influentialness can improve when the uninformed sender pools with middle interval types. Proposition 5 states that for some $p \in (0,1)$ the supremum of b that supports

N-step equilibrium is greater than that in the CS N-step equilibrium. This implies that the influentialness improves over the best CS equilibrium if an uninformed sender exists. Specifically, the improvement occurs when the uninformed sender pools with the mid-interval types, $[t_{\frac{N-1}{2}}, t_{\frac{N+1}{2}})$, for an odd number N and when she pools with the types $[t_{\frac{N}{2}}, t_{\frac{N}{2}+1})$ for an even number N. The following examples are the cases of $N = 3$ and 4.

Example 2. When $b \leq \frac{9-8p}{24(3-2p)}$, there exists a 3-step equilibrium in which $m^U = m_2$, where

$$t_1 = \frac{9 - 8p - 36b + 24bp}{3(9-8p)} \quad \text{and} \quad t_2 = \frac{18 - 16p - 36b + 24bp}{3(9-8p)}.$$

Example 3. When $b < \frac{9-8p}{24(6-5p)}$, there exists a 4-step equilibrium in which $m^U = m_3$, where

$$t_1 = \frac{1}{8p}\left(45 - 40p - 24bp - 3\sqrt{225 - 416p + 16bp + 192p^2 + 64b^2p^2}\right),$$

$$t_2 = \frac{1}{4p}\left(45 - 40p - 8bp - 3\sqrt{225 - 416p + 16bp + 192p^2 + 64b^2p^2}\right),$$

$$t_3 = \frac{1}{8p}\left(75 - 64p - 8bp - 5\sqrt{225 - 416p + 16bp + 192p^2 + 64b^2p^2}\right).$$

Example 2 shows that if the uninformed sender mimics the mid-interval types, then for $b \in [1/12, 1/8)$, a 3-step equilibrium exists as long as p is small enough. Formally speaking, if $p \leq 3/4$, then $\bar{b}_2^3(p) = \frac{9-8p}{24(3-2p)} \geq 1/12$ and for $b \in [1/12, \bar{b}_2^3(p)]$, there exists a 3-step equilibrium under the receiver uncertainty. As p gets close to 0, $\bar{b}_2^3(p)$ goes up to 1/8. Notice that for $b \in [1/12, 1/4)$, any equilibria induce at most two actions when the sender is informed for certain.

The supremum of b that supports 4-step equilibria in CS is 1/24. However, in Example 3, we can identify that $\bar{b}_3^4(p) = \frac{9-8p}{24(6-5p)} > 1/24$ for all $p \in (0, 1)$. Therefore, if the uninformed sender pools with $[t_2, t_3)$, then for $b \in [1/24, \bar{b}_3^4(p))$, a 4-step equilibrium exists for any $p \in (0, 1)$. As p gets close to zero, $\bar{b}_3^4(p)$ goes up to 1/16.

We have seen that the supremum of b is greater than in the CS model to support an N-step equilibrium when the uninformed sender pools with middle interval types. More influential equilibrium exists than the best CS equilibrium and thus, the presence of the uninformed sender can make the informed sender transmit more precise information to the decision maker. However, this does not mean that the decision maker receives more precise information. The quality of information received is coarser because the information is distorted by the uninformed sender.

1.5 WELFARE

In this section, we keep working with the uniform-quadratic case on welfare analysis. We denote the informed sender's and the uninformed sender's expected utilities by EU^{IS} and EU^{US} respectively. In an equilibrium in which $m^U = m_k$, they are calculated as

$$EU^{IS} = \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} -(a_i - (t+b))^2 \,\mathrm{d}t,$$

$$EU^{US} = \int_0^1 -(a_k - (t+b))^2 \,\mathrm{d}t,$$

and the sender's and the receiver's *ex ante* expected utilities are obtained as

$$\begin{aligned} EU^S &= pEU^{IS} + (1-p)EU^{US}, \\ EU^R &= p\sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} -(a_i - t)^2 \,\mathrm{d}t + (1-p)\int_0^1 -(a_k - t)^2 \,\mathrm{d}t, \end{aligned}$$

respectively. In this model, it still holds that $EU^S = EU^R - b^2$ in any equilibria and, hence, they are Pareto ranked. Thus, the sender and the receiver will coordinate on the best equilibrium at the *ex ante* state. However, it is possible that the best equilibrium is *ex ante* Pareto dominated by the best CS equilibrium even though it is more influential. This is because the introduction of uncertainty about the sender's informedness makes the environment of communication worse and the quality of information that the receiver receives is coarser. We can see an example in Table 1.1.

Table 1.1 shows the expected utilities in all equilibria when $p = 0.25$ and $b = 0.1$. EU^S_{CS} and EU^R_{CS} denote the sender's and the receiver's expected utilities, respectively, in a CS equilibrium. Note that a mixed strategy equilibrium is dominated by a pure strategy equilibrium, so only consider pure strategy equilibria. When $b = 0.1$, the 2-step equilibrium

Table 1.1: The expected utilities when $p = 0.25$ and $b = 0.1$

	EU^{IS}	EU^{US}	EU^{R}	EU^{S}_{CS}	EU^{R}_{CS}
1-step equilibrium	-0.0933	-0.0933	-0.0833	-0.0933	-0.0833
2-step equilibrium					
$m^U = m_1$	-0.0319	-0.1017	-0.0743		
$m^U = m_2$	-0.0702	-0.0894	-0.0746	-0.0408	-0.0308
$m^U = q_1 m_1 + (1-q_1)m_2$	-0.0338	-0.1018	-0.0748		
3-step equilibrium					
$m^U = m_2$	-0.0196	-0.0964	-0.0672	—	—
$m^U = q_2 m_2 + (1-q_2)m_3$	-0.0250	-0.0965	-0.0686		

is the best in the CS model in which the sender is informed for certain. If the uninformed sender is present with probability 0.75 ($= 1-p$), the best equilibrium is now of 3-step with the uninformed sender pooling with the mid-interval types. Thus, the informed sender's messages convey more precise information, and so the receiver might be better off by this informational effect. However, the received information is distorted by the uninformed sender, which dominates the informational effect and the receiver is worse off.

We may think of the informed sender and the uninformed sender as different agents in that informed experts and uninformed charlatans coexist in the real world. When the uninformed sender is present, the informed sender's expected utility can be higher than that in the best CS equilibrium. Consider again the case in which $b = 0.1$. Then, the best CS equilibrium is of 2-step and $EU^{S}_{CS} = -0.0408$. When the uninformed sender exists with probability $1-p$, a 3-step equilibrium exists for $p \leq 9/16$, in which the uninformed sender pretends to be one of the mid-interval types (See Example 2). The informed sender's expected utility is

$$EU^{IS} = -\frac{9.43407(p-1.125)(p^2-2.28392p+1.32286)}{(8p-9)^3},$$

which is decreasing in p, and $EU^{IS} = -0.0206$ when $p = 9/16$. Hence, $EU^{IS} > EU^{S}_{CS}$ for all $p \in (0, 9/16]$. That is, the informed sender can be better off by the uninformed sender pooling with the mid-interval types.

As Table 1.1 shows, when $p = 0.25$, the informed sender's expected utility is higher in both the 2-step equilibrium with $m^U = m_1$ and the 3-step equilibrium with $m^U = m_2$ than that in the best CS equilibrium if we only consider pure strategy equilibria. For the informed sender, the 3-step equilibrium is the best. However, for the uninformed sender, the 2-step equilibrium with $m^U = m_2$ is the best, which gives her higher expected utility than that in the uninformative equilibrium which would be unique if the informed sender were absent. Thus, they have a conflict over the preferred equilibria.

1.6 CONCLUSION

The main results of this chapter are summarized as follows. When an uninformed sender exists and the decision maker cannot distinguish her from the informed expert, the uninformed sender has an incentive to pretend to be informed. Since the informed sender is willing to distinguish herself from the uninformed sender, the informed sender's cheap talk message conveys more precise information. Because of this informational effect, the informed sender is better off by the presence of the uninformed sender. However, the information received by the decision maker is distorted and the decision maker is worse off if the distortion effect is large.

In this model with the uninformed sender, there are much more equilibria than in the CS model and the equilibrium selection problem is severe. If an N-step equilibrium exists, then for every integer $M \leq N$, various kinds of M-step equilibria exist. If we consider the informed sender and the uninformed sender as different agents, it is not easy to find which equilibrium is the best because there is a conflict over equilibria among the three agents. However, at the *ex ante* stage, all equilibria are Pareto ranked and the agents will coordinate on the best equilibrium.

This chapter is different from the existing studies in that the receiver is uncertain of how informed the sender is. However, we only considered an extreme case in which the sender is either fully informed or fully uninformed. The model can be generalized by introducing a partially informed sender but we conjecture that the results will not change qualitatively.

2.0 COSTLY CERTIFICATION OF PRODUCT QUALITY

2.1 INTRODUCTION

In a product market with uncertain quality, informed sellers have incentives to reveal the product information to increase the profits unless revelation cost is high. They are allowed to reveal possibly vague information directly but not allowed to misreport it. Such a feature on messages reflects the situations where the messages take the form of a certificate issued by an organization that has public's trust such as an ISO standard, where sellers display or demonstrate an object, or where lying is virtually impossible because it is very expensive by the law or future reputation.

When disclosure is costless, the private information is fully revealed as a unique equilibrium outcome (Grossman and Hart [16], Grossman [15], Milgrom [33], Jovanovic [23], Okuno-Fujiwara *et al*. [38], and Seidmann and Winter [43]). Consider a disclosure problem of product quality by sellers to buyers. Let θ be a random variable, which represents the quality of a product to be traded, the sellers' private information. A representative buyer's utility is increasing in θ, but he only knows the distribution of θ over the set of all possible qualities. Each seller may report or hide any information about the quality but is not allowed to misreport it. As is well known, if disclosure is costless, a seller informs buyers that her product quality is at least as good as its true value, $\bar{\theta}$, and the buyers infer that $\theta = \bar{\theta}$ with probability 1. In this manner, the product information is fully revealed.

