

**PERFORMANCE ANALYSIS OF LINK ADAPTIVE 802.11 WLANS WITH  
MULTIUSER DETECTION CAPABLE RECEIVERS**

by

**Mir Hamza Mahmood**

BSc (Hons.) Computer Engineering, Lahore University of Management Sciences, Pakistan, 2007

Submitted to the Graduate Faculty of  
Swanson School of Engineering in partial fulfillment  
of the requirements for the degree of  
Master of Science

University of Pittsburgh

2008

UNIVERSITY OF PITTSBURGH  
SWANSON SCHOOL OF ENGINEERING

This Thesis was presented

by

Mir Hamza Mahmood

It was defended on

October 31, 2008

and approved by

Dr. Zhi-Hong Mao, Assistant Professor, Department of Electrical and Computer Engineering

Dr. Marlin Mickle, Professor, Department of Electrical and Computer Engineering

Thesis Advisor: Dr. Heung-No Lee, Assistant Professor, Department of Electrical and  
Computer Engineering

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Mir Hamza Mahmood, M.S.

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Conventional IEEE 802.11 Medium Access Control (MAC) protocol does not allow simultaneous transmissions from stations at any moment, to avoid collisions. Namely, it employs measures such as carrier sense multiple access, collision avoidance, and distributed coordination function which are designed to discourage simultaneous transmissions. With the advent of sophisticated physical layer technologies, multi-user detection capable receivers become available. In this paper, therefore, we aim to investigate if the popular 802.11 MAC can be modified to exploit this innovation in the physical layer. We accomplish this by deriving a new throughput expression. Modern WLANs support multiple data rate transmissions via link adaptation for higher spectral efficiency. Thus, we include multi rate link adaptation in the analysis. We find that 802.11 can be modified slightly to support simultaneous transmissions and to obtain significant benefit from multi-user detection capable receivers.

**Keywords** – Multiuser Detection, 802.11, Carrier sense multiple access, collision avoidance, throughput optimization.

## TABLE OF CONTENTS

|  |           |
|--|-----------|
| <b>PREFACE.....</b>  | <b>IX</b> |
| <b>1.0 INTRODUCTION.....</b>   | <b>1</b>  |
| <b>1.1 MOTIVATION AND PROBLEM CONTEXT.....</b>                             | <b>1</b>  |
| <b>1.2 RELATED WORK.....</b>   | <b>3</b>  |
| <b>1.3 CONTRIBUTION AND THESIS STRUCTURE.....</b>                          | <b>5</b>  |
| <b>2.0 CONVENTIONAL 802.11 WIRELESS LOCAL AREA NETWORKS.....</b>           | <b>6</b>  |
| <b>2.1 INTRODUCTION .....</b>  | <b>6</b>  |
| <b>2.2 802.11 MAC PROTOCOL.....</b>  | <b>7</b>  |
| <b>2.2.1 Frame Reception Acknowledgement .....</b>                         | <b>7</b>  |
| <b>2.2.2 The Hidden Terminal Problem and RTS/CTS handshake.....</b>        | <b>8</b>  |
| <b>2.3 MODELING THE BACK-OFF MECHANISM IN DCF .....</b>                    | <b>12</b> |
| <b>2.3.1 The Distributed Coordination Function (DCF).....</b>              | <b>12</b> |
| <b>2.3.2 Transition Probabilities .....</b>                                | <b>15</b> |
| <b>2.3.3 Steady State Distribution and Transmission Probabilities.....</b> | <b>16</b> |
| <b>3.0 ANALYSIS OF MULTI RATE WLAN WITH MUD CAPABILITY .....</b>           | <b>18</b> |
| <b>3.1 SYSTEM AND PROBABILITY MODELING FOR 802.11-M MUD .....</b>          | <b>19</b> |
| <b>3.1.1 System Model .....</b>  | <b>19</b> |
| <b>3.1.2 Busy and Collision Probabilities .....</b>                        | <b>20</b> |

|                           |   |           |
|---------------------------|---|-----------|
| <b>3.2</b>                | <b>THROUGHPUT DERIVATION .....</b>                            | <b>22</b> |
| 3.2.1                     | Throughput definition.....                                    | 22        |
| 3.2.2                     | Idle Period .....   | 24        |
| 3.2.3                     | Busy Period .....   | 24        |
| 3.2.4                     | Successful Effective Data Transmission .....                  | 27        |
| 3.2.5                     | Network Throughput .....                                      | 28        |
| 3.2.6                     | Discussion .....  | 29        |
| <b>4.0</b>                | <b>NUMERICAL RESULTS AND DISCUSSION ON 802.11-M WLAN.....</b> | <b>30</b> |
| 4.1                       | INFLUENCE OF INITIAL CONTENTION WINDOW SIZE.....              | 31        |
| 4.2                       | THROUGHPUT OPTIMIZATION VS. FAIRNESS.....                     | 38        |
| 4.3                       | MODIFICATOINS TO EXPLOIT FULLY THE MUD CAPABILITY .....       | 41        |
| 4.3.1                     | Physical Layer Modifications .....                            | 41        |
| 4.3.2                     | MAC Layer Modifications.....                                  | 42        |
| <b>5.0</b>                | <b>CONCLUSION.....</b>  | <b>44</b> |
| <b>APPENDIX A .....</b>   |   | <b>45</b> |
| <b>BIBLIOGRAPHY .....</b> |   | <b>47</b> |

## LIST OF TABLES

|  |    |
|--|----|
| Table 4.1: System parameters used in numerical analysis..... | 30 |
|--|----|

## LIST OF FIGURES

|   |    |
|---|----|
| Figure 2.1: Packet exchange between transmitter and the receiver in the basic access scheme.....    | 8  |
| Figure 2.2: Hidden Terminal problem example.....  | 9  |
| Figure 2.3: Packet Exchange between the transmitter and the receiver in the RTS/CTS scheme.         | 10 |
| Figure 2.4: RTS/CTS mechanism mitigating the hidden terminal problem. ....                          | 11 |
| Figure 2.5: Markov chain (of an $i$ th group station) is constructed.....                           | 14 |
| Figure 3.1: The Basic Service Set system.....   | 19 |
| Figure 3.2: The renewal period of multi rate 802.11 MUD MAC protocol .....                          | 23 |
| Figure 4.1: Initial contention window sizes of group 3 and 4 are varied against throughput.....     | 32 |
| Figure 4.2: Network Throughput vs. $CW_{3,0}$ and $CW_{4,0}$ . ....                                 | 33 |
| Figure 4.3: Network Throughput vs. $M$ , the total number of stations. Only $M_4$ is increased..... | 34 |
| Figure 4.4: Network Throughput vs. $M$ , total number of stations and $CW_{4,0}$ . ....             | 36 |
| Figure 4.5: Network Throughput vs. Average length of frame. ....                                    | 37 |
| Figure 4.6: Network Throughput vs. $p_1$ . ....   | 39 |
| Figure 4.7: Throughput vs $p_1$ and $p_2$ . The plot is for 802.11-2 with only two groups. ....     | 40 |



## **PREFACE**

All praise to Almighty Allah who gave me courage, wisdom and ability to write this thesis. He always guided me in the difficult times and it is only due to Him what I am today.

I cannot thank much Professor Heung-No Lee under whose supervision, I have worked throughout my Masters Degree. He is an excellent human being who taught me how to investigate problems. It is he who taught me the essence of logic and the importance of communicating well what one is investigating to other people through effective writing. Professor Zhi-Hong Mao is another extraordinary human from whom I am honoured to learn during my stay at University of Pittsburgh. I am thankful for his contributions in the problem I investigated for my thesis.

I also thank Dr Marlin Mickle for serving on my defense committee and for giving me valuable inputs on the thesis write-up.

Cheng-Chun Chang, with whom I shared office ever since I joined University of Pittsburgh, is an excellent friend, guide and co-researcher. I had an opportunity of learning a lot from him both about research and about life in general. Other friends I made during from the University of Pittsburgh are Xaioyu, Sushaant, Hariram, Bobby, Ashish, Vikram and Mircea.

I thank my parents who not only supported my goals of getting graduate education, but also encouraged me to go for it. It is not only because of my interest but also due to tremendous

enthusiasm of my father that I will continue my journey of graduate education beyond MS. I thank Almighty Allah for making them my parents and thank them for the sacrifices that they have made to realize my educational dreams. I am in debt to my brother, sister, extended family and friends who have always given me support and much needed encouragement. May Allah bless all of them.

Mir Hamza Mahmood

October 2008

## 1.0 INTRODUCTION

### 1.1 MOTIVATION AND PROBLEM CONTEXT

The single packet reception constraint at the access node in conventional wireless multiple access networks can be relaxed using Multi User Detection (MUD) techniques which can decode information transmitted from multiple stations simultaneously. In the past, a MUD receiver [9] was viewed as an exclusive technique, perhaps deemed suitable only for high performance base stations in cellular networks, as high complexity operations were usually required for it. Recent advances in graph codes theory such as low-density parity-check codes and interleaved convolutional codes have made the MUD algorithms less computationally intensive via the use of turbo-iterative MUD algorithms, see [20][21][24][25]. This user differentiation via channel code alone has been proven to be more spectrum efficient than what the conventional approach of separated spreading and channel coding can provide [10].

The downside of such a MUD receiver is that the receiver complexity still increases, at least linearly to the number of users. Therefore, for an access node in a typical Wireless Local Area Network (WLAN) application, the number of users, say  $m$ , that a MUD receiver can detect simultaneously will highly likely be limited up to several signals at maximum. Let's call such an access network  $m$ -MUD enabled.

We are interested in examining the throughput behavior of a WLAN when the access node is  $m$ -MUD enabled. With employment of an  $m$ -MUD enabled access node, lesser collisions and an increased amount of effective information transmission are expected as more than one stations can communicate with the access node simultaneously [7]. With the employment of a centrally controlled MAC protocol, such as the Point Coordination Function in 802.11, polling of stations can be used and exactly  $m$  stations can be scheduled to access the channel simultaneously. This case is not of interests in this work.

We are rather concerned in analyzing the throughput behavior of a MAC when it is in distributive control mode, such as the Distributed Coordination Function (DCF) [3] in 802.11. In distributive control mode, it is not entirely clear how to encourage simultaneous transmission, and how the throughput would behave under  $m$ -MUD enabled access node because current MAC mechanisms are designed to discourage simultaneous transmissions. The fundamental access mechanism used in 802.11 MAC protocol, for example, is Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). Stations are not allowed to send frames whenever they sense signaling activity in the channel. On top of CSMA/CA, stations follow the Distributed Coordination Function (DCF) [3] with which they share the medium via randomized transmissions. Simultaneous transmissions from multiple stations to the access node is considered as collision and hence discouraged.

The discussion so far leads to the following questions which we intend to answer in this paper: Can the wide spread distributive MAC protocols like DCF in 802.11 be used to exploit the  $m$ -MUD capability? Would it be possible with simple modification? What are these modifications required? How does the throughput scale as  $m$  increases? Namely, how should the DCF be modified so that a right amount of simultaneous transmissions can be encouraged?