In many cases, a seller's verifiable message is costly: for example, product demonstrations[1], statements of objective inspections in a used-car market[2], and product specifications that can be obtained by experiments or quality testing. Revelation cost is one factor of failure of full revelation with voluntary disclosure and the revelation of uncertain information does not guarantee an increase in social welfare. The literature shows that the market outcome leads to an excessive disclosure in view of social welfare even though it is not full revelation (Jovanovic [23] and Cheong and Kim [7]). However, it goes too far because the social value of information is not considered in the models. We introduce a variable that generates social value of information, which is buyer's *action* such as the usage and maintenance of the product after purchase. The new variable explains that the seller is still willing to reveal the private information even if the revelation incurs a cost.

An important result of this chapter is that if the buyer is concerned about his action, more information is revealed by the seller. Even though the product is of high quality, inappropriate use prevents the buyers from enjoying the full benefits. Only buyers who have received full information can use it most effectively. The buyer's concern about the effective use affects the seller's incentive to reveal the private information. We show that the seller is more willing to reveal when the effectiveness of use matters. Furthermore, more information is revealed as the variance of the uncertain quality is larger or as the average quality is lower. Since the uncertainty lowers the price by the variance, the seller is more willing to reveal the quality in order to raise the price.

A certificate may prove either precise or partial information, but more commonly it only proves partial information as in Lipman and Seppi [30] and Wolinsky [46] because it is hard to prove claims precisely. Moreover, it is a reasonable assumption that the certification cost is increasing, that is, a better certificate is more costly. Certifying high quality is more costly because it needs more tests, more time, or higher technology. Nonetheless, the standard assumption in costly disclosure models is that sellers can either disclose the true quality at a flat cost whatever the quality is or simply conceal the information (for example, Jovanovic [23] and Cheong and Kim [7]). This environment leads to a unique equilibrium in which each seller discloses the true quality if it exceeds a threshold level and does not otherwise.

[1]As examples of costly product demonstration, Cheong and Kim [7] introduce free samples of cosmetics and demo versions of computer software.

[2]Grossman [15] takes this example as costly certificates.

We assume in the model that a seller certifies that the quality is at least a minimum level and the certification cost is increasing in the minimum quality. The seller is not allowed to forge any certificates stating that the minimum quality exceeds the true value because of a technical issue or an antifraud law. In this model, there are multiple equilibria unless the certification costs are too high or too low. We show that the least level of revelation is *ex ante* Pareto optimal and no-revelation is the best if it is one of the equilibrium outcomes. If the social value of information was not considered, the optimal level of revelation might be underestimated. However, with buyer's action involved in the model as a variable that generates social value of information, more information is revealed in the optimal equilibrium as compared with the case in which the buyer is only concerned about the quality.

There are many variations in models of costly disclosure. As in the above standard models, full revelation is not generally an equilibrium with voluntary disclosure. Grossman and Hart [16] analyze a takeover bid process with the relevance to the disclosure problem. Lizzeri [31] introduces a certification intermediary into a seller-buyer model. The intermediary chooses a flat certification fee and a disclosure rule, and then the seller decides whether to pay the fee and have the test results disclosed under the disclosure rule. Levin *et al.* [27] analyze the effect of competition on disclosure level. They show that disclosure level is higher under a monopoly cartel than under duopoly.

The models above as well as ours are concerned with how the information is revealed with disclosure itself through verifiable messages when disclosure cost is not too high. In Grossman [15], however, in the case that information is too costly to certify, sellers instead give warranties on *ex post* realized breakdown of products to buyers as an indirect means of conveying information. In this case, assuming that quality is different only in the probability of breakdown, the optimal warranty is full coverage for breakdown regardless of the quality, and buyers do not care about the quality with the full warranty. Hence, when disclosure is costly, private information is not revealed even with warranties.

Some studies show that full revelation may fail even if disclosure is costless. Board [6] and Hotz and Xiao [19] report competition among firms as alleviating the firms' incentive to disclose private information. In Board [6], when firms are involved in price competition in duopoly with vertical differentiation, the firm with a lower quality product may choose not to disclose the quality to consumers, depending on the quality level. Hotz and Xiao [19] analyze a circumstance in which products are differentiated horizontally as well as vertically. Quality disclosure makes the products more substitutable in a market in which they are already differentiated horizontally. This induces intensive price competition among firms, resulting in a decrease in their profits. Therefore, full disclosure is not the best for the firms. In their cases, mandatory disclosure laws ensure that consumers gain in markets with asymmetric information while the firms' profits decrease.

Also, uncertain environment prevents full revelation even with costless disclosure. If the informed party's information structure is uncertain, that is, the informed party is imperfectly informed with a positive probability (Shin [45]), if the informed party's preferences are uncertain (Wolinsky [46]), or if there are buyers who cannot interpret disclosed information (Fishman and Hagerty [12]), then full revelation may not occur. Harbaugh and To [18] present receiver's private information as another source of revelation failure. They show that no-revelation can be an equilibrium if the number of verifiable messages available to the sender is limited and the receiver has private information about the sender's type.

In most models of disclosure, sellers are assumed to be originally informed. On the other hand, there is similar but different literature in which sellers do not know the quality of their own products but can still decide to acquire the information at some or no cost. Once they acquire the information, they can disclose it without a cost. Matthews and Postlewaite [32] analyze costless information acquisition when the seller does not know the quality before testing. In Farrell [10] and Shavell [44], information acquisition is costly.

The rest of the chapter is organized as follows. Section 2.2 gives the basic model and Section 2.3 specifies the seller's optimal pricing rule. In Section 2.4, necessary conditions for equilibrium certification are derived. Sections 2.5 and 2.6 characterize fully revealing and non-revealing equilibrium, and the welfare aspects on equilibria follows in Section 2.7. In Section 2.8, we introduce buyer's action into the model and compare the seller's incentive to reveal information with the basic model. Lastly, Section 2.9 summarizes and concludes the chapter.

2.2 THE BASIC MODEL

Consider a market for a product with asymmetric information on the quality between a seller and a buyer. The seller owns a product for sale and knows its quality θ. She is concerned about the price $p \in \mathbb{R}_+$ that she can receive. The quality θ is relevant to the buyer's utility but the product is assumed to be an experience good[3] so that the buyer learns its quality only after he consumes it. Before the trade, the buyer only knows the distribution F over $\Theta = [0, 1]$, the set of all possible qualities. Once the seller charges a price p, the buyer decides whether or not to purchase the product at that price.

The seller can certify the product quality at some cost c before she charges a price. A certificate (verifiable message) is denoted by $m \in M = [0, 1]$, which proves that the true quality θ belongs to $[m, 1]$. In words, the seller can only certify that the quality is at least some value, m. This reflects the situation in which it is really hard to prove what the true quality is exactly. For instance, for $\theta \in \Theta$, the accurate certificate stating $\theta \in \{\theta\}$ is in fact impossible except for the highest quality, $\theta = 1$. If the product passes a high-quality test, its quality is definitely high, but the high quality product could pass a low-quality test as well. Thus, by the low-quality test, we cannot conclude that the quality is low. Since the seller cannot certify that her product quality exceeds the true value, the feasible certificates of the seller of $\theta \in \Theta$ is $M(\theta) = [0, \theta]$.[4] Denoting by $c(m)$ the certification cost of m, we assume that $c(\cdot)$ is differentiable, $c'(m) > 0$ for all $m \in M$, and $c(0) = 0$, so that a better certificate is more costly.

Until Section 2.7, we analyze the model without buyer's action after purchase. The sequence of the game in a benchmark is summarized as follows:

1. The seller of $\theta \in \Theta$ reports a certificate $m \in M(\theta)$ to the buyer at cost of $c(m)$.
2. The seller charges a price $p \in \mathbb{R}_+$.
3. The buyer decides whether or not to purchase the product.

If the seller sells her product at price p, then the seller's profit is $\pi = p - c$. If the trade

[3] This classification follows Nelson [36].

[4] With these feasible certificates, product quality is only partially provable except for the highest quality. It is common as shown in Austen-Smith [1], Lipman and Seppi [30], and Wolinsky [46].

fails, assuming that just holding the product does not benefit the seller, she only loses c. On the other hand, the buyer enjoys benefit θ by consuming a product of quality θ. The buyer's utility is the net benefit from the purchase, that is, $u = \theta - p$, and the reservation utility is assumed to be zero.

2.3 PRICING RULE

We will use perfect Bayesian equilibrium as the equilibrium concept for this model. The seller reports information about her product quality to the buyer through a certificate. Given the certificate, the price is determined as the buyer's expected quality. Let $p(\cdot)$ be a seller's pricing rule under which $p(m)$ is the price that the seller charges for the product after reporting a certificate m. Let E_μ represent the buyer's expectation associated with his belief μ, which is a probability distribution over Θ.

According to the game theory, the pricing rule should be depicted by a function of $(\theta, m) \in \Theta \times M(\theta)$ and the belief be conditional on $(m, p) \in M \times \mathbb{R}_+$. In this model, however, prices never signal the product quality and thus do not influence the buyer's expectation on the quality because once the seller reports a certificate, then she charges the same price regardless of the quality. The reasoning is as follows. Suppose that the seller charges different prices depending on qualities after reporting m. If the seller of high quality product charges a higher price and the buyer purchases the product at that price, then the low quality seller profitably deviates to that price. If the buyer does not purchase at the higher price, then the seller lowers the price so that the buyer may purchase. Thus, the equilibrium pricing rule does not depend on qualities and, consequently, the buyer's belief about the quality does not depend on prices. For out-of-equilibrium prices, we assume that the buyer forms the same belief. Therefore, in a perfect Bayesian equilibrium, the optimal pricing rule and the buyer's posterior expectations only depend on m, and we denote them without any loss by $p(m)$ and $E_\mu[\theta|m]$ instead of $p(\theta, m)$ and $E_\mu[\theta|m, p]$.[5]

[5] The situation is equivalent to a model in which there are at least two buyers and the buyers bid a price for the product simultaneously after observing a certificate m instead of the seller asking a price. In this case, under the assumption that each buyer has the same valuation on the product, the buyer's bidding rule

Assume that the buyer buys the product if the purchase gives him nonnegative expected utility. Then, given a buyer's belief μ, the seller's equilibrium pricing rule $p(\cdot)$ is such that $p(m) = E_\mu[\theta|m]$ if she reports m to the buyer. To prove it, suppose that the seller has reported a certificate m at cost of $c(m)$. If the buyer purchases the product at price p after observing m, his expected utility conditional on m is $E_\mu[u|m] = E_\mu[\theta|m] - p$. Any $p > E_\mu[\theta|m]$ is dominated by lower prices for the seller because the buyer does not purchase the product at that price and the seller only loses the certification cost. Thus, for given m, the profit is maximized when the seller charges $p(m) = E_\mu[\theta|m]$.

The seller charges the price equal to the buyer's expectation on the quality so that the buyer's expected utility from the purchase is zero. This implies that the seller takes the buyer's expected surplus. With the optimal pricing rule, the seller wants to choose a certificate m in order to maximize the buyer's expected quality conditional on her report less the certification cost, $E_\mu[\theta|m] - c(m)$.

2.4 EQUILIBRIUM CERTIFICATION

In the above, we have specified the equilibrium pricing rule as $p(\cdot) = E_\mu[\theta|\cdot]$, under which the buyer always purchases the product. Given the pricing rule and a buyer's belief μ, now the seller must choose a certificate to maximize the profit. That is, for each $\bar{\theta} \in \Theta$, the seller faces the following problem:

$$\max_{m \in M(\bar{\theta})} \pi = E_\mu[\theta|m] - c(m). \tag{2.1}$$

Let σ be a seller's certification strategy, where $\sigma(\bar{\theta}) \in M(\bar{\theta})$ for each $\bar{\theta} \in \Theta$, which solves (2.1) in an equilibrium. For $m \in M$, let $T(m;\sigma) := \{\theta \in \Theta : \sigma(\theta) = m\}$, the set of θ for which the seller reports m according to σ. The equilibrium belief μ is required to follow Bayes' rule whenever possible so that $E_\mu[\theta|m] = E[\theta|\theta \in T(m;\sigma)]$ if $T(m;\sigma)$ is nonempty. Since the pricing rule $p(\cdot)$ and the buyer's decision rule depend on beliefs, an equilibrium is

$p(\cdot)$ and expectation $E_\mu[\theta|\cdot]$ are functions of m, and the optimal bidding just follows the expectation of the quality.

characterized by simply (σ, μ). In this section, we present necessary conditions for equilibria regarding certification. We begin by assuming a tie-breaking rule for the choice between indifferent alternatives.

Assumption 1. Given μ, if $E_\mu[\theta|m] - c(m) = E_\mu[\theta|m'] - c(m')$ and $c(m) < c(m')$, then the seller prefers to choose m rather than m'.

If alternatives are indifferent to the seller given a buyer's belief, she prefers to choose one with the lowest cost. Thus, given a certification strategy, if deviation to a less costly certificate gives at least the same profit to the seller as she follows the strategy, then the seller deviates. The results do not change even if it is assumed that the seller chooses one with the highest cost. With this assumption, we simply exclude mixed strategies of certification.

Lemma 3. *For any equilibrium strategy σ, $T(m;\sigma)$ is a subinterval in Θ for all $m \in M$.*

Proof. If $T(m;\sigma)$ is an empty set or a singleton, it is an interval by definition. Suppose that θ_1, $\theta_2 \in T(m;\sigma)$ and $\theta_1 < \theta_2$. Since σ is an equilibrium strategy, it must be that for θ_2,

$$E_\mu[\theta|m] - c(m) \geq E_\mu[\theta|m'] - c(m') \tag{2.2}$$

for all $m' \in M(\theta_2)$ (the inequality is strict for $m' < m$ by Assumption 1). Then, for any $\theta' \in (\theta_1, \theta_2)$, (2.2) still holds for all $m' \in M(\theta')$ because $m \in M(\theta')$ and $M(\theta') \subset M(\theta_2)$. Thus, $\theta' \in T(m;\sigma)$. □

In any equilibria, $T(m;\sigma)$ has the minimum if it is nonempty. Let $\hat{\theta} = \inf T(m;\sigma)$. Intuitively, due to the convexity of $T(m;\sigma)$, by reporting m, $\hat{\theta}$ gets the buyer to believe that the quality is at least $\hat{\theta}$, but with any other available m' to $\hat{\theta}$, the buyer believes that the quality is at most $\hat{\theta}$. Thus, $\hat{\theta}$ should be in $T(m;\sigma)$. If $T(m;\sigma)$ does not have its minimum, this implies that $\hat{\theta}$ is (weakly) better off with some other m' rather than m. Then, that means all $\theta \in T(m;\sigma)$ are (weakly) better off with m', which is available to all $\theta \in T(m;\sigma)$. Thus, σ does not constitute an equilibrium. Moreover, m must be the minimum quality of $T(m;\sigma)$ in equilibria with Assumption 1.

Lemma 4. *Suppose that σ constitutes an equilibrium and $T(m;\sigma)$ is nonempty for $m \in M$. Then,* $\min T(m;\sigma) = m$.

Proof. Let $\hat{\theta} = \min T(m;\sigma)$, then $m \leq \hat{\theta}$. Since σ constitutes an equilibrium, for $\hat{\theta}$, (2.2) must hold for all $m' \in M(\hat{\theta})$ (the inequality is strict for $m' < m$ by Assumption 1). If $m < \hat{\theta}$, then any $\theta' \in [m, \hat{\theta})$ (weakly) profitably deviates to m because $m \in M(\theta')$ and $M(\theta') \subset M(\hat{\theta})$ so that (2.2) still holds for all $m' \in M(\theta')$. Thus, m must be equal to $\hat{\theta}$ because then $m = \hat{\theta}$ is unavailable to $\theta < \hat{\theta}$. □

Lemma 4 implies that in order for the seller to make the buyer believe that the quality is at least the true quality, she must certify it, which the lower quality sellers cannot pretend. In other words, the seller of quality θ must report $m = \theta$ if she wants to distinguish herself from the lower quality sellers. We can identify this fact in Example 4. Furthermore, for θ in any separating segment of qualities, it must be that $\sigma(\theta) = \theta$. From the above two lemmas, we know that any equilibrium strategy $\sigma(\cdot)$ is monotone. It is nondecreasing and, consequently, equilibrium prices $p(\sigma(\cdot)) = E_{\mu}[\theta|\sigma(\cdot)]$ are also nondecreasing in quality level. Thus the seller of higher quality never charges a lower price.

2.5 FULL REVELATION

Obviously, if the marginal benefit from more disclosure (a better certificate) is greater than the marginal cost, then the seller discloses more. Thus, if it is true for all available m, then full revelation occurs. There are infinitely many separating strategies in a signaling model with a continuum of messages. In this model, any strictly increasing function $\sigma(\cdot)$ with $0 \leq \sigma(\theta) \leq \theta$ for all $\theta \in [0,1]$ is a separating strategy. However, Lemma 4 implies that the only candidate of separating strategies for an equilibrium is $\sigma(\theta) = \theta$ for all $\theta \in [0,1]$.

Proposition 6. *There exists a fully revealing equilibrium if and only if $c'(m) < 1$ for all $m \in [0,1]$, where $\sigma(\theta) = \theta$ for all $\theta \in [0,1]$.*

Proof. Given μ and $p(\cdot) = E_{\mu}[\theta|\cdot]$, the buyer always purchases the product, and thus for each $\bar{\theta} \in [0,1]$, $\sigma(\bar{\theta})$ must solve (2.1). By Lemma 4, if an equilibrium is fully revealing, it must be that $\sigma(\bar{\theta}) = \bar{\theta}$ for all $\bar{\theta} \in [0,1]$. With this strategy, $E_{\mu}[\theta|\sigma(\bar{\theta})] = \bar{\theta}$ and $c(\sigma(\bar{\theta})) = c(\bar{\theta})$ for all $\bar{\theta} \in [0,1]$. Thus, in order for the fully revealing strategy to be optimal, it must be that for all $\bar{\theta} \in [0, 1]$ and for all $\theta' \in [0, \bar{\theta})$, $\bar{\theta} - c(\bar{\theta}) > \theta' - c(\theta')$, or

$$\frac{c(\bar{\theta}) - c(\theta')}{\bar{\theta} - \theta'} < 1.$$

This is equivalent to that $c'(m) < 1$ for all $m \in [0, 1]$, and the inequality is strict by Assumption 1. □

The seller is willing to certify some information if the seller can get a higher price than the certification cost by doing so. If the seller reveals the true value of the product quality, the price will be charged at the true value because it is assumed that the quality level is equal to the buyer's valuation on the product. Thus, if the quality difference between any two types of the product is greater than the difference in the costs between two certificates which respectively guarantee the true quality of each type, then for any quality level the seller has an incentive to certify that the quality is at least the true value. As a result, full revelation can be supported as an equilibrium outcome as long as the marginal costs of certification are sufficiently low.

2.6 NO REVELATION

Now we find a no-revelation condition, under which for all $\theta \in \Theta$ the seller reports $m = 0$ at zero cost. For the non-revealing equilibrium, we must specify the buyer's posterior beliefs off the equilibrium path, that is, the beliefs conditional on each certificate $m \in (0, 1]$. A perfect Bayesian equilibrium requires that given a strategy of certification, the buyer update his belief by Bayes' rule after observing a certificate. However, Bayes' rule is not defined for certificates the seller does not report under the strategy. One conceivable out-of-equilibrium inference of the buyer is by *skepticism*. That is, $E_\mu[\theta|m] = m$ for any certificate m off the equilibrium path.

Assumption 2. Given σ, if $T(m; \sigma) = \emptyset$ for $m \in M$, then $E_\mu[\theta|m] = m$.

For a certificate off the equilibrium path, the buyer believes that with probability 1 the quality is the possible minimum value the certificate guarantees. This assumption seems to be arbitrary because the seller of higher quality product would report a better certificate to get a higher price only if the benefit from doing so covered the increase in the cost.[6] However, from the buyer's perspective, the skeptical inference is beneficial to him because it would lower the price. Moreover, this assumption is supported by the following lemma.