On the other aspect, modern 802.11 WLANs allow stations to use multiple data rates. Stations choose a data rate depending on underlying channel condition, a process called *link adaptation* [22][23]. As the Signal to Noise Ratio (SNR) available to a station increases, it uses higher data rates to transmit its frame, hence reaching closer to channel capacity. Link adaptation in WLAN changes the throughput behavior significantly [1][18]. Thus, it is meaningful to include link adaptation in our throughput analysis and see its impact on the throughput.

## 1.2 RELATED WORK

Significant amount of work exists in the literature, which attempts to analyze the network throughput of IEEE 802.11 using Markov chains. Bianchi in [10] and [3] modeled 802.11 DCF using Markov chains and evaluated the performance of WLANs using single transmission rate. Improvements were then presented by several researchers which attempt to include all the details of 802.11 DCF in the Markov chain model for performance analysis. It should be pointed out that while substantial research is conducted on throughput analysis assuming single data rate, not comparable attention is given to the scenario where stations are allowed to use different data transmission rates, a typical case in modern 802.11 WLANs.

More recently conducted research carried out in [1], [4], [5] and [17] accommodate multi rate transmission assumption. Yang et al. [1], assume exponential backoff procedure, as outlined in the IEEE 802.11 DCF. The model, based on improved versions of the one presented in [3], accommodates multi rate transmission. With employment of multi rate transmissions, “performance anomaly” problem was observed: low rate users hold the channel longer in time

and thus network throughput suffers. The authors have tried to address this problem by controlling access parameters of stations such as initial contention window size, the frame size, and maximum backoff stage. All these parameters are well defined in the IEEE 802.11 standard. This problem of performance anomaly was also observed in [17]. In [5], the authors analyze multi rate 802.11 WLANs, where they assume two Markov models for stations and channels. However, the remedial solution for the performance anomaly was not addressed.

The authors in [2] attempted to apply a MUD capable access node in IEEE 802.11 WLANs and proposed a modified MAC. They assumed a simple scenario that all nodes have the same SNR and use geometrically distributed random backoff interval as compared to the exponential backoff in DCF. In [6] [7], the authors attempted to implement Multiple Packet Reception (MPR) in 802.11 WLANs using DCF. CSMA was modified for MPR scenario in [18] and a modified cross layer CSMA for MPR, named XL-CSMA was proposed. Although decentralized, it did not assume DCF mechanism for backoff. *We point out that all these prior works with 802.11 and MUD mentioned here assume single transmission rate for throughput analysis.*

All the works compared so far start with the assumption of the classic CSMA/CA framework where simultaneous transmissions are discouraged. Among the pioneering works on Multi Packet Reception (MPR) in random access networks, the authors of the paper [12] studied Slotted ALOHA (SA) systems under infinite number of users and single buffer assumption. Stations under SA transmit at the start of each frame whenever they have a frame to send. Thus, simultaneous transmission is easier than CSMA/CA to be encouraged via manipulating the transmission probabilities. However, there are downsides as well which make SA less attractive than 802.11. The throughput of SA is poor. For example, the throughput is only  $e^{-1}$  (for single

user detection receivers); the throughput vanishes as the offered load increases. SA is known to have instability problem as the offered load increases requiring a separate remedial treatment [12]. Furthermore, most of the existing MPR MAC protocols [12],[13], assume the existence of a central coordinator that schedules transmissions from stations. Hence, these prior works are not applicable to our 802.11-based distributed approach here.

### **1.3 CONTRIBUTION AND THESIS STRUCTURE**

The contributions of this work are as follows: First, we derive the throughput expression for the multi rate 802.11 MAC protocol with the support of MUD capable access node. To the best of our knowledge, this is novel (see our comparative literature survey in the next section.). Second, we show how this analytic throughput expression can be used in optimal control of 802.11 DCF.

The rest of the thesis is organized as follows. In chapter 2, we provide an introduction to MAC layer protocol of conventional 802.11 WLAN. The hidden terminal problem and its remedy is discussed. The Distributed Coordination Function (DCF) is discussed and mathematically modeled. In chapter 3, the proposed system with MUD support is discussed in detail and its performance is analyzed. Numerical results are presented in chapter 4 including discussion of tuning the current MAC protocols to maximally exploit the MUD capability and enhance system throughput, following which we conclude in chapter 5.

## 2.0 CONVENTIONAL 802.11 WIRELESS LOCAL AREA NETWORKS

In this chapter, we provide an introduction to the MAC layer protocols used in conventional 802.11 WLAN. We start with describing the transmission modes used in the protocol and collision avoidance mechanisms as provided in a typical system. The Distributed Coordination Function (DCF), which is used to manage the backoff process of the stations is described in detail and modeled as a Markov Chain in the last section of this chapter.

### 2.1 INTRODUCTION

The basic building block of a 802.11 WLAN is called Basic Service Set (BSS). It contains stations (one or more) who are contending for network resources in order to transmit their frames, and an Access Point (AP), through which the wireless stations connect and communicate with the larger network or the internet. This kind of configuration is called *infrastructure WLAN*. When wireless stations group together in the absence of an AP and no central control, an *ad hoc network* is said to be formed.



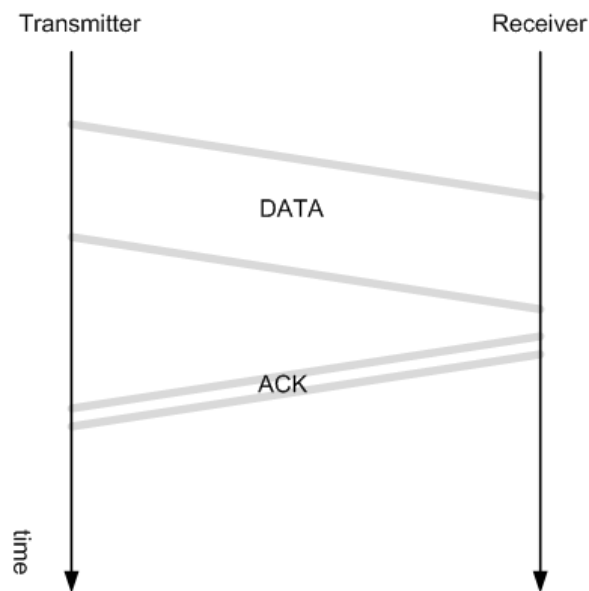
## 2.2 802.11 MAC PROTOCOL

In this section, we will briefly review the key features of Medium Access Control (MAC) Protocol that is deployed in 802.11 WLANs. As mentioned earlier, a station when associates itself to an AP, becomes part of the corresponding BSS. Then that station can communicate with other stations in the network through that AP. Since more than one station may attempt to communicate with the AP at the same time, we need a protocol to effectively manage these communications. Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) is the protocol used in WLANs under discussion to manage transmissions between the access point (AP) and stations. This protocol requires all stations to *sense* the channel before start transmitting their frames. The stations should avoid transmission if the channel is sensed busy. The collision avoidance part comes from the fact that before starting transmission, each station chooses to wait a random amount of time before it makes an attempt to transmit its frame. This random wait time is chosen using a number of ways. As stated earlier, this work concentrates on WLANs where such backoff process is DCF, as used in 802.11. This backoff process is implemented using Distributed Coordination Function (DCF), which is detailed in section 2.3. Next we discuss how the MAC protocol ensures that the frame is correctly received by the receiver.

### 2.2.1 Frame Reception Acknowledgement

When a station has a frame to transmit, it waits a random amount of time by going through the backoff process and then transmits its frame to the receiver, which is AP in our case. The receiver, upon successful reception of the frame, sends an ACKnowledgment (ACK) packet back to the source station if as shown in Figure 2.1. ACK is sent after waiting for a fixed period of

time, known as Short Interframe space (SIFS). The transmission of ACK by the receiver completes what we call one *renewal period*. Each station in the network who is contending for network resources will then wait for a fixed period of time, called DIFS, before making an attempt to transmit their data again.

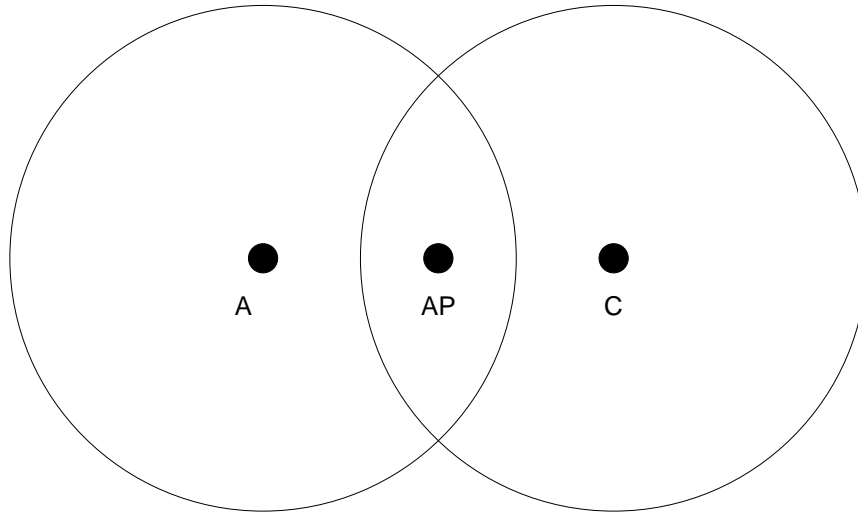


**Figure 2.1:** Packet exchange between transmitter and the receiver in the basic access scheme.

ACK packet is an indication to the source station that its data has been successfully transmitted. It will enter the backoff process again to retransmit the frame if it does not receive the ACK from the receiver.

### **2.2.2 The Hidden Terminal Problem and RTS/CTS handshake**

The MAC protocol provides a mechanism to mitigate the negative effects of the classic hidden terminal problem. In this subsection, we will first describe the hidden terminal problem and then the mechanism that 802.11 MAC protocol proposes to minimize its effects, using RTS/CTS handshake, will be elaborated. Figure 2.3 shows a hidden terminal example.

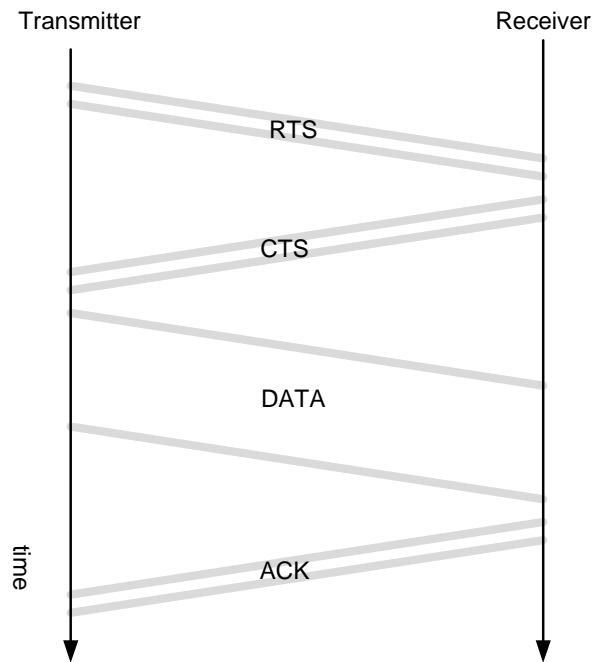


**Figure 2.2:** Hidden Terminal problem example.

As shown in Figure 2.3, suppose station A starts to transmit its frame to the AP to which another station, station C, is also associated. Since CSMA/CA is used, if C has a frame to transmit, it will sense the channel to see if any other station is talking to AP. We note that C is far from A and does not fall in its *coverage area*, it will not be able to sense A's transmission. Hence C will attempt to send its frame to the AP assuming no other station is talking to AP at this point in time and a collision will occur at the AP. We say that both A and C are *hidden* to each other.