Lemma 5. *Suppose that (σ, μ) is an equilibrium for some μ. Then, there exists an equilibrium (σ, μ') where $E_{\mu'}[\theta|m] = m$ for $m \in M$ such that $T(m;\sigma) = \emptyset$.*

Proof. Consider an equilibrium (σ, μ). Then, for all $\bar{\theta} \in \Theta$ and for any out-of-equilibrium $m \in M(\bar{\theta})$, $E_\mu[\theta|\sigma(\bar{\theta})] - c(\sigma(\bar{\theta})) \geq E_\mu[\theta|m] - c(m)$ (the inequality is strict for $m < \sigma(\bar{\theta})$ by Assumption 1). With the same certification strategy, σ, $E_\mu[\theta|\sigma(\bar{\theta})] = E_{\mu'}[\theta|\sigma(\bar{\theta})]$ but $E_\mu[\theta|m] \geq E_{\mu'}[\theta|m]$. Thus, (σ, μ') is also an equilibrium. □

If there exists an equilibrium, then there also exists an outcome-equivalent equilibrium with the skeptical belief off the equilibrium path. Thus, to find the necessary and sufficient condition for a specific equilibrium outcome, we can assume innocuously that the buyer is skeptical off the equilibrium path because any equilibrium outcome can be supported by such a belief system.

For the non-revealing equilibrium, if the seller reports certificate $m = 0$ regardless of the quality, then the buyer believes that the quality should be m' if he observed $m' \in (0, 1]$. With this belief system off the equilibrium path, we have the following no-revelation condition.

Proposition 7. *There exists a non-revealing equilibrium if and only if for all $m \in [0, 1]$,*

$$E[\theta] \geq m - c(m), \tag{2.3}$$

where $\sigma(\theta) = 0$ for all $\theta \in \Theta$.

The following lemma is convenient to check equilibria with a pooling segment of qualities. We shall use it to prove Proposition 7.

[6] Cho and Kreps' [8] Intuitive Criterion or D1 places no restrictions on out-of-equilibrium beliefs in this model. However, without certification costs, the skeptical belief is the only one that passes the Intuitive Criterion.

Lemma 6. *Given σ, if for some $\theta \in T(m;\sigma)$, $\sigma(\theta)$ is incentive compatible, then for all $\theta' \in T(m;\sigma)$ with $\theta' < \theta$, so is $\sigma(\theta')$.*

Proof. This follows from the facts that the seller's profit does not depend on the value of θ directly so every $\theta \in T(m;\sigma)$ makes the same profit and that $M(\theta') \subset M(\theta)$ for any θ', $\theta \in \Theta$ with $\theta' < \theta$. □

Proof of Proposition 7. The only completely pooling strategy in the model is that $\sigma(\theta) = 0$ for all $\theta \in \Theta$. Given the certification strategy, the buyer's conditional expectation on the quality is $E_\mu[\theta|m = 0] = E[\theta]$ associated with the prior distribution and $E_\mu[\theta|m] = m$ for $m \in (0, 1]$ with the skeptical beliefs. For the specified strategy to be optimal, it is enough that $\sigma(\bar{\theta})$ solves (2.1) for $\bar{\theta} = 1$, that is,

$$E_\mu[\theta|0] - c(0) \geq E_\mu[\theta|m] - c(m)$$

for all $m \in (0, 1]$ because then, by Lemma 6, $\sigma(\bar{\theta})$ is incentive compatible for all $\bar{\theta} \in [0, 1)$ as well. For $m = 0$, (2.3) holds trivially. □

Corollary 2. *Suppose that a fully revealing equilibrium exists. Then, there exists a non-revealing equilibrium if and only if $E[\theta] \geq 1 - c(1)$.*

Proof. If there exists a fully revealing equilibrium, $m - c(m)$ is increasing in m and reaches the maximum when $m = 1$ because $c'(m) < 1$ for all $m \in [0, 1]$ by Proposition 6. Then, by Proposition 7, a non-revealing equilibrium exists if and only if (2.3) holds for $m = 1$. □

If the average quality, which is equal to the price when the seller does not certify any information, is greater than the benefit from any revelation net of the cost, then there exists another equilibrium in which the seller just conceals the private information altogether. Thus, if either high qualities are common in the industry or the certification cost is sufficiently high, then no-revelation can be supported as an equilibrium. The former case is similar to Harbaugh and To [18] in which no-revelation is an equilibrium even without disclosure cost when good news is common.

Corollary 2 depicts the situation in which both a fully revealing equilibrium and a non-revealing equilibrium exist, where the average quality is greater than the profit of the highest quality seller from revelation. Besides the fully revealing equilibrium and the non-revealing equilibrium, there may exist other equilibria in between. A simple example with three qualities is presented below.

Example 4. Consider a three-quality example. $\Theta = \{\theta_1, \theta_2, \theta_3\}$, where $\theta_1 = 1$, $\theta_2 = 2$, $\theta_3 = 3$, which are uniformly distributed. Let $M = \{m_1, m_2, m_3\}$, where $m_1 = \{\theta_1, \theta_2, \theta_3\}$, $m_2 = \{\theta_2, \theta_3\}$, $m_3 = \{\theta_3\}$, and let $c(m_1) = 0$, $c(m_2) = 0.6$, $c(m_3) = 1.2$. In this example, there are four (pure strategy) equilibria including both a fully revealing equilibrium and a non-revealing equilibrium. The certification strategies constituting the other two partially revealing equilibria are

$$\begin{cases} \sigma(\theta_1) = m_1 \\ \sigma(\theta_2) = m_1 \\ \sigma(\theta_3) = m_3 \end{cases} \quad \text{and} \quad \begin{cases} \sigma(\theta_1) = m_1 \\ \sigma(\theta_2) = m_2 \\ \sigma(\theta_3) = m_2 \end{cases}$$

respectively. Notice that in all equilibria, if θ_i is separating from θ_{i-1}, $\sigma(\theta_i) = m_i$ as Lemma 4 states.

2.7 WELFARE ASPECTS

In this model, the buyer does not care about which equilibrium should be realized because his entire surplus is extracted by the seller in any cases. From the seller's perspective, full concealment is the best equilibrium and full revelation is the worst because the revelation incurs a cost.

Let $\pi(\theta)$ and $u(\theta)$ be the equilibrium profit and utility respectively when the quality is θ, and let $E\pi$ and Eu be the *ex ante* equilibrium payoffs. In any equilibrium (σ, μ), the product is always traded at the price equal to the buyer's expectation on the quality. Thus, for each $\bar{\theta} \in [0, 1]$,

$$\pi(\bar{\theta}) = E_\mu[\theta | \sigma(\bar{\theta})] - c(\sigma(\bar{\theta})) \quad \text{and} \quad u(\bar{\theta}) = \bar{\theta} - E_\mu[\theta | \sigma(\bar{\theta})];$$

and with the prior belief F on θ,

$$E\pi = \int_0^1 \pi(\theta)\,\mathrm{d}F(\theta) \quad \text{and} \quad Eu = \int_0^1 u(\theta)\,\mathrm{d}F(\theta).$$

$Eu = 0$ in any equilibria, where the monopoly seller extracts the buyer's entire expected net benefit.

In order to compare equilibria, first suppose that both a fully revealing equilibrium and a non-revealing equilibrium exist and note that (2.3) holds for all $m \in [0,1]$. In the fully revealing equilibrium, $\sigma(\bar{\theta}) = \bar{\theta}$ and $E_\mu[\theta|\sigma(\bar{\theta})] = \bar{\theta}$ for all $\bar{\theta} \in [0,1]$. Thus, for all $\bar{\theta} \in [0,1]$,

$$\pi(\bar{\theta}) = \bar{\theta} - c(\bar{\theta}) \quad \text{and} \quad u(\bar{\theta}) = 0.$$

In the non-revealing equilibrium, $\sigma(\bar{\theta}) = 0$ and $E_\mu[\theta|\sigma(\bar{\theta})] = E[\theta]$ for all $\bar{\theta} \in [0,1]$. Thus, for all $\bar{\theta} \in [0,1]$,

$$\pi(\bar{\theta}) = E[\theta] \quad \text{and} \quad u(\bar{\theta}) = \bar{\theta} - E[\theta].$$

For any quality level, the non-revealing equilibrium is better than the fully revealing equilibrium for the seller by (2.3), and therefore this is also true at the *ex ante* stage. Since at most the seller of $\theta = 1$ is indifferent between them, the seller strictly *ex ante* prefers the non-revealing equilibrium to the fully revealing equilibrium. The private information itself is not valuable to the seller and the certification is costly, which makes the seller prefer the concealment to the revelation. This is another corollary of Proposition 7.

Corollary 3. *Suppose that there exist both a fully revealing equilibrium and a non-revealing equilibrium. Then, for all $\theta \in \Theta$ the non-revealing equilibrium gives at least as high profit to the seller as the fully revealing equilibrium for any distribution F supporting the non-revealing equilibrium. At most the seller of $\theta = 1$ is indifferent between them.*

Proof. For all $\bar{\theta} \in [0,1]$, $\pi(\bar{\theta}) = E[\theta]$ in the non-revealing equilibrium and $\pi(\bar{\theta}) = \bar{\theta} - c(\bar{\theta})$ in the fully revealing equilibrium, and we know by Proposition 7 that $E[\theta] \geq \bar{\theta} - c(\bar{\theta})$ for all $\bar{\theta} \in [0,1]$. Since, by Proposition 6, the RHS is increasing in $\bar{\theta}$, the equality is possible only for $\bar{\theta} = 1$. □

Table 2.1: The equilibrium payoffs in Example 4

σ	(m_1, m_1, m_1)	(m_1, m_1, m_3)	(m_1, m_2, m_2)	(m_1, m_2, m_3)
$\pi(\theta)$	$(2, 2, 2)$	$(1.5, 1.5, 1.8)$	$(1, 1.9, 1.9)$	$(1, 1.4, 1.8)$
$u(\theta)$	$(-1, 0, 1)$	$(-0.5, 0.5, 0)$	$(0, -0.5, 0.5)$	$(0, 0, 0)$
$E\pi$	2	1.6	1.6	1.4
Eu	0	0	0	0

In the non-revealing equilibrium, the seller's profit equals the average quality regardless of the quality levels. Given the non-revealing certification strategy, any other certificate than $m = 0$ is not profitable and the deviation gives at most full revelation profit with the skeptical buyer off the equilibrium path. This is true for any quality level because the quality itself is valueless to the seller. Thus, the seller makes higher profit by concealing the information rather than revealing. For the buyer, the non-revealing equilibrium is better with high qualities above the average whereas the fully revealing equilibrium is better with low qualities. However, at the *ex ante* stage, any equilibria are indifferent to the buyer because the expected utility is always zero by the monopolist's pricing rule.

Table 2.1 shows both the *ex post* and *ex ante* payoffs for all equilibria in Example 4. In this example with three qualities, there exist four (pure strategy) equilibria: a fully revealing equilibrium, a non-revealing equilibrium, and two partially revealing equilibria. As the table shows, for all quality levels, the non-revealing equilibrium with the certification strategy (m_1, m_1, m_1) gives the highest profit to the seller and the fully revealing equilibrium with (m_1, m_2, m_3) gives the lowest profit among all equilibria.

The non-revealing equilibrium is socially optimal if it exists, when it is evaluated at the *ex ante* stage. Since the monopoly seller extracts the surplus from the buyer, the buyer's benefit from revelation only transfers to the seller. At the *ex ante* stage, the amount of the benefit is the same as the average quality over all equilibria but the revelation incurs a social cost.

Proposition 8. *An equilibrium* (σ, μ) *is ex ante Pareto efficient if* σ *minimizes*

$$\int_0^1 c(\sigma(\theta))\,\mathrm{d}F(\theta) \tag{2.4}$$

over the set of equilibria.

Proof. Since $\int_0^1 E_\mu[\theta|\sigma(\bar{\theta})]\,\mathrm{d}F(\bar{\theta}) = E[\theta]$, the *ex ante* payoffs are calculated as $E\pi = E[\theta] - \int_0^1 c(\sigma(\theta))\,\mathrm{d}F(\theta)$ for a given equilibrium strategy σ and $Eu = 0$ for any equilibria. Thus, only $E\pi$ varies over equilibria and efficient σ minimizes (2.4). □

Corollary 4. *Suppose that there exists a non-revealing equilibrium. Then, it ex ante Pareto dominates any other equilibria.*

Proof. (2.4) is zero for the non-revealing equilibrium and otherwise it is strictly positive. Hence, the non-revealing equilibrium is *ex ante* Pareto efficient. □

The social welfare can be measured by $E\pi$ because $Eu = 0$ in any equilibria. The revelation cost is calculated at the *ex ante* stage as (2.4), which is the welfare loss caused by certification. Since the seller's expected revenue is $E[\theta]$ whatever the strategy is, the social welfare is only affected by the revelation cost and the equilibrium with the least cost is socially optimal. Note that (2.4) is largest when information is fully revealed and it is zero when no information is revealed. Thus, when certification is costly, full revelation of the product quality is the worst and full concealment is the best for the market if they are both an equilibrium. This result is in the stream of the existing literature on costly disclosure such as Jovanovic [23] and Cheong and Kim [7]. They show that equilibrium level of information disclosure is socially excessive when disclosure is costly.[7]

However, disclosure laws can reduce the buyer's risk of the purchase at the expense of the seller's profits. The buyer's expected utility is zero in any equilibria but the risk from purchase is different over the equilibria. Although the non-revealing equilibrium is the best, it is risky to the buyer because no information is revealed and the same price is charged for

[7]However, they propose different policies to prevent the excessive disclosure. Jovanovic [23] argues that the inefficiency can be eliminated by a subsidy to sale without disclosure and a tax on disclosure is an inferior policy, whereas Cheong and Kim [7] are in favor of taxation on disclosure.

all quality levels. Hence, the buyer gets negative utility with some positive probability. On the other hand, full revelation gives him zero utility for certain.

Alternatively, we might think about warranties in this model. Consider again Example 4 and see Table 2.1 for the equilibrium payoffs. If the seller gives a warranty to the buyer that covers buyer's *ex post* negative utility, then in the non-revealing equilibrium the equilibrium payoffs for each quality are

$$\pi(\theta_1) = 1, \quad \pi(\theta_2) = 2, \quad \pi(\theta_3) = 2,$$
$$u(\theta_1) = 0, \quad u(\theta_2) = 0, \quad u(\theta_3) = 1.$$

Thus, the non-revealing equilibrium is the best *ex ante* for both the seller and the buyer. Instead of disclosure laws, forcing sellers to make warranties increases consumers' welfare without welfare loss caused by the revelation cost. However, in food or health industry, disclosure laws might be more appropriate than *ex post* warranties to protect consumers.

2.8 EFFECTIVE USE OF THE PRODUCT

So far, a market for a product with uncertain quality has been analyzed, in which the buyer is only concerned about the quality. However, when the buyer is more informed of the quality, he can use or maintain the product more effectively and reduce welfare loss from an inappropriate use. In this section, by modifying the buyer's utility function, we analyze a market where buyer's effective use of the product matters.

Now suppose that if the buyer purchases the product, he takes an action $a \in \mathbb{R}$ on it and gets utility

$$u = \theta - (a - \theta)^2 - p. \tag{2.5}$$

As in the benchmark, the quality θ itself gives a value to the buyer, which is the first term of the utility function. In addition, the utility depends on how effectively the buyer uses and maintains the product, which we call the buyer's *action*. The optimal action is different across the quality levels. Even though the product is of high quality, inappropriate use may do harm to the consumers like medicine and food for diet. A low quality car user should have the car inspected more often. Only buyers who have received full information of the product can use it most effectively. This is shown by the second term, the quadratic loss form. When the effectiveness of use matters, the seller is more willing to reveal the product information. Furthermore, more information is revealed as the variance of the uncertain quality is larger because the buyer becomes more cautious about the risky consumption.

Since the buyer's expectation on the quality is $E_\mu[\theta|m]$ with belief μ on observing m, his best action given m after he purchases the product is $a(m) = E_\mu[\theta|m]$. Given m, p, and the best action function $a(\cdot)$, the buyer's expected utility from purchase is derived as

$$E_\mu[u|m] = E_\mu[\theta|m] - \mathrm{Var}_\mu(\theta|m) - p.$$

Thus, the expected utility now additionally depends on the variance of the quality conditional on a certificate. By adding the effectiveness of use into the model, the uncertain product quality lowers the buyer's expected utility. Therefore, the seller charges a lower price than she would in the case in which effective use does not matter. Formally, the seller's pricing rule after reporting a certificate m is $p(m) = E_\mu[\theta|m] - \mathrm{Var}_\mu(\theta|m)$.[8] Then, in an equilibrium, for all $\bar{\theta} \in \Theta$, $\sigma(\bar{\theta})$ solves

$$\max_{m \in M(\bar{\theta})} \; E_\mu[\theta|m] - \mathrm{Var}_\mu(\theta|m) - c(m).$$

Note that all the previous lemmas hold with the modified utility function. Thus, the full revelation condition is the same as in Proposition 6, that is, $c'(m) < 1$ for all $m \in [0,1]$ because $\mathrm{Var}_\mu(\theta|m) = 0$ for any $m \in M$ with the certification strategy $\sigma(\theta) = \theta$ for all $\theta \in \Theta$. However, no-revelation condition is slightly different. First, we should specify out-of-equilibrium beliefs, but the fact that Lemma 5 still holds in this version implies that we can just assume the buyer is skeptical off the equilibrium path. This is because any equilibrium outcome can be supported by such a belief system. The following is the lemma for this version.

[8]If for some m, $E_\mu[\theta|m] < \mathrm{Var}_\mu(\theta|m)$, then the market might fail for the product whose seller reported m because the buyer's expected utility would be negative for any price. However, with the set of qualities $[0,1]$, we can safely avoid the market failure. Since $\theta \geq \theta^2$ for $\theta \in [0,1]$, $E[\theta] \geq E[\theta^2]$ for any distribution on any subset of $[0,1]$. Thus, $E[\theta] \geq E[\theta^2] - (E[\theta])^2$ (the inequality is strict unless $\theta = 0$ with probability 1), where the right-hand side is another expression of $\mathrm{Var}(\theta)$. Therefore, for any μ and any m, $E_\mu[\theta|m] \geq \mathrm{Var}_\mu(\theta|m)$ and the market never fails.

Lemma 7. *Suppose that the utility function is given as* (2.5) *and* (σ, μ) *is an equilibrium for some* μ. *Then, there exists an equilibrium* (σ, μ') *where* $E_{\mu'}[\theta|m] = m$ *for* $m \in M$ *such that* $T(m; \sigma) = \emptyset$.

Proof. Consider an equilibrium (σ, μ). Then, for all $\bar{\theta} \in \Theta$ and for any out-of-equilibrium $m \in M(\bar{\theta})$,

$$E_{\mu}[\theta|\sigma(\bar{\theta})] - \mathrm{Var}_{\mu}(\theta|\sigma(\bar{\theta})) - c(\sigma(\bar{\theta})) \geq E_{\mu}[\theta|m] - \mathrm{Var}_{\mu}(\theta|m) - c(m),$$

where the inequality is strict for $m < \sigma(\bar{\theta})$ by Assumption 1. Since with the same certification strategy, σ,

$$E_{\mu}[\theta|\sigma(\bar{\theta})] - \mathrm{Var}_{\mu}(\theta|\sigma(\bar{\theta})) = E_{\mu'}[\theta|\sigma(\bar{\theta})] - \mathrm{Var}_{\mu'}(\theta|\sigma(\bar{\theta})),$$

(σ, μ') is an equilibrium if

$$E_{\mu}[\theta|m] - \mathrm{Var}_{\mu}(\theta|m) - c(m) \geq E_{\mu'}[\theta|m] - c(m).$$

Since $E_{\mu'}[\theta|m] = m$, we want to show that $E_{\mu}[\theta|m] - \mathrm{Var}_{\mu}(\theta|m) \geq m$. Let a random variable $x = \theta - m$. Then, $E_{\mu}[x|m] = E_{\mu}[\theta|m] - m$ and $\mathrm{Var}_{\mu}(x|m) = \mathrm{Var}_{\mu}(\theta|m)$. Following the argument in footnote 8, $E_{\mu}[x|m] \geq \mathrm{Var}_{\mu}(x|m)$ because x is distributed over a subset of $[0, 1]$. This completes the proof. □

Now the no-revelation condition follows as Proposition 9.