802.11 use two modes for data transmission, *Basic Access* and *RC* mode. Basic access mode is depicted in Figure 2.1 and discussed in section 2.2.1 where a DATA frame is followed by and ACK. In such scheme, a situation as shown in Figure 2.2 and described in the preceding paragraph results in a collision. The *RC* mode, on the other hand, is described in Figure 2.3. In this scheme, two additional control packets, Request To Send (RTS) and Clear to Send (CTS) are used.

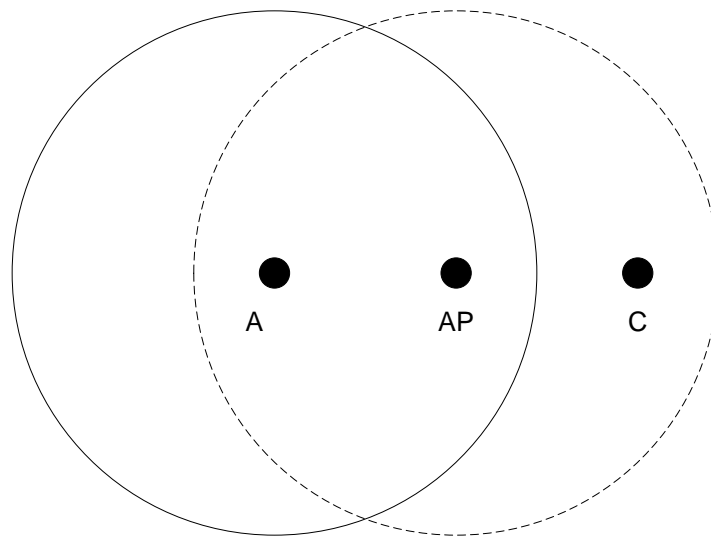
When a station has a frame to transmit, it enters into the backoff process, as was the case in Basic Access mode. After its backoff process ends, the station sends an RTS to the AP. This packet contains information such as length of the data packet the station intends to transmit. After successful reception and decoding of the RTS packet, the AP will transmit a CTS packet back to the sender. This CTS packet is received by all the stations that are in the coverage area of the AP. The sender then transmits its data packet to the AP, who in return ACK if the data is received successfully. The wait time between RTS-CTS, CTS-DATA and DATA-ACK is equal to SIFS whereas all stations wait for time equal to DIFS when one data frame is transmitted. This completes the renewal period when the network is working in RC mode.



**Figure 2.3:** Packet Exchange between the transmitter and the receiver in the RTS/CTS scheme.

Figure 2.4 shows how the RTS/CTS handshake is effective in making the hidden terminal(s) realize about current transmission even if they are not in the coverage area of the

source station. Here C is hidden to A, but when AP transmits the CTS packet, it is also received by C since it falls in its coverage area (shown by dotted circle). The length of the data frame to be sent by A is contained in CTS packet. When the packet is decoded by C, it will refrain from starting its transmission for the time that will be taken by the data packet of A and the corresponding ACK.



**Figure 2.4:** RTS/CTS mechanism mitigating the hidden terminal problem.

Although there is still a chance of collision at the level of RTS, considering a situation that both A and C transmit their RTS packets simultaneously, we note that the RTS packets are much smaller in length than Data packets and hence the time wasted due to collision will be much smaller in length. So RTS/CTS mechanism mitigates the hidden terminal effect to a large extend.

## 2.3 MODELING THE BACK-OFF MECHANISM IN DCF

In this section, we use a discrete-time Markov chain model to describe the exponential back-off process defined in the Distributed Coordination Function (DCF) mode of operation of IEEE 802.11. Yang et al in [1], an extended work of [3] with the inclusion of link rate adaptation in the analysis, also used a discrete-time Markov chain. Pictorial view of DCF as Markov Chain is shown in Figure 2.5.

We start by describing the DCF in detail as outlined in the 802.11 standard. The details are used to construct a Markov chain which is used to derive its steady state probability distribution. Steady state distribution is then used to describe transmission probabilities which are employed in subsequent throughput analysis carried out in the next chapter.

### 2.3.1 The Distributed Coordination Function (DCF)

Under the DCF, a station senses channel before transmitting. If it is idle, it just starts transmission (after waiting an inter frame space (IFS) and finding the channel idle again). If it is sensed busy, the station defers its transmission for a time equal to DCF Inter frame space (DIFS). After the DIFS, the station chooses a random back-off counter and takes an additional deferral period before transmitting.

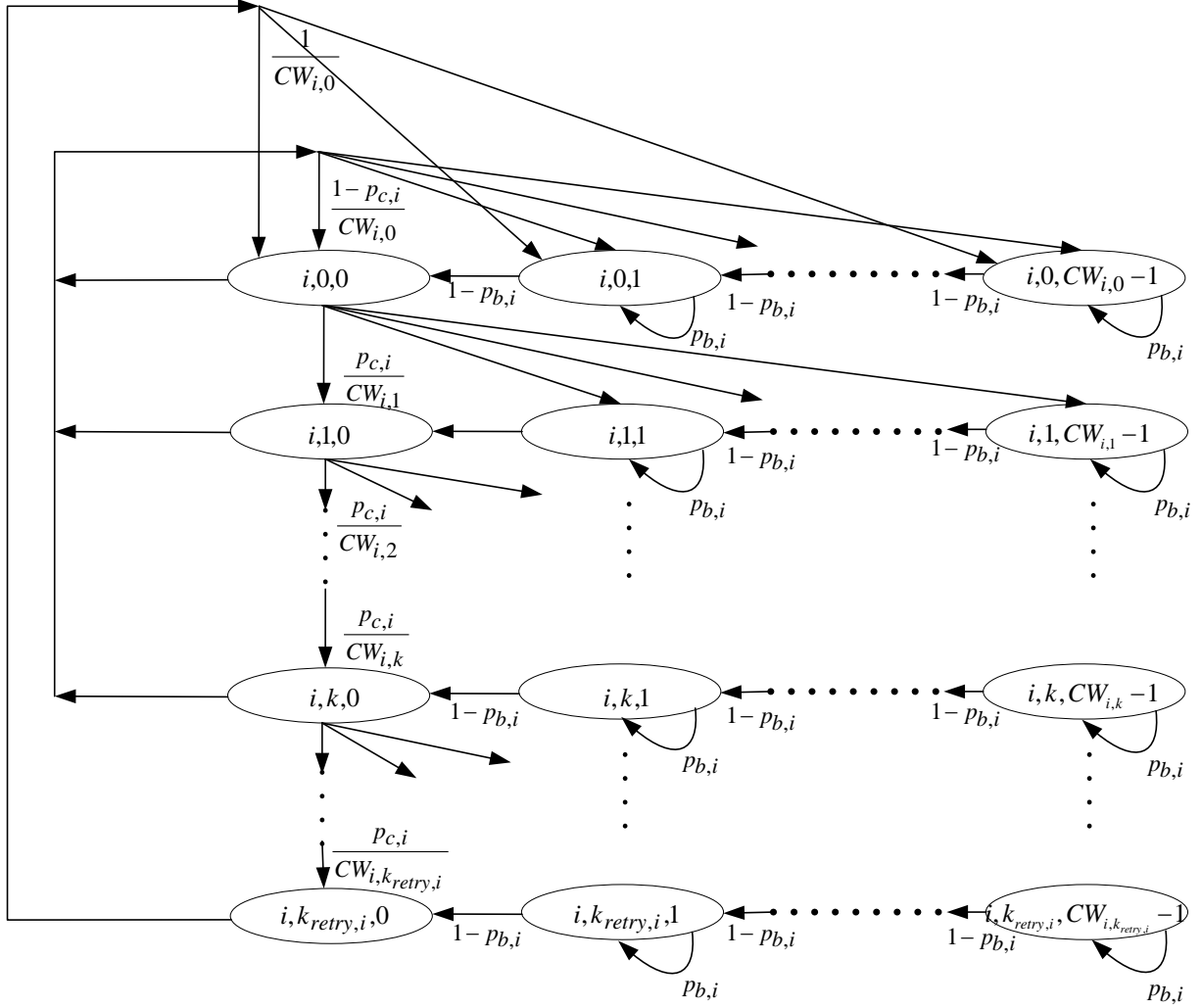
The value of the backoff counter is chosen randomly from a uniform distribution over the interval  $[0, CW_{i,0} - 1]$ , where  $CW_{i,k}$  is defined as the size of the contention window (CW) of a station in the  $i$ th group in the  $k$ th stage. The stage index starts at zero and is incremented by 1 each time collision occurs. Hence  $CW_{i,0}$  is the size of initial contention window. Whenever a

timeslot is sensed idle, the backoff counter is decremented. If the slot is sensed busy at anytime, the process is suspended, and starts again when an idle slot is encountered. When the counter reaches zero, the frame is transmitted.

When collision occurs, those stations involved in the collision enter into their own back-off process. For an example of the 1<sup>st</sup> collision, the stage index is incremented by one, i.e.,  $k = 1$ . In addition, the size of the contention window is increased by

$$CW_{i,k} = \begin{cases} 2^k CW_{i,0} & 0 < k \leq k_{\max,i} - 1 \\ 2^{k_{\max,i}} CW_{i,0} & k_{\max,i} \leq k \leq k_{\text{retry},i} \end{cases} \quad (2.1)$$

With each collision, the size of CW is increased until it reaches up to its maximum, i.e.,  $CW_{\max,i} = 2^{k_{\max,i}} CW_{i,0}$  after which it remains the same. The frame is dropped after  $k_{\text{retry},i}$  attempts.



**Figure 2.5:** Markov chain (of an  $i$ th group station) is constructed according to the IEEE Distributed Coordination Function for the multi rate 802.11 MUD MAC protocol.