Proposition 9. *Suppose that the utility function is given as* (2.5). *Then, there exists a non-revealing equilibrium if and only if for all* $m \in [0, 1]$,

$$E[\theta] - \mathrm{Var}(\theta) \geq m - c(m), \tag{2.6}$$

where $\sigma(\theta) = 0$ *for all* $\theta \in \Theta$.

Proof. Following the certification strategy, $\sigma(\theta) = 0$ for all $\theta \in \Theta$, the seller's profit is $E[\theta] - \mathrm{Var}(\theta)$, and by deviating to $m \in (0, 1]$ if it is available, the profit is $m - c(m)$ with the buyer's skeptical beliefs. For the specified strategy to be optimal, by Lemma 6, it is enough that $\sigma(\theta)$ is incentive compatible for $\theta = 1$. Hence, we need $E[\theta] - \mathrm{Var}(\theta) \geq m - c(m)$ for all $m \in (0, 1]$. For $m = 0$, the inequality holds trivially. □

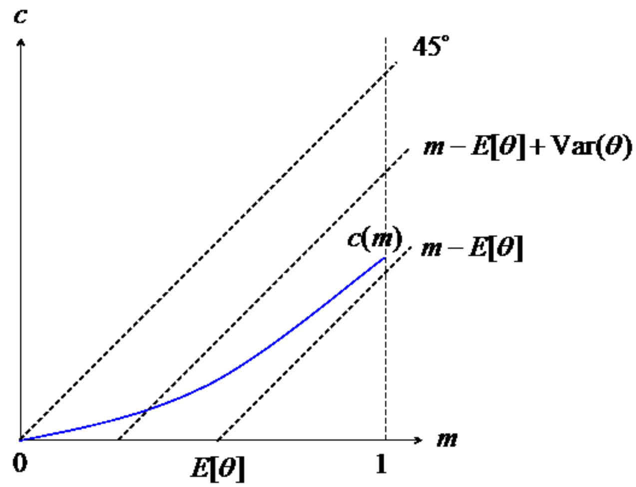

Figure 2.1: No-revelation condition

Since the least revelation (the least cost equilibrium) *ex ante* Pareto dominates any other equilibria, Propositions 7 and 9 provoke the comparison of the amount of information revealed between the two environments. In order to compare the no-revelation conditions between the original model and the modified model, we rearrange the terms in conditions (2.3) and (2.6) for the cost function as $c(m) \geq m - E[\theta]$ and $c(m) \geq m - E[\theta] + \text{Var}(\theta)$ respectively. In Figure 2.1, the line of $m - E[\theta] + \text{Var}(\theta)$ should be located between $m - E[\theta]$ and 45° line because $E[\theta] > \text{Var}(\theta) > 0$ for any distribution F. For the existence of a non-revealing equilibrium, the cost function should be located above the line of $m - E[\theta]$ in the original model and above the line of $m - E[\theta] + \text{Var}(\theta)$ in the modified model. This implies that given a cost function, the seller is more willing to reveal the quality when the effectiveness of use matters.

In the figure, there exists a non-revealing equilibrium, which is socially optimal, when the utility depends on product quality alone, while there should be at least some degree of revelation when effective use matters to the buyer. Furthermore, as the variance of the quality is larger or as the average quality is lower, more information is revealed in the optimal equilibrium. The uncertainty increases the probability that the buyer deals with the product ineffectively and lowers the price by the variance of the quality. Therefore, as the variance is larger, the seller is more willing to reveal the information in order to raise the price.

2.9 CONCLUSION

One of the factors of the failure of full revelation is the existence of certification cost. Only if the marginal benefit from certification is greater than the marginal cost, full revelation can be supported as an equilibrium outcome. In this chapter, the buyer is uncertain of product quality and the seller certifies the quality at a cost. The quality is relevant to the buyer's utility and the buyer decides if he will purchase the product at a price the seller charges. By the assumption that the seller is a monopolist, the buyer always purchases the product and the seller extracts the entire surplus from the buyer. With the monopolist model, we remove allocative distortion caused by a certification cost and only focus on the informational issue.

The existence of a fully revealing equilibrium depends on the cost structure of certification. In this model, the certification cost is increasing in the sense that a better certificate is more costly. The seller can charge a higher price if she can persuade the buyer that the product is of good quality. Thus, if the marginal benefit from certification is greater than the marginal cost, that is, if the seller can charge a sufficiently high price relative to the certification cost, then the seller certifies information and discloses it to the buyer. However, full revelation of information is the worst equilibrium because of the certification cost whereas full concealment is the best if it exists. Full concealment occurs when the average quality or certification cost is sufficiently high. In general, there are multiple equilibria and the equilibrium with the least revelation is *ex ante* Pareto optimal. This is because the private information itself is valueless to the seller and the revelation only incurs a cost.

If the buyer is concerned about his action as well as the product quality, the seller has more incentive to reveal the quality even if the certification cost is high. As the buyer is more informed, he can deal with the product more effectively. If we incorporate this fact into the model, more information is revealed in the optimal equilibrium as compared with the case in which the buyer is only concerned about the quality. Furthermore, more information is revealed as the variance of the quality is larger or as the average quality is lower. Since the buyer is uncertain of the quality, the price falls by the variance because of the risky consumption. Therefore, the seller is more willing to reveal the private information in order to raise the price.

We could define the set of all possible qualities as any compact interval in $\mathbb{R}$. By allowing for negative qualities, we can analyze a market for a product that is undesirable to the buyer with a positive probability, for example, cigarettes, medicine, food, etc. They may do harm to the consumers by side effects or food poisoning. In this case, it is conceivable that the seller tends to conceal the information and the role of disclosure laws is more important. Some other issues remain for further work. In this chapter, we analyzed the model with an increasing cost function of certification without further assumptions. However, the cost structure varies over industries: convex or concave functions, high or low marginal costs. The characterization of equilibria depending on cost structures will make possible cross-industry comparison. Also, the model can be extended by introducing heterogeneous buyers or competition among multiple sellers.

3.0 QUALITY DISCLOSURE AND COMPARATIVE ADVERTISEMENT

3.1 INTRODUCTION

We study firms' voluntary disclosure in an oligopoly market for differentiated products, where firms are allowed to advertise a rival's product as well as their own product. When consumers are uncertain of product qualities, Board [6] and Hotz and Xiao [19] show that price competition among firms alleviates firms' incentives to disclose the quality of their own product. However, firms also have incentives to distinguish their own product from others to attract more consumers. A comparative advertisement is a useful way to distinguish products and thus, a rival's advertisement can lead to a disclosure of a firm's product information.

Comparative or negative advertisements are used in many industries and political campaigns. In the advertisement of the mobile phone 3G network service on the TV commercial and their Internet website, Verizon Wireless compare the service coverage with that of their competitor, AT&T.[1] They have a broader service area than AT&T and use a comparative advertisement to show that their service is superior. Through the negative advertisement by Verizon Wireless on the AT&T's service, the information on the AT&T's mobile phone service is revealed even though AT&T do not disclose their nation-wide service coverage in a picture. This contrasts with that AT&T show the map of their nation-wide 3G service coverage of iPad.[2]

This chapter allows firms to advertise a rival's product. We show that the qualities of

[1]See the map at http://vzwmap.verizonwireless.com/dotcom/coveragelocator/images/maps/3Gcomparison.pdf. "3G takes raw, untamed signal and adds an extra, turbo-charged boost. It helps you transmit and receive impressively fast, with average download speeds ranging from 600Kbps to 1.4Mbps" (http://phones.verizonwireless.com/3g/).

[2]See the map at http://www.wireless.att.com/coverageviewer/.

all the products in a market are fully revealed by a high quality firm's comparative advertisement and full revelation is the unique equilibrium outcome. Each firm's advertisement (message) can convey information on either its own product quality or a rival's or both. Other than traditional models that consider advertisement as signaling product qualities (Nelson [37], Schmalensee [42], Grossman and Shapiro [17], and Kihlstrom and Riordan [24]), we consider it as a truthful claim about qualities. That is, it conveys information directly to the consumers. The restriction of firms' messages to truthful claims can be justified by the argument that an untruthful claim, such as an overstatement on their own product or an understatement on their rival's product, could be challenged in a court of law. If the claim was found to be untruthful, the firm that sent the untrue message might have to pay a fine or worse, and the true quality would be revealed as the result.

In this model, full revelation occurs as the unique equilibrium outcome. If firms disclose no information and consumers do not distinguish the qualities among products, then the firms' profits are zero by price competition. By revealing some information and having consumers perceive that the product is differentiated from a rival's, the firms can increase the profits. With two firms, full information on all products is revealed by the high quality firm as in the above example. First, comparing the products between the firms is good for the high quality firm because then it can attract more consumers. Thus, the high quality firm can increase the profit by advertising the rival's low quality product negatively. Since lying is not allowed, the true quality of the rival's product is revealed. Additionally, the high quality firm reveals the true quality of its own product to get a high price. Since the advertisement that fully reveals both firms' product qualities is a dominant strategy for the high quality firm, full revelation is the unique outcome.

In general, full revelation fails without advertisement on a rival's product. However, literature shows that full revelation increases consumers' welfare, and so argues in favor of the introduction of mandatory disclosure laws (Fishman and Hagerty [12], Board [6], and Hotz and Xiao [19]). The results of this chapter imply that a market can lead to full revelation with voluntary disclosure by allowing for negative advertisement on rivals' product instead of mandatory disclosure laws which entail an enforcement cost and a deadweight loss. Also, negative advertisement plays a role in a political campaign because full revelation of candidates' information is a very important issue. Therefore, although there is a debate, recent studies such as Polborn and Yi [41] are in favor of prevailing negative campaigning to facilitate a more informed choice by the electorate.

Lipman and Seppi [30] analyze a similar but different model in which informed senders have conflicting preferences among them. The senders sequentially move and each of them sends either a cheap talk message or a partially provable message which excludes some states of nature. They show that full revelation occurs because conflicting preferences guarantee that someone will have an incentive to correct any mistaken inference.