Now, we define  $X(i, t) = \{K(i, t), L(i, t)\}$  as a discrete-time Markov Chain. We use the same assumptions made in [1] that the conditional busy probability  $p_{b,i}$ , that the channel is sensed busy by an  $i$ th group station, and the conditional collision probability  $p_{c,i}$ , that the transmitted frame of  $i$ th group station collides with any other station, are assumed independent of the backoff mechanism. Here,  $K(i, t)$  and  $L(i, t)$  are random processes representing the backoff stage of a station in the  $i$ th group and the size of the backoff counter of a station in the  $i$ th group



respectively. We define  $(i, k, l)$  as a state in the Markov chain which represents the  $i$ th group,  $k$ th stage and the value of random backoff counter  $l$ . Therefore, the ranges of these indexes are given by  $1 \leq i \leq N, 0 \leq k \leq k_{\text{retry},i}, 0 \leq l \leq CW_{i,k} - 1$ .

### 2.3.2 Transition Probabilities

For the description of the transition probabilities of the Markov chain, it is useful to see Figure 2.5 again. The state transition probability of the Markov chain  $X(i, t)$  defined by

$$P\{i, k_1, l_1 | i, k_0, l_0\} := P\{K(i, t) = k_1, L(i, t) = l_1 | K(i, t) = k_0, L(i, t) = l_0\}, \quad (2.2)$$

can be obtained from *the busy and collision probabilities*, in the following way:

$$P\{i, k, l-1 | i, k, l\} = 1 - p_{b,i} \quad \text{for } 1 \leq l \leq CW_{i,k} - 1 \quad (2.3)$$

$$P\{i, k, l | i, k, l\} = p_{b,i} \quad \text{for } 1 \leq l \leq CW_{i,k} \quad (2.4)$$

$$P\{i, k+1, l | i, k, 0\} = \frac{p_{c,i}}{CW_{i,k+1}} \quad \text{for } 0 \leq k \leq k_{\text{retry},i}, 0 \leq l \leq CW_{i,k+1} - 1 \quad (2.5)$$

$$P\{i, 0, l | i, k, 0\} = \frac{1 - p_{c,i}}{CW_{i,0}} \quad \text{for } 0 \leq k \leq k_{\text{retry},i}, 0 \leq l \leq CW_{i,0} \quad (2.6)$$

$$P\{i, 0, l | i, k_{\text{retry},i}, 0\} = \frac{1}{CW_{i,0}} \quad \text{for } 0 \leq l \leq CW_{i,0} \quad (2.7)$$

Eq. (2.3) shows that the back-off counter is reduced by 1 each time the channel is sensed idle, whose probability is  $1 - p_{b,i}$ . Eq. (2.4) shows that the chain halts reducing the back-off counter when the channel is sensed busy (happens with the busy probability  $p_{b,i}$ ). Eq. (2.5) shows that the stage index  $k$  of the chain is incremented by 1 when collision occurs. The size of contention window determines the range of the counter value. Since uniformly distributed, each

counter value  $l$  in the range  $0 \leq l \leq CW_{i,k+1} - 1$  is equally probable. Eq. (2.6) shows that the chain jumps downward from  $k$ th stage to 0-th stage with a successful transmission. Again a uniform randomly selected counter value from the range  $0 \leq l \leq CW_{i,0}$  will determine the next counter state  $l$ . Eq. (2.7) is for the event of frame drop. These equations are constructed to model the back-off process of 802.11.

### 2.3.3 Steady State Distribution and Transmission Probabilities

Let us now define a steady state distribution,  $s_{i,k,l}$ , of the Markov chain where

$s_{i,k,l} = \lim_{t \rightarrow \infty} P\{K(i,t) = k, L(i,t) = l\}$ . Using the definition of probability, we have

$$\sum_{k=0}^{k_{\text{retry},i}} \sum_{l=0}^{CW_{i,k}-1} s_{i,k,l} = 1 \quad (2.8)$$

Using the balance equations on the Markov chain, we obtain

$$s_{i,k,0} = p_{c,i}^k s_{i,0,0} \quad (2.9)$$

and

$$s_{i,k,l} = \frac{CW_{i,k} - l}{CW_{i,k}} \frac{1}{1 - p_{b,i}} s_{i,k,0} \quad \text{for } 0 \leq k \leq k_{\text{retry},i}, 0 \leq l \leq CW_{i,k} - 1. \quad (2.10)$$

Since a station transmits its frame when its backoff counter reaches zero, the probability  $p_i$  that a station in the  $i$ th group transmits its frame can then be calculated as the sum of the probabilities of such events, i.e.,

$$p_i = \sum_{k=0}^{k_{\text{retry},i}} s_{i,k,0} = \sum_{k=0}^{k_{\text{retry},i}} p_{c,i}^k s_{i,0,0} = \frac{1 - p_{c,i}^{k_{\text{retry},i}}}{1 - p_{c,i}} s_{i,0,0}. \quad (2.11)$$

Using (2.8), (2.10), and (2.9) in the respective order, we can find  $s_{i,0,0}$

$$\begin{aligned}
s_{i,0,0} &= \left[ \sum_{k=0}^{k_{\text{retry},i}} \sum_{l=0}^{CW_{i,k}-1} \frac{CW_{i,k}-l}{CW_{i,k}} \frac{1}{1-p_{b,i}} p_{c,i}^k \right]^{-1} \\
&= \left[ \sum_{k=0}^{k_{\text{retry},i}} \frac{p_{c,i}^k}{(1-p_{b,i})} \left( 1 + \frac{CW_{i,k}-1}{2} \right) \right]^{-1}.
\end{aligned} \tag{2.12}$$

Using (2.11) and (2.12),  $p_i$  can be obtained, i.e.

$$p_i = \frac{1-p_{c,i}^{k_{\text{retry},i}}}{1-p_{c,i}} \left[ \sum_{k=0}^{k_{\text{retry},i}} \frac{p_{c,i}^k}{(1-p_{b,i})} \left( 1 + \frac{CW_{i,k}-1}{2} \right) \right]^{-1} \text{ for } i = 1, 2, \dots, N. \tag{2.13}$$

*The transmission probabilities  $p_i$  prove to be the key statistics for further analysis. Since they dictate how much access any station from the  $i$ th group will have on the network resources, it directly influences the system throughput.*

### 3.0 ANALYSIS OF MULTI RATE WLAN WITH MUD CAPABILITY

In the last chapter, the DCF as used in the conventional 802.11 WLAN was analyzed and modeled as Markov chain. The conventional WLAN discourages simultaneous transmissions from multiple stations at the same time. In this chapter, we perform throughput analysis of our proposed system, which allows more than one stations to transmit their data simultaneously to the Access Point (AP). We use the DCF of the conventional system with slight modifications which is then able to accommodate our MUD assumption. In chapter 4, comparisons are made with the performance of conventional WLANs and modifications are discussed which can be employed to maximally exploit the MUD capability to enhance system throughput.

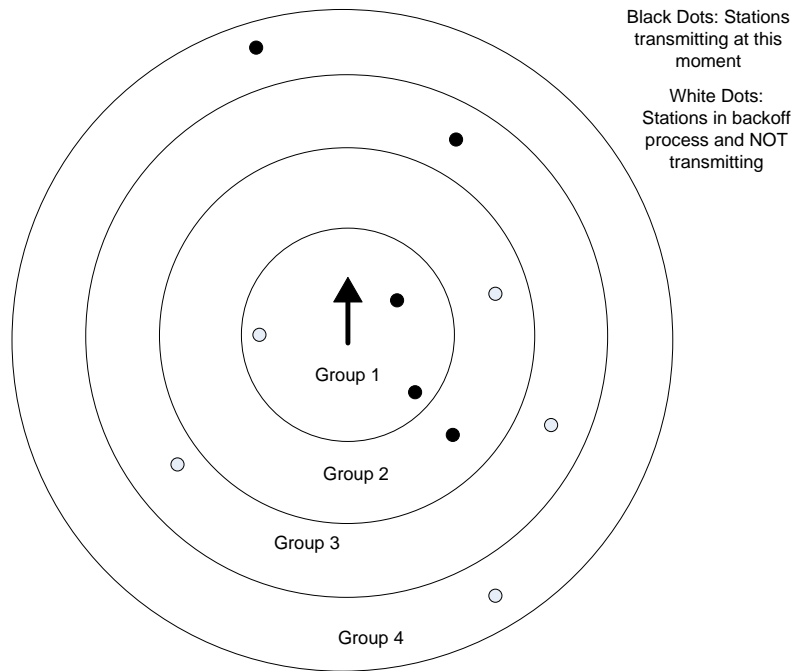
We use throughput of a network as a performance metric. In this chapter, the system model of the proposed WLAN is presented and a sample renewal period of such network is described. Using that renewal period and the result of last chapter, equation (2.13), which describes the transmission probability of a station belonging to the  $i$ th group, the throughput expression is derived.

## 3.1 SYSTEM AND PROBABILITY MODELING FOR 802.11-M MUD

### 3.1.1 System Model

Figure 3.1 depicts a Basic Service Set (BSS) system in consideration. All the stations in the BSS are divided into  $N$  groups; those belonging to each group transmit their frames at the data rate of  $r_i$ , for  $i = 1, 2, \dots, N$ . It is further assumed that there are a total of  $M$  stations in the network, each group having  $M_i$  number of stations having their frames ready to be transmitted, i.e.

$$M = \sum_{i=1}^N M_i .$$



**Figure 3.1:** The Basic Service Set system: There are four groups  $N = 4$ . There are total of 10 stations,  $M = 10$ . The number of stations in each group is  $M_1 = 3$ ,  $M_2 = 2$ ,  $M_3 = 3$ , and  $M_4 = 2$ . There are total five stations transmitting at the moment,  $n = 5$ . Among them two are in group 1,  $n_1 = 2$ , one in group 2,  $n_2 = 1$ , three in group 3  $n_3 = 1$ , and one in group 4,  $n_4 = 1$ .

Since we are assuming MUD, there are  $n_i$  stations from the  $i$ th group which start their transmission simultaneously at the start of the same timeslot. Hence, overall, there are  $n$  number of stations starting transmission simultaneously, where  $n = \sum_{i=1}^N n_i$ .

### 3.1.2 Busy and Collision Probabilities

As it was argued before, the Distributed Coordination Function (DCF), as used in the conventional system can be utilized even with our proposed physical layer modification, with minor changes. These changes are related with the way the collision and busy probabilities are defined. Hence we are going to rely on the analysis of DCF as done for conventional 802.11 and redefine the probabilities in order to accommodate the modifications.

Now we define  $P_n$  the probability that  $n$  number of stations start transmitting their frames simultaneously. Recall that  $n = \sum_{i=1}^N n_i$  where  $n_i$  is the random variable denoting the number of stations from the  $i$ th group making simultaneous transmissions in a time slot. Note that all  $n_i$  are mutually independent with each other.

To obtain  $P_n$ , we first determine the probability mass function (pmf) of the random variable  $n_i$ . We note that with  $p_i$  known, the pmf under discussion can be obtained as the binomial distribution:

$$P\{n_i = k_i\} = \binom{M_i}{k_i} p_i^{k_i} (1 - p_i)^{M_i - k_i}. \quad (3.1)$$

Since all  $n_i$  are independent of each other, the joint pmf can be written as:

$$P\{n_1 = k_1, \dots, n_N = k_N\} = \prod_{i=1}^N \binom{M_i}{k_i} p_i^{k_i} (1 - p_i)^{M_i - k_i} \quad (3.2)$$

Using (3.2),  $P_n$  can be found to be

$$P_n = \sum_{S_1} P\{n_1 = x_1, \dots, n_N = x_N\}, \quad (3.3)$$

$$\text{where } S_1 \triangleq \left\{ (x_1, \dots, x_N) : \sum_{i=1}^N x_i = n, 0 \leq x_i \leq M_i \right\}.$$

Given  $P_n$ , we can obtain the probabilities  $p_{b,i}$  and  $p_{c,i}$ . We note that an  $i$ th group station will sense the channel whether it is busy or not. Reserving one station from the  $i$ th group as the station sensing the channel, therefore, the conditional busy probability  $p_{b,i}$  is calculated. In the similar fashion, conditioning upon the event that an  $i$ th group station transmits its frame, the conditional collision probability  $p_{c,i}$  is calculated. Namely, assuming an  $m$ -MUD, a collision occurs if  $m$  or more *other* stations are also transmitting their frames in that time slot. For equations (3.4) and (3.5) where we define  $p_{b,i}$  and  $p_{c,i}$ , therefore, we will use  $P_n(M_i - 1)$  to imply the probability  $P_n$  in (3.3) obtained from excluding exactly one station from  $i$ th group; and then we have

$$p_{b,i} = \sum_{n=1}^{M_i-1} P_n(M_i - 1), \text{ for } i = 1, 2, \dots, N, \quad (3.4)$$

and

$$p_{c,i} = \sum_{n=m}^{M_i-1} P_n(M_i - 1), \text{ for } i = 1, 2, \dots, N. \quad (3.5)$$

Similarly, we can calculate the probability that a channel is busy,  $p_b$  and the probability that a channel is idle,  $p_{idle}$ .

$$p_b = \sum_{n=1}^M P_n \quad (3.6)$$

$$p_{idle} = 1 - p_b. \quad (3.7)$$

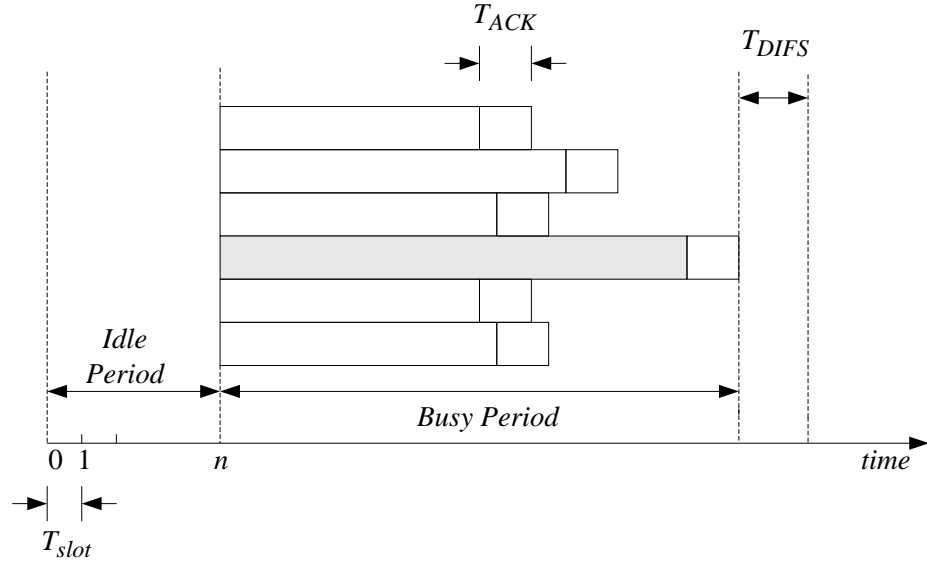
Given the set of network parameters, such as contention windows  $\{CW_{i,k}\}$ , number of stations in each group  $\{M_i\}$ , and the backoff stage limits  $\{k_{\max,i}, k_{\text{retry},i}\}$ , we notice that the transmission probabilities  $\{p_i\}$ , the busy probabilities and the collision probabilities are fixed. Thus, they are obtained from numerical evaluation of equations (2.13), (3.3), (3.4), and (3.5).

## 3.2 THROUGHPUT DERIVATION

### 3.2.1 Throughput definition

Given the transmission probabilities obtained in the previous section, we may carry our analysis with the renewal theorem. The Markov chain renews itself every time a successful transmission is made. Then, the average throughput can be obtained by analyzing a single renewal period as shown in Figure 3.2





**Figure 3.2:** The renewal period considered for the throughput analysis of multi rate 802.11 MUD MAC protocol

As described earlier, each station enters the exponential back-off process upon sensing the channel busy or experiencing a collision. There may be more than one stations that finish their backoff process earlier than others and hence start transmitting their frames simultaneously. Therefore the shared communication medium is idle at the start of each renewal period until the time a back-off process of any station(s) comes to an end. Those stations whose back-off counter reaches zero then start transmitting their frames simultaneously. Since CSMA/CA protocol requires any station to *sense* the medium before transmitting its frame, no station can start communication as long as the channel is occupied. It is clear from Figure 3.2 that once a busy period starts, the medium remains occupied until the transmission of the maximum time consuming frame finishes. The data transmitted in one busy period is taken to be the sum of effective data transmitted by all the stations. We further explain each of them, idle period, busy period, and equivalent successful data transmission in the subsequent pages.

We can now define the network throughput as follows:

$$\text{Throughput} = \frac{\bar{U}}{\bar{I} + \bar{B}} \quad (3.8)$$

where  $\bar{U}$  denotes the average amount of successful data transmission,  $\bar{I}$  the average idle period, and  $\bar{B}$  the average busy period.

### 3.2.2 Idle Period

The average idle period is defined as the time before any of the stations start their frame transmission. We obtained the probability when no station is transmitting its frame in a slot,  $p_{idle}$  in the last section. The average idle period (number of slots in fact) is calculated as the average of the geometric distribution, i.e.,

$$\bar{I} = \sum_{n=0}^{\infty} n p_{idle}^n (1 - p_{idle}) = \frac{p_{idle}}{1 - p_{idle}} \quad [\text{slots}]. \quad (3.9)$$

### 3.2.3 Busy Period

The busy period (in the unit of slots) is defined as the duration of the maximum time-consuming frame transmitted. Note that the frame may be generated from any station in any group. The frame length of any stations regardless of membership to a group is identically distributed, and it is geometrically distributed with parameter  $q$ . The probability that the packet length is  $l$  is given by

$$P\{L = l\} = q(1 - q)^{l-1} \quad \text{for } l = 1, 2, \dots \quad (3.10)$$

The distribution function is then given by

$$P\{L \leq l\} = 1 - (1 - q)^l \quad \text{for } l = 1, 2, \dots \quad (3.11)$$

Note that average frame length is  $1/q$  [bits].

We now move on to model the length of the maximum frame in the  $i$ th group, which is transmitted at the rate  $r_i$  bit-per-second [bps]. Note that with multi-rate link adaptation, the rate is different for each group  $i = 1, 2, \dots, N$ . After that, the maximum time consuming frame is modeled. Note that the maximum time consuming frame may be transmitted by any station from any group. Let's define the unit for frame lengths as the number of bits.

We define  $L_{\max,i}$  as the maximum size of a frame generated by a station in  $i$ th group, where  $L_{j,i}$  is defined as the frame length by  $j$ th station in the  $i$ th group. Since it is assumed that there are  $n_i$  stations transmitting their frames simultaneously from the  $i$ th group, the maximum length given  $n_i$  can be written as

$$L_{\max,i}(n_i) := \max\{L_{1,i}, L_{2,i}, L_{3,i}, \dots, L_{n_i,i}\}. \quad (3.12)$$

Then, we can find the distribution function for this random variable  $L_{\max,i}$ , given that there are  $n_i$  number of stations from the  $i$ th group which are transmitting their data. It is given by

$$\begin{aligned} P\{L_{\max,i}(n_i) \leq l\} &= P\{L_{1,i} \leq l, L_{2,i} \leq l, \dots, L_{n_i,i} \leq l\} \\ &= \prod_{j=1}^{n_i} P\{L_{j,i} \leq l\} \\ &= [1 - (1 - q)^l]^{n_i} \end{aligned} \quad (3.13)$$

The second line in (3.13) is due to the fact that the frame lengths of all the stations are mutually independent with each other, and the third line is from (3.11).

Let's measure time in the unit of the number of slots again. Without loss of generality, take the slot duration is equal to 1sec. We use (3.12) to get the distribution function of  $t_{\max,i}$ , the maximum time consumed by a frame transmitted by an  $i$ th group station, where  $L_{\max,i} = r_i t_{\max,i}$  [bits]:

$$\begin{aligned}
P\{t_{\max,i}(n_i) \leq \tau\} &= P\{L_{\max,i}(n_i) \leq r_i \tau\} \\
&= \left[1 - (1-q)^{r_i \tau}\right]^{n_i}
\end{aligned} \tag{3.14}$$

Similarly, since the lengths of frames of all the stations are independent of each other,  $t_{\max,i}$  are also independent of each other for all groups. Therefore, the maximum time taken by a frame transmitted by a station belonging to *any* group,  $t_{\max}$ , can be defined, given that there are a total of  $n_i$  stations belonging to the  $i$ th group transmitting their frames and the total number of stations transmitting is  $n = \sum_{i=1}^N n_i$ . The time  $t_{\max}$  can be defined as

$$t_{\max}(\mathbf{n}) = \max\{t_{\max,1}(n_1), t_{\max,2}(n_2), \dots, t_{\max,N}(n_N)\} \tag{3.15}$$

where  $\mathbf{n} := (n_1, n_2, \dots, n_N)$ . Now the distribution of  $t_{\max}$  can be found, i.e.,

$$\begin{aligned}
P\{t_{\max}(\mathbf{n}) \leq \tau\} &= P\{t_{\max,1}(n_1) \leq \tau, t_{\max,2}(n_2) \leq \tau, \dots, t_{\max,N}(n_N) \leq \tau\} \\
&= \prod_{i=1}^N \left[1 - (1-q)^{r_i \tau}\right]^{n_i}
\end{aligned} \tag{3.16}$$

With the above distribution function, we can calculate the expected value of  $t_{\max}$ , i.e.,

$$\begin{aligned}
E_{\tau}(t_{\max}(\mathbf{n})) &= \int_{\tau=0}^{\tau=\infty} P\{t_{\max}(\mathbf{n}) > \tau\} d\tau \\
&= \int_{\tau=0}^{\tau=\infty} \left[1 - \prod_{i=1}^N \left[1 - (1-q)^{r_i \tau}\right]^{n_i}\right] d\tau \\
&= \int_{\tau=0}^{\tau=\infty} \left[1 - \prod_{i=1}^N \sum_{x_i=0}^{n_i} \binom{n_i}{x_i} (-1-q)^{r_i \tau x_i}\right] d\tau \\
&= \int_{\tau=0}^{\tau=\infty} \left[-\prod_{i=1}^N \sum_{x_i=0}^{n_i} \binom{n_i}{x_i} (-1-q)^{r_i \tau x_i}\right] d\tau, \quad \text{for } \forall x_i \neq 0 \\
&= \int_{\tau=0}^{\tau=\infty} \left[-\sum_{x_1=1}^{n_1} \dots \sum_{x_N=1}^{n_N} \binom{n_1}{x_1} \dots \binom{n_N}{x_N} (1-q)^{(r_1 x_1 + \dots + r_N x_N) \tau}\right] d\tau, \quad \text{for } \forall x_i \neq 0
\end{aligned} \tag{3.17}$$

where the third line is due to the use of binomial expansion. Note that when  $x_1 = \dots = x_N = 0$ , the first term of binomial expansion is equal to 1. Excluding such a case, the last two steps are obtained. We note that (3.17) can be evaluated by the following

$$E_\tau \{t_{\max}(\mathbf{n})\} = \sum_{x_1}^{n_1} \dots \sum_{x_N}^{n_N} \frac{(-1)^\alpha \prod_{i=1}^N \binom{n_i}{x_i}}{\left( \sum_{i=1}^N x_i r_i \right) \ln(1-q)} \quad (3.18)$$

where  $\alpha \triangleq \sum_{i=1}^N x_i$ .