There are other competition models in which the informed parties can only disclose their own information. Ivanov [20] shows that informed sellers' information can be fully revealed in a sufficiently competitive market if each seller's message is privately observed by the consumer. However, full revelation does not guarantee the full market efficiency. Levin *et al.* [27] and Cheong and Kim [7] analyze the effect of price competition of firms on the incentive to disclose quality when the disclosure is costly. In Levin *et al.* [27], disclosure level is higher under a monopoly cartel than under duopoly. Furthermore, Cheong and Kim [7] argue that firms tend to conceal their private information as the number of competing firms becomes larger even though the disclosure cost is very low.

All the above models study information transmission from the informed party to the uninformed party. On the other hand, there is literature that studies information sharing among competitors. Jansen [22] shows that if innovation is not protected by intellectual property rights and the imitation of the technology is costless, at most one firm reveals some of its technologies. Gill [13] analyzes the incentives to disclose intermediate research results in an R&D competition. The leading innovator's disclosure may help the follower by knowledge spillovers. However, the leader may disclose the results in order to signal commitment to the project and induce the rival to exit the competition.

The rest of the chapter is organized as follows. In Section 3.2, we present an oligopoly model with price competition between two firms and derive the firms' profit functions from the equilibrium pricing rules and the associated demand functions. With the profit functions, in Section 3.3, we analyze the firms' advertisement strategies and show that full information is revealed by the high quality firm. Section 3.4 concludes the chapter by discussing extendible issues.

3.2 MODEL

We follow a general setting for a differentiated product market. Two firms, 1 and 2, compete in prices with product qualities uncertain to a continuum of consumers. The quality of firm i's ($i = 1, 2$) product is θ_i, which is drawn from $[0, 1]$ according to a distribution F independently. Each firm knows the qualities of both its own product and the rival's product. The consumers only know the distribution and do not observe the quality until they purchase it. Assuming that the production cost is zero, the firm i's profit is $\pi_i = p_i D_i$, where $p_i \in \mathbb{R}_+$ is the product price of firm i and D_i is the demand for i's product. A consumer can purchase at most one unit and her utility is given as $u = \gamma\theta_i - p_i$ if she purchases a product from firm i. We denote by γ the degree of strength of consumers' preference, which is uniformly distributed on $[0, 1]$. The reservation utility is assumed to be zero.

Before the firms compete in prices, each of them can advertise both on its own product and on the rival's product. We denote firm i's advertisement by a message $m_i = (m_{i1}, m_{i2})$, where $m_{ij} \in \mathcal{P}([0, 1])$ is firm i's message on firm j's product. Messages are restricted to be truthful so that m_{ij} must contain the true value of θ_j or otherwise it is empty. The firms simultaneously choose messages. Let μ be consumers' belief on the qualities, then the consumers' expectation on the quality of the firm i's product associated with μ is $E_\mu[\theta_i|\mathbf{m}]$ after observing $\mathbf{m} = (m_1, m_2)$.

In order to analyze advertisement strategies, we first derive the demand function and the optimal pricing rule for each firm. Since it is a zero probability event that $\theta_1 = \theta_2$, we assume $\theta_1 < \theta_2$ without loss of generality. Also, we exclude the case that $E_\mu[\theta_1|\mathbf{m}] = E_\mu[\theta_2|\mathbf{m}]$ because then, by price competition, each firm's profit goes down to zero, which is not in an equilibrium. Given $\mathbf{m}$, a consumer with γ purchases firm 2's product if

$$\gamma E_\mu[\theta_2|\mathbf{m}] - p_2 \geq \gamma E_\mu[\theta_1|\mathbf{m}] - p_1$$

$$\Rightarrow \quad \gamma \geq \frac{p_2 - p_1}{E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]} \ ;$$

and she purchases firm 1's product if

$$\gamma E_\mu[\theta_1|\mathbf{m}] - p_1 \geq 0 \quad \text{and} \quad \gamma E_\mu[\theta_1|\mathbf{m}] - p_1 > \gamma E_\mu[\theta_2|\mathbf{m}] - p_2$$

$$\Rightarrow \quad \frac{p_1}{E_\mu[\theta_1|\mathbf{m}]} \leq \gamma < \frac{p_2 - p_1}{E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]}.$$

Thus, the demand functions are derived as

$$\begin{aligned} D_1(p_1, p_2) &= \text{Prob}\left(\frac{p_1}{E_\mu[\theta_1|\mathbf{m}]} \leq \gamma < \frac{p_2 - p_1}{E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]}\right) \\ &= \frac{p_2 - p_1}{E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]} - \frac{p_1}{E_\mu[\theta_1|\mathbf{m}]} \end{aligned}$$

and

$$\begin{aligned} D_2(p_1, p_2) &= \text{Prob}\left(\gamma \geq \frac{p_2 - p_1}{E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]}\right) \\ &= 1 - \frac{p_2 - p_1}{E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]}. \end{aligned}$$

In deriving the demand functions, we assumed implicitly

$$0 \leq \frac{p_1}{E_\mu[\theta_1|\mathbf{m}]} \leq \frac{p_2 - p_1}{E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]} \leq 1.$$

Otherwise, at least one firm's demand would be zero and give zero profit to the firm, which does not occur in equilibrium. With the demand functions, each firm's equilibrium pricing rule is uniquely determined as

$$\begin{aligned} p_1(\mathbf{m}) &= \frac{E_\mu[\theta_1|\mathbf{m}](E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])}{4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]}, \\ p_2(\mathbf{m}) &= \frac{2E_\mu[\theta_2|\mathbf{m}](E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])}{4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}]}, \end{aligned}$$

and the firms' profit functions under the pricing rules are derived as

$$\begin{aligned} \pi_1(p_1(\mathbf{m}), p_2(\mathbf{m})) &= \frac{E_\mu[\theta_1|\mathbf{m}]E_\mu[\theta_2|\mathbf{m}](E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])}{(4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])^2}, \\ \pi_2(p_1(\mathbf{m}), p_2(\mathbf{m})) &= \frac{4(E_\mu[\theta_2|\mathbf{m}])^2(E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])}{(4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])^2}. \end{aligned}$$

Since profits are nonnegative, it must be that $E_\mu[\theta_2|\mathbf{m}] > E_\mu[\theta_1|\mathbf{m}]$ and $p_2(\mathbf{m}) > p_1(\mathbf{m})$ in equilibrium. Then, we can now analyze the firms' strategies on advertisement by considering the profit functions.

3.3 ADVERTISEMENT STRATEGIES

Profits depend on the consumers' expectation on qualities. If the firms reveal nothing, this advertisement strategy is not an equilibrium because $p_1 = p_2 = 0$ and $\pi_1 = \pi_2 = 0$ by price competition. Revealing information in a market for differentiated products reduces the price competition between firms and increases the profits.

First, consider the low quality firm 1. Given the consumers' expectation on θ_2, consider how the profit of firm 1 varies with the expectation on θ_1. We can derive the partial derivative as

$$\frac{\partial \pi_1(p_1(\mathbf{m}), p_2(\mathbf{m}))}{\partial E_\mu[\theta_1|\mathbf{m}]} = \frac{(E_\mu[\theta_2|\mathbf{m}])^2(4E_\mu[\theta_2|\mathbf{m}] - 7E_\mu[\theta_1|\mathbf{m}])}{(4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])^3}.$$

The sign of the derivative depends on the expectations on the qualities and this implies that firm 1's advertisement on its own product is strategic as long as firm 2 does not reveal θ_1. This is shown in Board [6]. On the other hand, the partial derivative with respect to the expectation on θ_2 is

$$\frac{\partial \pi_1(p_1(\mathbf{m}), p_2(\mathbf{m}))}{\partial E_\mu[\theta_2|\mathbf{m}]} = \frac{(E_\mu[\theta_1|\mathbf{m}])^2(E_\mu[\theta_1|\mathbf{m}] + 2E_\mu[\theta_2|\mathbf{m}])}{(4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])^3} > 0.$$

Thus, firm 1 has an incentive to reveal the rival's high quality because the profit of firm 1 is increasing in the expectation on the rival's product quality. However, the low quality firm's advertisement has no effects on the outcome because all information on the both firms' products is revealed by the high quality firm.

Proposition 10. *Let $\theta_1 < \theta_2$. Then, θ_1 and θ_2 are revealed by firm 2 in equilibrium and full revelation is the unique outcome.*

The proposition is confirmed by considering the profit function of the high quality firm 2. The partial derivatives of the function with respect to the consumers' expectations are

$$\frac{\partial \pi_2(p_1(\mathbf{m}), p_2(\mathbf{m}))}{\partial E_\mu[\theta_1|\mathbf{m}]} = -\frac{4(E_\mu[\theta_2|\mathbf{m}])^2(2E_\mu[\theta_2|\mathbf{m}] + E_\mu[\theta_1|\mathbf{m}])}{(4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])^3} < 0$$

and

$$\frac{\partial \pi_2(p_1(\mathbf{m}), p_2(\mathbf{m}))}{\partial E_\mu[\theta_2|\mathbf{m}]} =$$

$$\frac{4E_\mu[\theta_2|\mathbf{m}](4(E_\mu[\theta_2|\mathbf{m}])^2 - 3E_\mu[\theta_1|\mathbf{m}]E_\mu[\theta_2|\mathbf{m}] + 2(E_\mu[\theta_1|\mathbf{m}])^2)}{(4E_\mu[\theta_2|\mathbf{m}] - E_\mu[\theta_1|\mathbf{m}])^3} > 0.$$

The profit of firm 2 is decreasing in the consumers' expectation on θ_1 and increasing in the expectation on θ_2. Thus, firm 2's best strategy on the rival's product is the negative advertisement, which rules out all the higher values than θ_1. Then, the consumers rationally believe that the true quality is the highest value that is not ruled out. Similarly, firm 2's best strategy on its own product is to rule out all the lower values than θ_2 and the consumers infer exactly the true quality, which is the lowest value that is not ruled out. Thus, all information on the products is revealed by the high quality firm's comparative advertisement.

The firm 2's advertisement strategies on its own product and the rival's product that fully reveal the qualities are dominant strategies because they are the best regardless of firm 1's strategies. Therefore, full revelation of the uncertain qualities is the unique equilibrium outcome.