Averaging the expected value (31) over all possible vectors  $\mathbf{n}$  will give us the average busy period :

$$\bar{B} = \sum_{S_B} P\{n_1 = k_1, \dots, n_N = k_N\} E_\tau \{t_{\max}(k_1, k_2, \dots, k_N)\} \quad [\text{slots}] \quad (3.19)$$

where  $S_B \triangleq \{(k_1, \dots, k_N) : 0 \leq k_i \leq M_i\}$ . The protocol overheads such as  $T_{ACK}$  and  $T_{DIFS}$  in terms of number of slots can be added to the busy period.

### 3.2.4 Successful Effective Data Transmission

The effective data transmitted in one renewal period is defined as the averaged data being successfully served by the access node within a single renewal period.

For  $m$ -MUD, the access node is capable of detecting up to  $m$  frames simultaneously; a collision occurs if more than  $m$  stations simultaneously attempt to transmit their frames. Hence the effective data transmitted is zero. We define a random variable  $U$  to represent the good data successfully served. Recall that  $n_i$  is the total number of stations transmitting their frames

simultaneously from the  $i$ th group. The random variable is defined as a function of the vector

$\mathbf{n} := (n_1 n_2 \cdots n_N)$ , i.e.,

$$U \triangleq \begin{cases} \sum_{i=1}^N \sum_{x=1}^{n_i} L_{x,i} & \text{for } n = \sum_{i=1}^N n_i \leq m \\ 0 & \text{for } n = \sum_{i=1}^N n_i > m \end{cases} \quad (3.20)$$

Then the average effective data being transmitted,  $\bar{U}$ , can be found by averaging over vector  $\mathbf{n}$  as was done for the case for the average busy period analysis, as well as over the frame length. It is then given by

$$\bar{U} = \sum_{S_U} P\{n_1 = k_1, \dots, n_N = k_N\} \sum_{i=1}^N n_i \bar{L} \quad [\text{bits}] \quad (3.21)$$

where  $\bar{L} = 1/q$  is the average frame length [bits] and the set  $S_U$  is defined

as  $S_U \triangleq \{(k_1, \dots, k_N) : \sum_{i=1}^N k_i \leq m \text{ and } 0 \leq k_i \leq M_i\}$ .

### 3.2.5 Network Throughput

Finally, by substituting (3.21), (3.19), and (3.9) into (3.8), we obtain

$$\text{Throughput} = \frac{\frac{1}{q} \sum_{S_U} P(\mathbf{n}) \sum_{i=1}^N n_i}{\frac{P_{idle}}{(1 - P_{idle})} + \sum_{S_B} P(\mathbf{n}) E\{t_{\max}(\mathbf{n})\} + \sum_{S_U} P(\mathbf{n}) T_{ACK} + T_{DIFS}} \quad [\text{bits/slot}] \quad (3.22)$$

where  $S_U \triangleq \{(k_1, \dots, k_N) : \sum_{i=1}^N k_i \leq m \text{ and } 0 \leq k_i \leq M_i\}$  and  $S_B \triangleq \{(k_1, \dots, k_N) : 0 \leq k_i \leq M_i\}$ .

The protocol overhead should be given in the unit of the number of slots.

### 3.2.6 Discussion

Looking closely at equation (3.22), we observe that it is dependent on the average frame length, number of stations in each group and transmission probability of each group. We already know from equation (2.13) that transmission probabilities are determined by two parameters, initial contention window sizes and maximum retry limits of each group. Hence we see that throughput is dependent on some key transmission parameters which gives us a chance to tune them in order to study its behaviour.

In the next chapter, the numerical results based on the analytical model presented in this chapter will be discussed. The key parameters will be identified which largely dominate the network throughput and then its variations against those parameters will be studied. Identification of such parameters is important as it will lead us towards system performance optimization. The issue of fairness will also be briefly discussed when performance optimization is performed.

## 4.0 NUMERICAL RESULTS AND DISCUSSION ON 802.11-M WLAN

In previous chapter, we have obtained the throughput expression for the multi-rate 802.11  $m$ -MUD. Let's call 802.11- $m$  from now on. In this chapter, we use these expressions to evaluate and compare the performance of 802.11- $m$  ( $m \geq 2$ ) system with the conventional one, i.e., 802.11-1 ( $m = 1$ ). Our results prove that our analysis of 802.11- $m$  is generic enough to cater the conventional system as well. Throughput optimization is then discussed and it is shown that the performance of 802.11- $m$  network increases significantly with increasing  $m$ .

We assume that the stations are divided into four groups; each group uses a different data rate. Group 1 uses the highest available data rate while Group 4 employs the lowest. They are 11, 5.5, 3 and 1 Mbps respectively. The other parameters used for the generation of numerical results are listed in Table 1.

**Table 4.1:** System parameters used in numerical analysis

| Parameters           | Value     |
|----------------------|-----------|
| Slot Duration        | $10\mu s$ |
| DIFS Duration        | $20\mu s$ |
| SIFS Duration        | $5\mu s$  |
| ACK                  | 14 bytes  |
| Average frame length | 1000 bits |

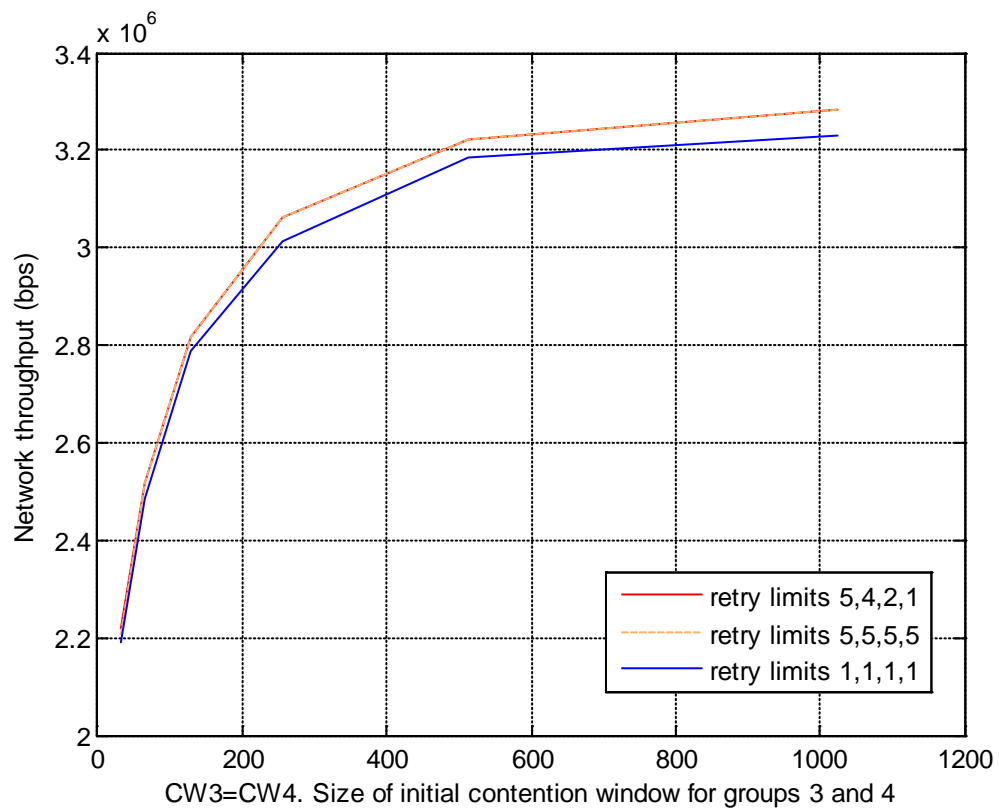


## 4.1 INFLUENCE OF INITIAL CONTENTION WINDOW SIZE

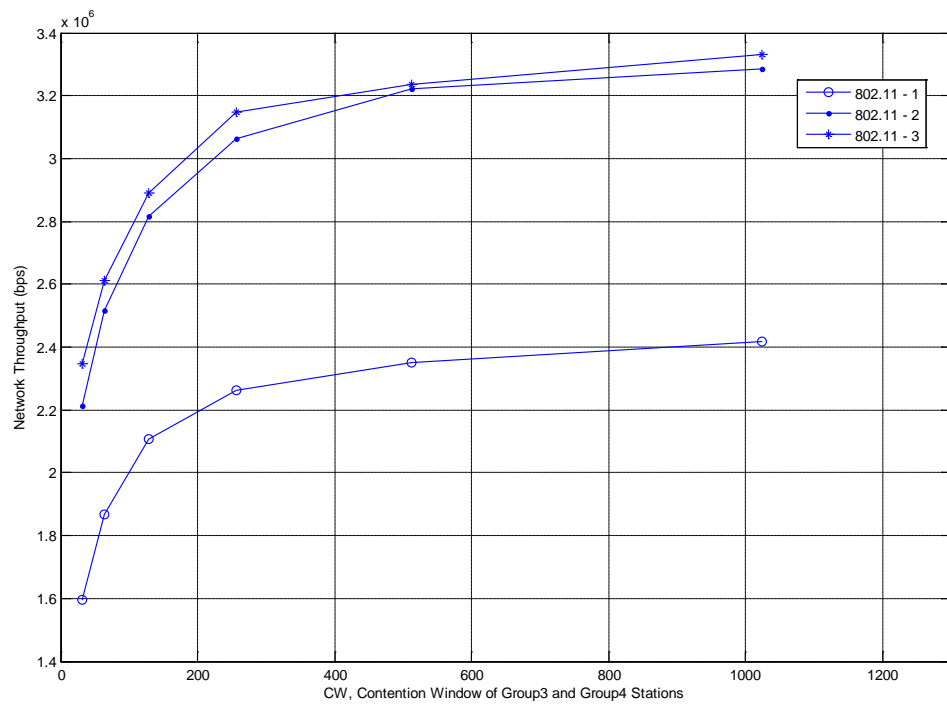
In this subsection, we study the effect of initial contention window  $CW_{i,0}$  on network throughput of our 802.11- $m$  system. The effect of maximum backoff stage,  $k_{\max,i}$  is negligible, a fact that is verified by our numerical computations as shown in Figure 4.1. This view is also shared by Yang et al [1] who have reported minimal impact of  $k_{\max,i}$  on network throughput of conventional 802.11.

Figure 4.2 aims to show how throughput varies as the sizes of initial contention windows of stations belonging to group 3 and group 4 are increased. The number of stations in each group we have used are  $M_1 = 3, M_2 = 2, M_3 = 3$  and  $M_4 = 2$  respectively. These values are also used for all other graphs except when they are varied in Figure 6 and 7. Initial contention window sizes of the faster rate groups are fixed at 32. Since these stations are using the lowest data rates, as the size of their initial CW increases, their chances to transmit frames--  $p_i$  the transmission probability of stations belong to group  $i$ --decrease. We note that the throughput - the sum of all throughputs of individual groups is largely dominated by faster rate group stations. Hence the throughput is expected to increase.

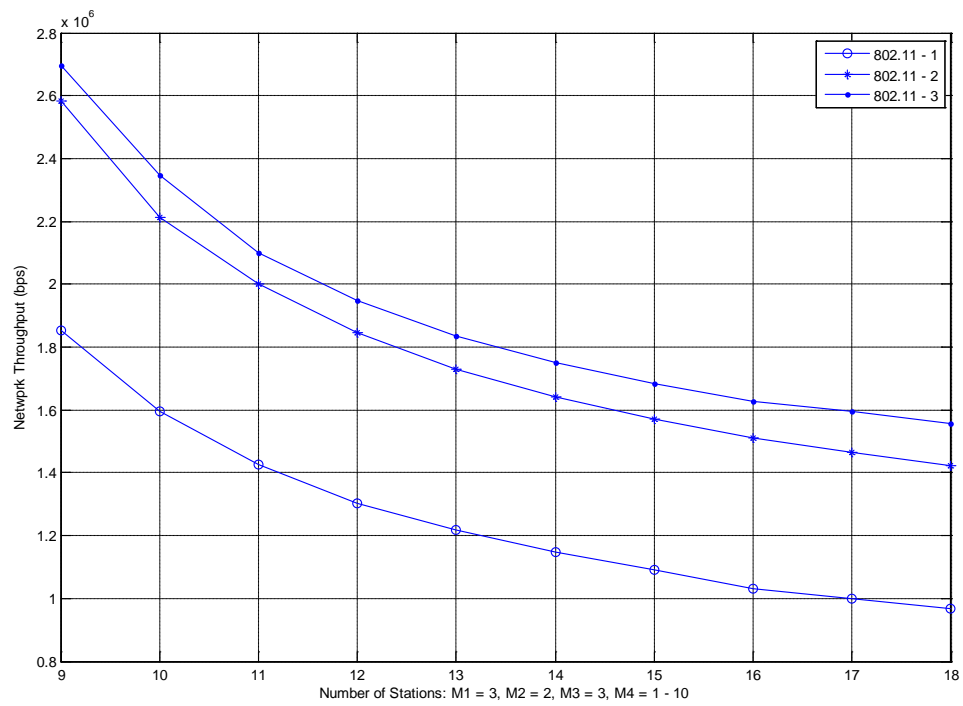
A considerable increase in throughput, about 32%, is shown for 802.11-2 in Figure 4.2, as compared to the conventional system; while that for 802.11-3 is only about 3.0%. Note that the throughput increase for  $m = 3$  is very small in this setting. We will talk about the issue of throughput maximization as  $m$  changes in the next subsection.



**Figure 4.1:** Initial contention window sizes of group 3 and 4 are varied against throughput. Graphs are obtained for different retry limits.



**Figure 4.2:** Network Throughput vs.  $CW_{3,0}$  and  $CW_{4,0}$  plotted for 802.11-1, 802.11-2 and 802.11-3 MAC.

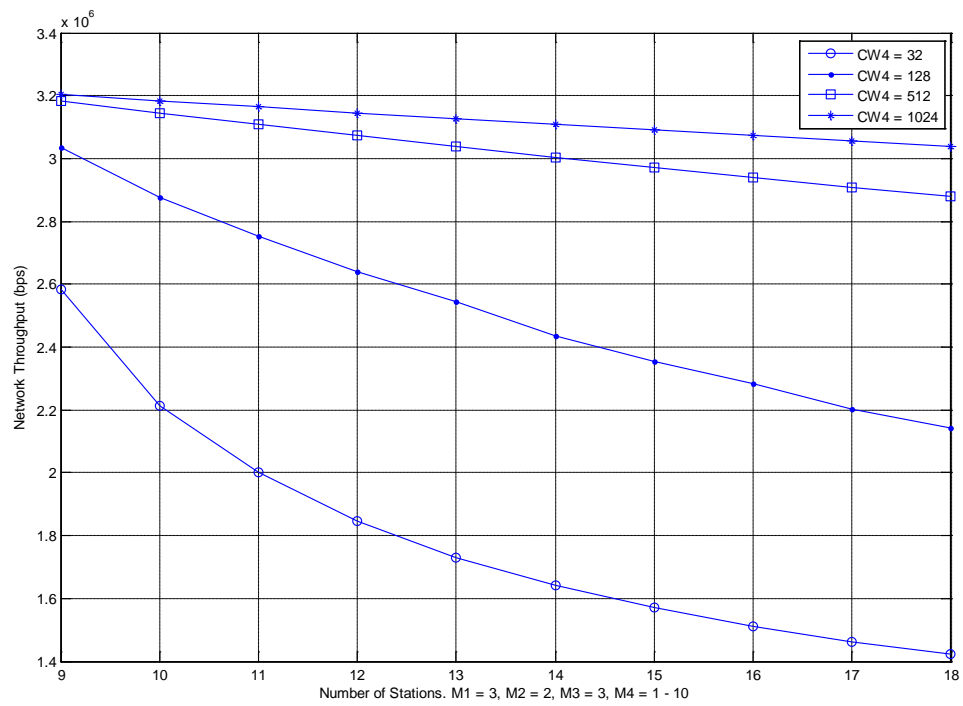


**Figure 4.3:** Network Throughput vs.  $M$ , the total number of stations. Only  $M_4$  is increased here. Graphs are plotted for 802.11-1, 802.11-2 and 802.11-3 MAC.

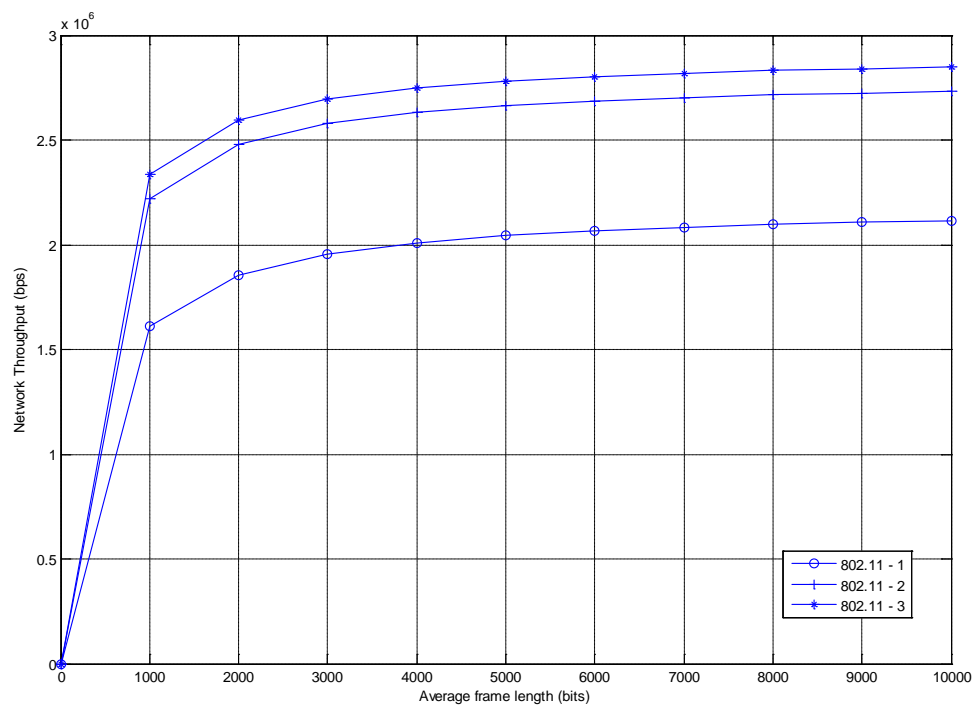
Next we analyze what happens when the number of stations of the lowest rate group  $M_4$  increases, from 1 to 10. As shown in Figure 4.3, the throughput now decreases for a fixed initial contention window size, set at 32 for all groups. This can be explained as follows: with the increase in  $M_4$ , the number of stations contending for network resources belonging to group 4 increases. Hence more and more stations using the lowest data rate get a chance to grab the channel and hold the channel longer. This will obviously reduce the throughput. Another factor which reduces the throughput is that as the total number of stations increase, the chance to collisions increase as well. Hence the effective amount of data successfully transferred will reduce.

However, by increasing the initial contention window size of group-4 stations, their probability of transmission,  $p_4$  can be made smaller. Hence the effect of increasing  $M_4$  can be mitigated. Figure 4.4 indicates this situation. As the contention window size is increased from 32 to 1024, we notice that, the throughput can hold out well.

The frame size is another important parameter which influences the throughput heavily. In the earlier section, we have modeled the frame size of each station as a geometric distribution with parameter  $q$ . We now show in Figure 4.5 how throughput varies as the average frame length, or  $1/q$  is increased. The protocol overheads, shown in Table I, will play bigger role when smaller average frame lengths are used. The throughput will thus increase as the average frame length is increased. Average frame length also needs to be carefully chosen as the trend shows saturation behavior after 6000 bits. Using a larger average frame length will cause larger busy period and hence stations need to wait longer for their turn to transmit, so the quality of service suffers.



**Figure 4.4:** Network Throughput vs.  $M$ , total number of stations. Only  $M_4$  is increased here. Graphs are plotted for four values of initial contention window of the fourth group, which are 32, 128, 512 and 1024.