3.4 CONCLUSION

All information is fully revealed by a high quality firm's comparative advertisement and, moreover, full revelation is the unique equilibrium outcome. However, full revelation fails in general without negative advertisement in a competitive market. Some studies argue in favor of the introduction of mandatory disclosure laws because revelation increases consumers' welfare. This chapter shows that even without mandatory disclosure laws, the market outcome can lead to full revelation if negative advertisement on rivals' product is allowed.

The model can be easily extended to an oligopoly with more than two firms. It is conceivable that full revelation still occurs by the firms' comparative advertisements. The highest quality firm will reveal itself and each firm including the highest quality firm will advertise negatively the next high quality firm's product because the firms gain more as the difference of the qualities is larger.

If firms do not observe rivals' product qualities, it is probabilistic whether a claim about a rival's quality was truthful or not. Thus, we should consider the probability representing the likelihood of being charged by a court by misreporting the rival's information. In this case, the revelation will depend on the cost of misreporting and the joint distribution of qualities. Recalling the partial derivatives of the profit functions, if a firm's product is inferior to the rival's, then the firm gains more as the rival's product is perceived more superior. An overstatement on the rival's product will not be challenged because it is profitable for the high quality firm as well. Only the problematic case is a high quality firm's understatement on the rival's product because the low quality firm's advertisement on its own product is strategic. We leave this issue for future work.

APPENDIX

PROOFS

Proof of Proposition 3. For $N \geq 2$, consider a partition $\langle t_0 = 0, t_1, \ldots, t_N = 1 \rangle$ such that $m^I(t) = m_i$ for $t \in [t_{i-1}, t_i)$, $i = 1, 2, \ldots, N$, and $m^U = m_1$. Then, the receiver's optimal action rule is:

$$a_1 = \phi(m_1)\frac{t_1}{2} + [1 - \phi(m_1)]\frac{1}{2},$$

$$a_i = \frac{t_{i-1} + t_i}{2}, \qquad i = 2, 3, \ldots, N,$$

where

$$\phi(m_1) = \frac{pt_1}{pt_1 + 1 - p}.$$

By the informed sender's arbitrage conditions, we have

$$2t_1 = \frac{pt_1{}^2 + 1 - p}{2(pt_1 + 1 - p)} + \frac{t_1 + t_2}{2} - 2b, \tag{.1}$$

$$2t_i = \frac{t_{i-1} + t_i}{2} + \frac{t_i + t_{i+1}}{2} - 2b, \qquad i = 2, 3, \ldots, N-1. \tag{.2}$$

Solving the difference equation (.2), we get a sequence $\{t_1, t_2, \ldots, t_N = 1\}$ such that

$$t_i = t_1 + (t_2 - t_1)(i - 1) + 2b(i - 1)(i - 2), \quad i = 1, 2, \ldots, N. \tag{.3}$$

Since $t_N = 1$, by (.3), we get

$$t_2 = \frac{1 - 2b(N-1)(N-2) + t_1(N-2)}{N-1}. \tag{.4}$$

We need $t_1 < t_2$, and replacing t_2 with (.4), we have

$$t_1 < 1 - 2b(N-1)(N-2). \tag{.5}$$

Plugging (.4) into (.1) and solving for t_1 such that $t_1 > 0$, we get

$$t_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A},$$

where

$$\begin{aligned} A &= pN, \\ B &= 2bpN(N-1) - 2pN + (2N-1), \\ C &= 2b(1-p)N(N-1) - (1-p)N. \end{aligned}$$

Hence, we can determine the partition $\langle t_0 = 0, t_1, \ldots, t_N = 1\rangle$ and $m^I(\cdot)$ is the best response to the action rule.

For the uninformed sender, the incentive compatibility, together with (.5), requires that

$$\frac{1}{2} \leq t_1 < 1 - 2b(N-1)(N-2), \tag{.6}$$

and (.6) holds when

$$b \leq \frac{2 - pN}{4(2-p)N(N-1)}.$$

□

Proof of Proposition 4. For $N \geq 3$, consider a partition $\langle t_0 = 0, t_1, \ldots, t_N = 1\rangle$ such that $m^I(t) = m_i$ for $t \in [t_{i-1}, t_i)$, $i = 1, 2, \ldots, N$, and $m^U = m_N$. Then, the receiver's optimal action rule is:

$$a_i = \frac{t_{i-1} + t_i}{2}, \qquad i = 1, 2, \ldots, N-1,$$

$$a_N = \phi(m_N)\frac{t_{N-1} + 1}{2} + [1 - \phi(m_N)]\frac{1}{2},$$

where

$$\phi(m_N) = \frac{p(1 - t_{N-1})}{p(1 - t_{N-1}) + 1 - p}.$$

By the informed sender's arbitrage conditions, we have

$$2t_i = \frac{t_{i-1} + t_i}{2} + \frac{t_i + t_{i+1}}{2} - 2b, \qquad i = 1, 2, \ldots, N-2, \tag{.7}$$

$$2t_{N-1} = \frac{t_{N-2} + t_{N-1}}{2} + \frac{1 - p{t_{N-1}}^2}{2(1 - pt_{N-1})} - 2b. \tag{.8}$$

Solving the difference equation (.7), we get a sequence $\{t_0 = 0, t_1, \ldots, t_{N-1}\}$ such that

$$t_i = t_1 i + 2bi(i-1), \quad i = 0, 1, \ldots, N-1. \tag{.9}$$

Take t_{N-1} and t_{N-2} from (.9) and solve (.8) for t_1 such that $t_{N-1} < 1$. We get the solution as

$$t_1 = \frac{-B - \sqrt{B^2 - 4AC}}{2A},$$

where

$$A = pN(N-1),$$
$$B = 2bpN(N-1)(2N-3) - (2N-1),$$
$$C = 4b^2pN(N-1)^2(N-2) - 4b(N-1)^2 + 1.$$

If $t_1 > 0$, we can determine the partition $\langle t_0 = 0, t_1, \ldots, t_N = 1\rangle$ and $m^I(\cdot)$ is the best response to the action rule.

Now we need the uninformed sender's incentive compatibility, which requires that $t_{N-1} \leq 1/2$. From (.9), $t_{N-1} \leq 1/2$ implies that

$$t_1 \leq \frac{1 - 4b(N-1)(N-2)}{2(N-1)}, \tag{.10}$$

and the strictly positive t_1 satisfies (.10) when

$$\frac{pN-2}{4(2-p)N(N-1)} \leq b < \frac{N-1-\sqrt{(1-p)N(N-2)+1}}{2pN(N-1)(N-2)}.$$

□

Proof of Proposition 5. In Example 2, we constructed the 3-step equilibrium with $m^U = m_2$ and showed that $\bar{b}_2^3(p) = \frac{9-8p}{24(3-2p)}$ is greater than $\bar{b}_{CS}^3 = 1/12$ if $p < 3/4$.

Suppose that $N \geq 5$ and N is an odd number. Let $M \geq 2$ be an integer and consider a partition $\langle t_0 = 0, t_1, \dots, t_M, t'_0, t'_1, \dots, t'_M = 1 \rangle$ so that $N = 2M+1$, where the uninformed sender pools with $[t_M, t'_0)$, that is, $m^U = m_{M+1}$. Using the same way as in the above proofs, by the informed sender's arbitrage conditions and the receiver's optimal actions, we have

$$t_{i+1} - t_i = t_i - t_{i-1} + 4b \quad \text{and} \quad t'_{i+1} - t'_i = t'_i - t'_{i-1} + 4b$$

for $i = 1, 2, \dots, M-1$. Solving the two difference equations with $t_0 = 0$ and $t'_M = 1$, we get

$$t_1 = \frac{t_M - 2bM(M-1)}{M} \quad \text{and} \quad t'_1 = \frac{1 - 2bM(M-1) + t'_0(M-1)}{M}.$$

Since $0 < t_1$ and $t'_0 < t'_1$, we must have $2bM(M-1) < t_M$ and $t'_0 < 1 - 2bM(M-1)$. By these two inequalities and the uninformed sender's incentive compatibility condition, the specified partition supports an equilibrium if

$$2bM(M-1) < t_M \leq \frac{1}{2} \leq t'_0 < 1 - 2bM(M-1). \tag{.11}$$

With the help of the arbitrage conditions for $t = t_M$ and $t = t'_0$, we can solve for t_M and t'_0 as

$$t_M = \frac{4pM^2(M+1)[b(2M+1)-1] - M(2M+1)^2[2b(M+1)-1]}{(2M+1)[4pM(M-1)+(2M+1)]},$$

$$t'_0 = t_M + \frac{1}{2M+1}.$$

Plugging t_M and t'_0 into (.11) and replacing M with $(N-1)/2$, we can derive the following from each inequality:

$$b < \frac{N^2 - p(N-1)(N+1)}{N(N-1)[p(N^2-7N+8)+2N]},$$

$$b \geq \frac{N^2(N-2) - p(N-1)(2N^2-3N-1)}{N(N-1)(N+1)[N(1-p)+p]},$$

$$b \leq \frac{N(N^2-2N+2) - p(N-1)(2N^2-5N+5)}{N(N-1)(N+1)[N(1-p)+p]},$$

$$bN[p(N-1)(N^2-5N+10)-4N] < p(N-1)(3N-5) - N(N-2).$$

Thus, when the above four inequalities are satisfied simultaneously, the N-step equilibrium exists in which $m^U = m_{\frac{N+1}{2}}$. Given p and N, the supremum, $\bar{b}^N_{\frac{N+1}{2}}(p)$, of b that satisfies all the above inequalities is greater than $\bar{b}^N_{CS} = 1/[2N(N-1)]$ if

$$N = 5 \quad \text{and} \quad \frac{1}{4} < p \leq \frac{35}{54} \ ; \quad \text{or}$$

$$N \geq 7 \quad \text{and} \quad \frac{N(N-2)(N^2-4N-1)}{(N-1)(2N^3-10N^2+9N+5)} < p \leq \frac{N(2N-3)}{4N^2-11N+9}.$$

For an even number $N \geq 4$, we can prove it similarly by constructing an equilibrium in which $m^U = m_{\frac{N}{2}+1}$. We omit the proof. □

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