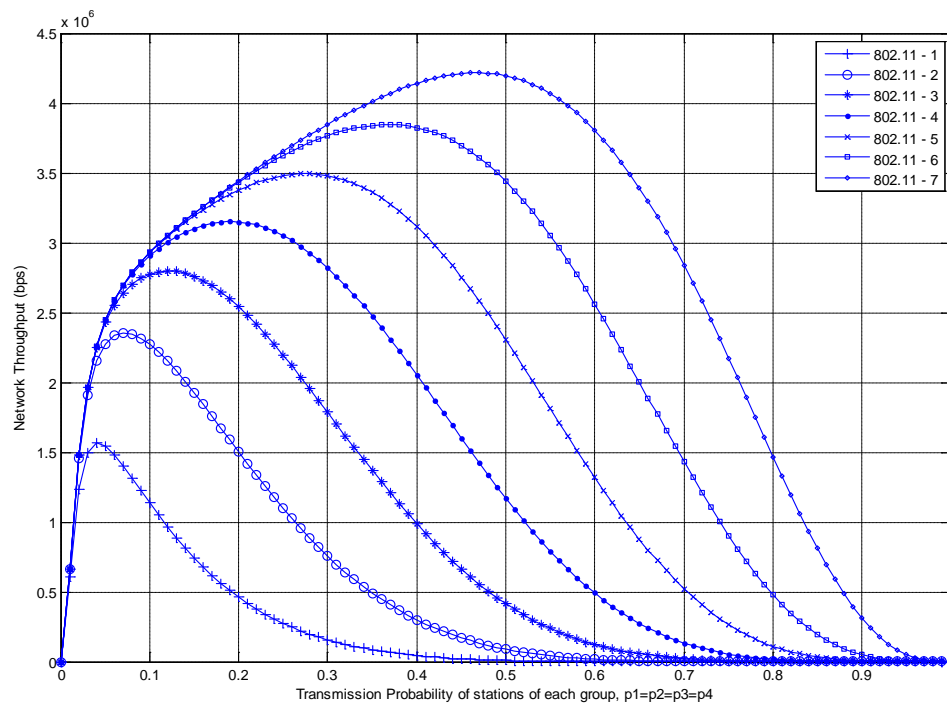
**Figure 4.5:** Network Throughput vs. Average length of frame. Graphs are plotted for 802.11-1, 802.11-2 and 802.11-3 MAC.

## 4.2 THROUGHPUT OPTIMIZATION VS. FAIRNESS

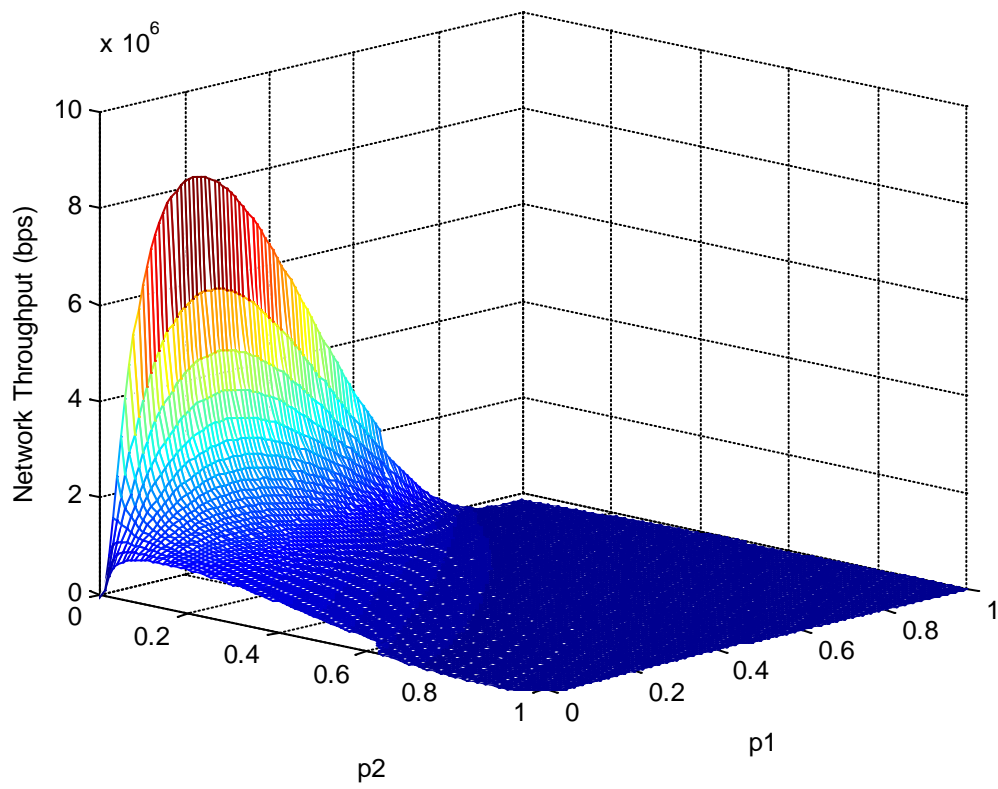
Now we study how throughput can be optimized for the multi rate 802.11- $m$  system. As we learned from earlier examples in this paper, the initial contention window size determines the transmission probability  $p_i$  and hence the throughput. One trivial optimization solution with maximum throughput can be obtained when casting all of the network resources to the fastest rate group stations. However, then the other lower rate group stations will not be able to transmit any frames. This can be observed in Figure 4.7. We consider a BSS consisting of only two groups and throughput is plotted as function of the transmission probabilities,  $p_1$  being the probability for the faster rate group. Note that the optimum point occurs when the transmission probability of the slower rate group set to zero, i.e.,  $p_2 = 0$ .

Therefore, the throughput maximization must be investigated with some form of fairness. Different criteria can be used for fairness. In this paper, we consider *proportional fairness* and see if non trivial optimal points are available. Under the *proportional fairness* criteria, the transmission probabilities of different groups are varied together in proportion. As we have done so far, let's divide the stations into 4 groups, and make  $p_i$  for each group equal to each other, i.e.,  $p_1 = p_2 = p_3 = p_4$ . Every group has equal chance of transmitting frames. We observe from Figure 4.6 that there exists optimal  $p_1$  for each  $m$  and substantial throughput enhancement can be made as  $m$  increases.





**Figure 4.6:** Network Throughput vs.  $p_1$ --the probability that station of group 1 transmits. Graphs are plotted for 802.11-1, 802.11-2, ..., 802.11-7 MAC.



**Figure 4.7:** Throughput vs.  $p_1$  and  $p_2$ . The plot is for 802.11-2 with only two groups.

Another point of interest that can be studied from Figure 4.6 is that as  $m$  increases, the optimal transmission probabilities also increase, hence giving the stations more chance to transmit their frames. This makes sense. Hence as  $m$  increases, simultaneous transmissions are encouraged by increasing their transmission probabilities (by decreasing initial contention window sizes).

### **4.3 MODIFICATOINS TO EXPLOIT FULLY THE MUD CAPABILITY**

Having established that the 802.11- $m$  system as discussed in this paper do provide significant throughput enhancement even in distributed MAC, we now discuss enabling system components by which this enhancement can be realized.

#### **4.3.1 Physical Layer Modifications**

At the physical layer, we want to use non spreading based techniques to perform MUD to improve on spectrum spectral efficiency. It has been shown that Low Density Parity Check (LDPC) codes can be used to identify and decode information from several sources transmitted simultaneously [19] [20] using message passing iterative decoders. As for the multi rate scenario, all the stations transmit their physical layer header using 1 Mbps. The signal field of header contains information about the rate at which the station is going to transmit [1]. After extracting the data rate information, the access node can start decoding at that data rate.

### 4.3.2 MAC Layer Modifications

At the MAC layer, all stations will use DCF with CSMA/CA to acquire network resources. It is possible that more than one stations finish their backoff process simultaneously. They will hence start transmission. A collision will occur in multi rate 802.11- $m$  system when more than  $m$  station finish their backoff process at the same time and start transmission. We are assuming no hidden terminals in this paper and all the stations start frame transmission at the start of a time slot. Thus, we are using basic access DCF without Request-to-Send and Clear-to-Send (RTS/CTS) procedure. The use of RTS/CTS will be useful for solving the hidden terminal problem. We plan to extend the current work to this situation in our future contribution.

Our work presented here show that the throughput can be optimized for a given network setting. For example, each station can get the estimate of number of stations contending for network resources, it can tune its initial contention window size to achieve the optimal throughput point as in Figure 4.6. After the optimal probabilities are obtained, the initial contention window sizes can be back calculated using (13). These optimum probabilities and initial contention window sizes can be pre-calculated for different network loads  $M$  and a loop-up table can be used. For example, considering 802.11-2 system, the optimal probabilities from Figure 4.6 can be observed as 0.07. This will result in initial contention window sizes of 13 if we set maximum backoff stage of all groups equal to 5. Similar exercise can be performed for other values of  $m$  to populate a look up table.

Therefore, our answers to the questions stated in the introduction of the paper have been clearly answered so far. Namely, the DCF in 802.11 can indeed be used to exploit the  $m$ -MUD capability. The required modification is very simple: use optimal initial set of contention windows obtained from the optimal transmission probabilities via a look-up table. The

throughput scales reasonably well as we have seen from Figure 4.6 worth a further study on this problem.

## 5.0 CONCLUSION

In this work, we have analyzed a proposed multi rate 802.11- $m$  MAC system where up to  $m$  stations can transmit their frames simultaneously using different data rates. We have obtained new analytic expression for the network throughput. The analysis result is general enough to subsume previous throughput expressions for 802.11 protocol. While the use of a MUD capable receiver at the physical layer has the potential to significantly improving performance, it is shown that controlling MAC parameters is very important to be able to actually achieve the benefit. The only goal should not be throughput maximization which will inadvertently ignore low rate group stations. We have shown that the proposed framework can tell us how the throughput can be maximized under the proportional fairness setting.

In the current work, we have assumed that every station can hear every other station, i.e. no hidden terminals. Possible research directions include conducting analysis with hidden terminals and then studying their effect on the performance. The RTS/CTS mechanism can then be included to investigate its effectiveness in combating hidden terminal interference in our proposed 802.11- $m$  system.

## APPENDIX A

### LIST OF KEY VARIABLES

$N$  : the total number of groups

$M_i$  : the number of  $i$ th group stations,  $i = 1, 2, \dots, N$

$M$  : the total number of stations, i.e.,  $M = \sum_{i=1}^N M_i$

$n_i$  : the random variable representing the number of stations from  $i$ th group starting transmission simultaneously

$n$  : a random variable representing the total number of stations starting transmission, i.e.,  $n = \sum_{i=1}^N n_i$

$CW_{i,0}$  : the size of the initial contention window of a station in  $i$ th group

$k_{retry,i}$  : the maximum number of retransmission attempts allowed for a station in  $i$ th group

$k_{max,i}$  : the number of retransmission attempts after which the size of contention windows will cease to increase

$CW_{i,k}$  : the size of the contention window of a station in  $i$ th group at the  $k$ th stage

$p_{b,i}$  : the probability that channel is sensed busy by an  $i$ th group station

$p_{c,i}$  : the probability that frame of an  $i$ th group station collides with frame of a station belonging to any other group

$\bar{I}$  : the average length of idle period (in slots)

$\bar{B}$  : the average length of busy period (in slots)

$\bar{U}$  : average successful effective data being transmitted (in bits)



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